Elastodynamic single-sided homogeneous Green's function representation:

Theory and examples

Christian Reinicke, Kees Wapenaar

Contents

1. Introduction
2. Homogeneous Green's function
3. Elastodynamic double-sided homogeneous Green's function representation
4. Elastodynamic single-sided homogeneous Green's function representation
5. Conclusions

TU Delft

WAVES

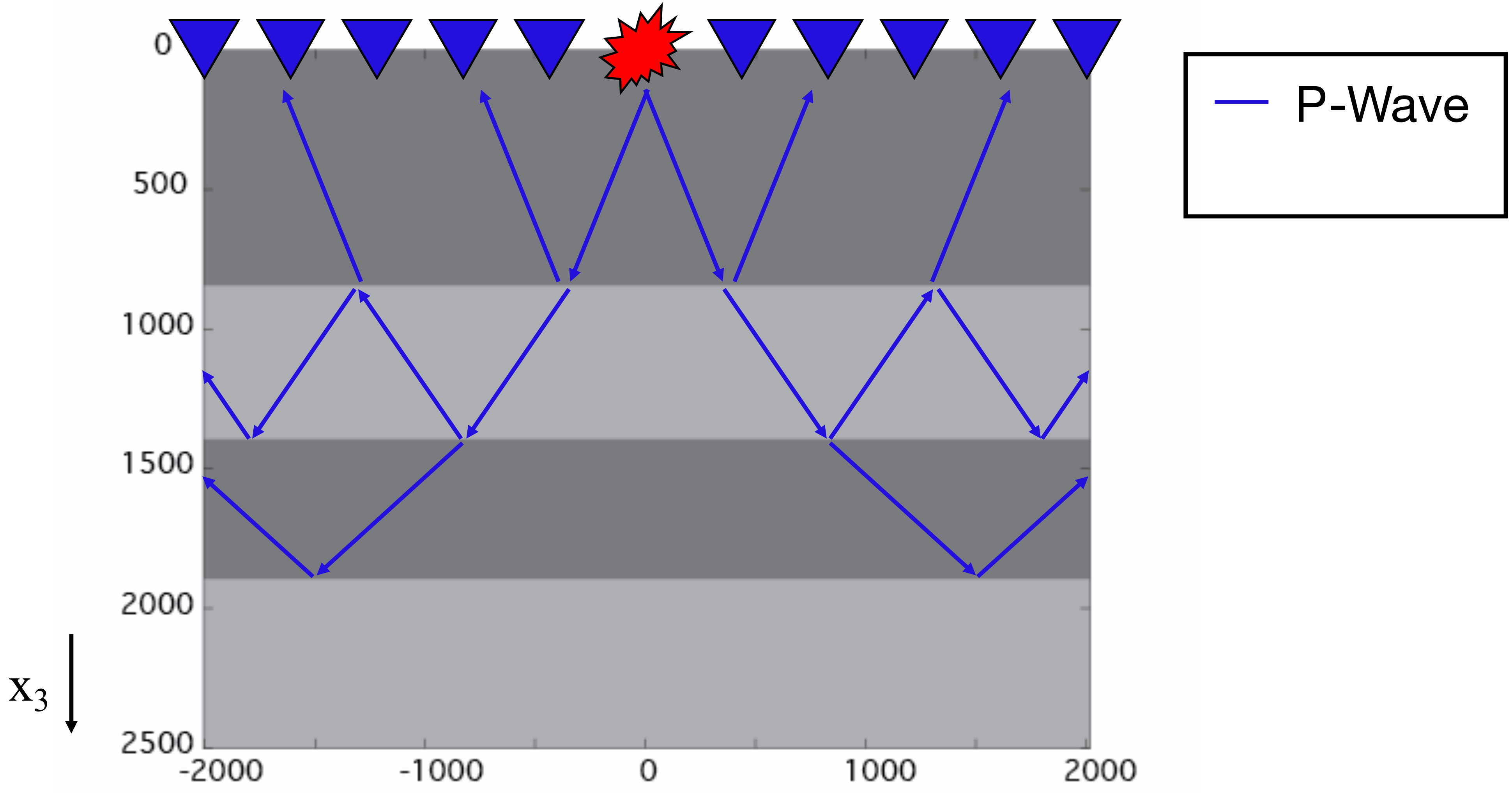
Contents

1. **Introduction**
2. Homogeneous Green's function
3. Elastodynamic double-sided homogeneous Green's function representation
4. Elastodynamic single-sided homogeneous Green's function representation
5. Conclusions

TUDelft

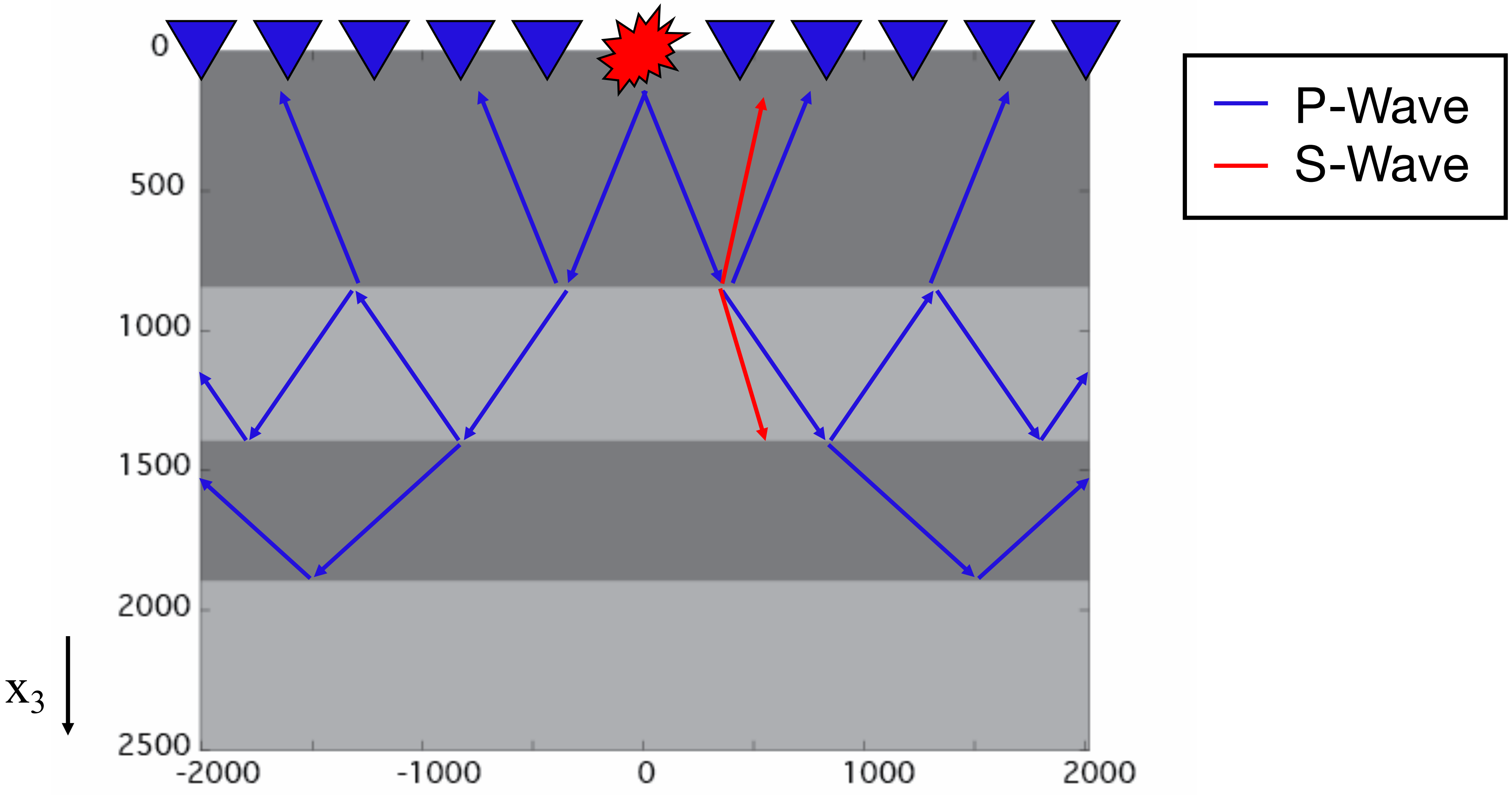
WAVES

Introduction



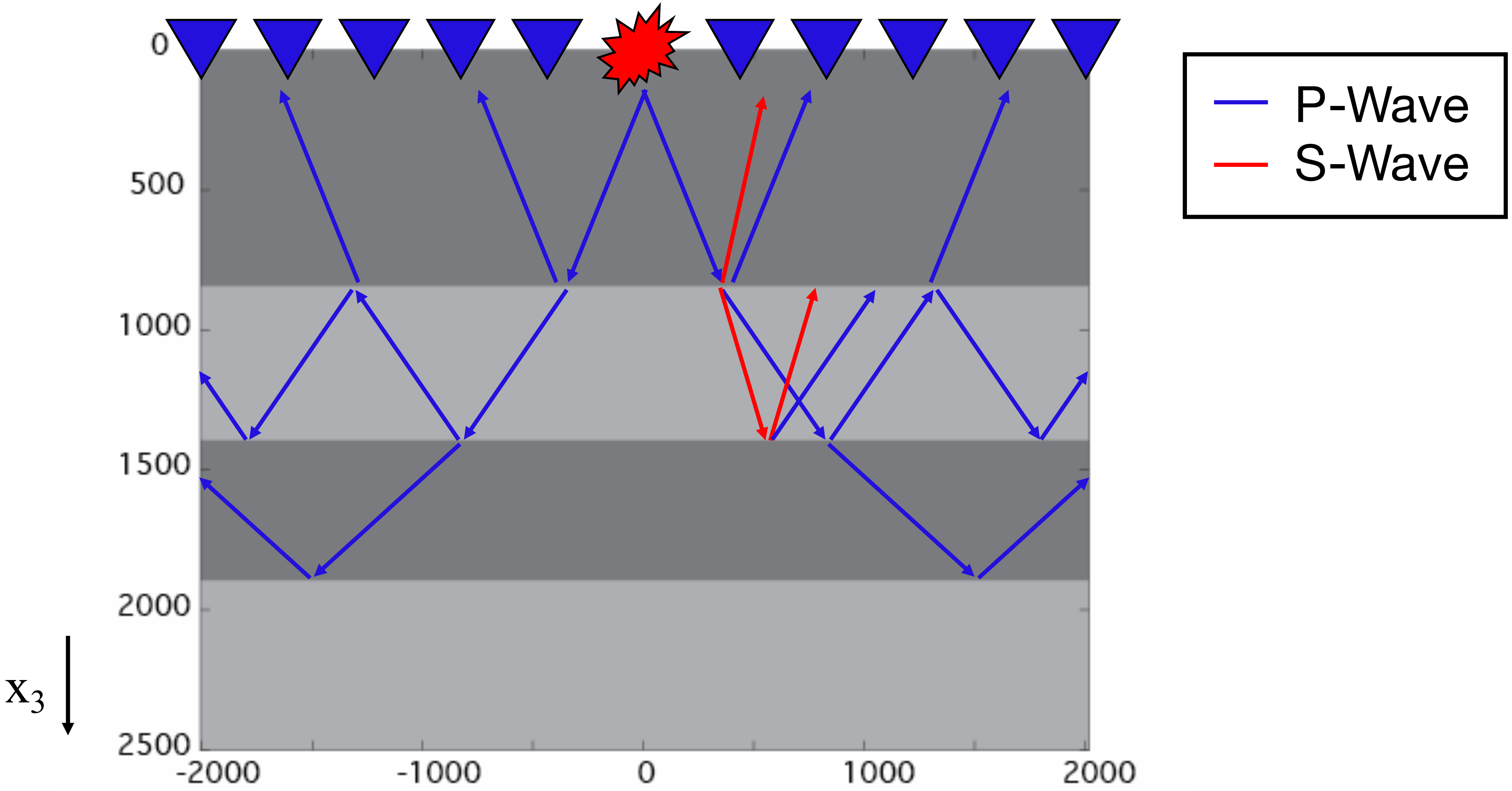
Reflection response

Introduction



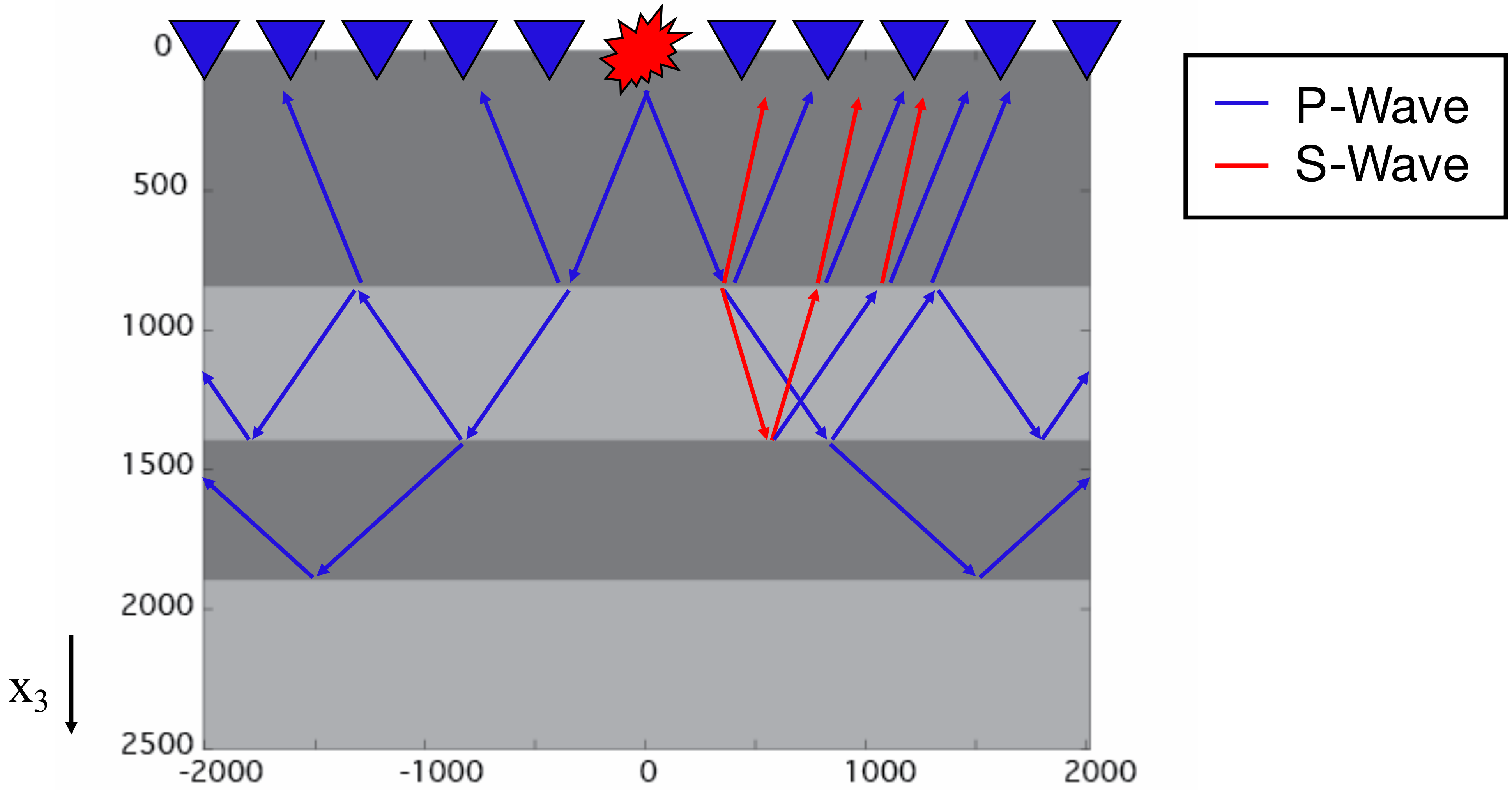
Reflection response

Introduction



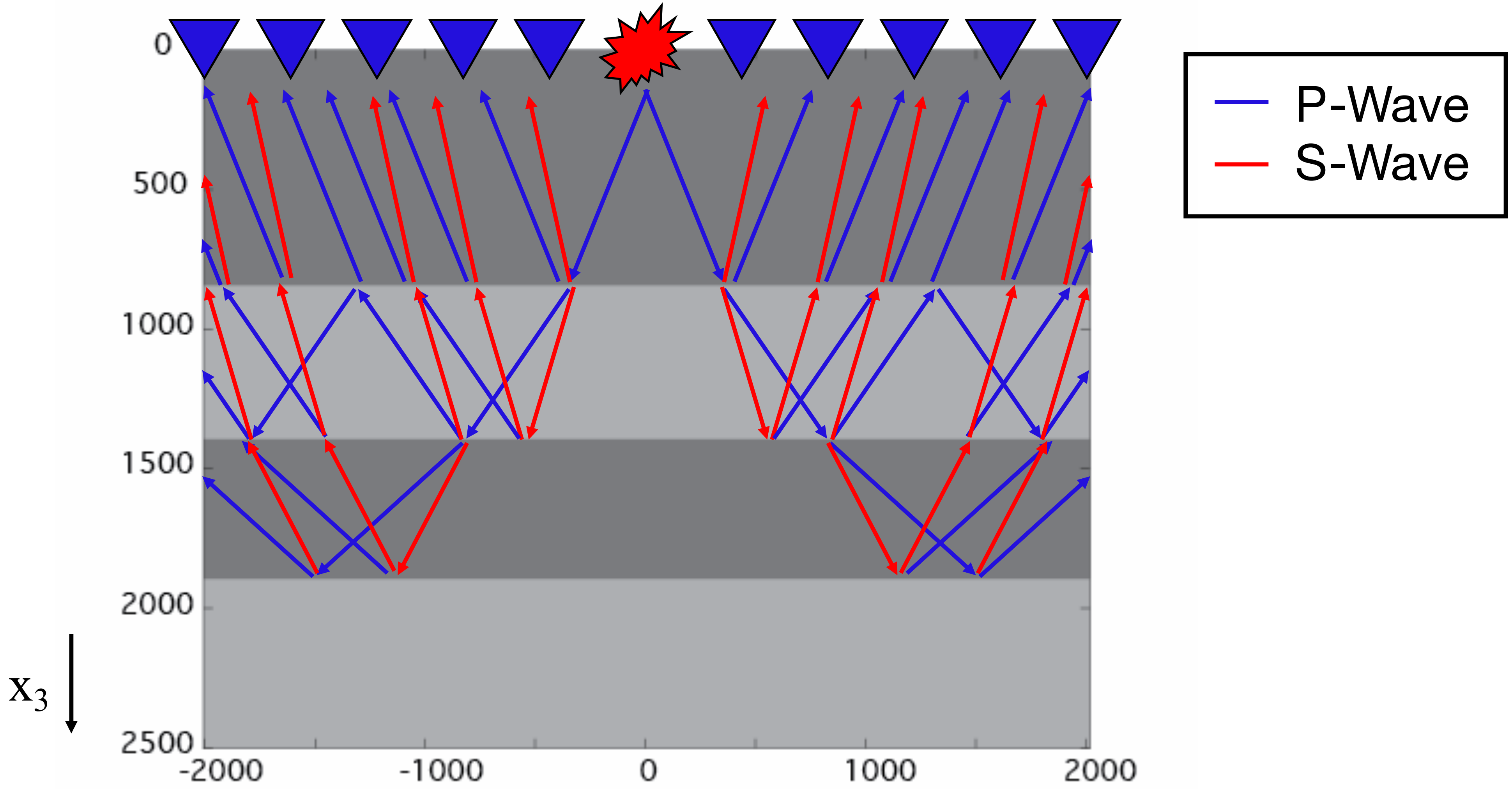
Reflection response

Introduction



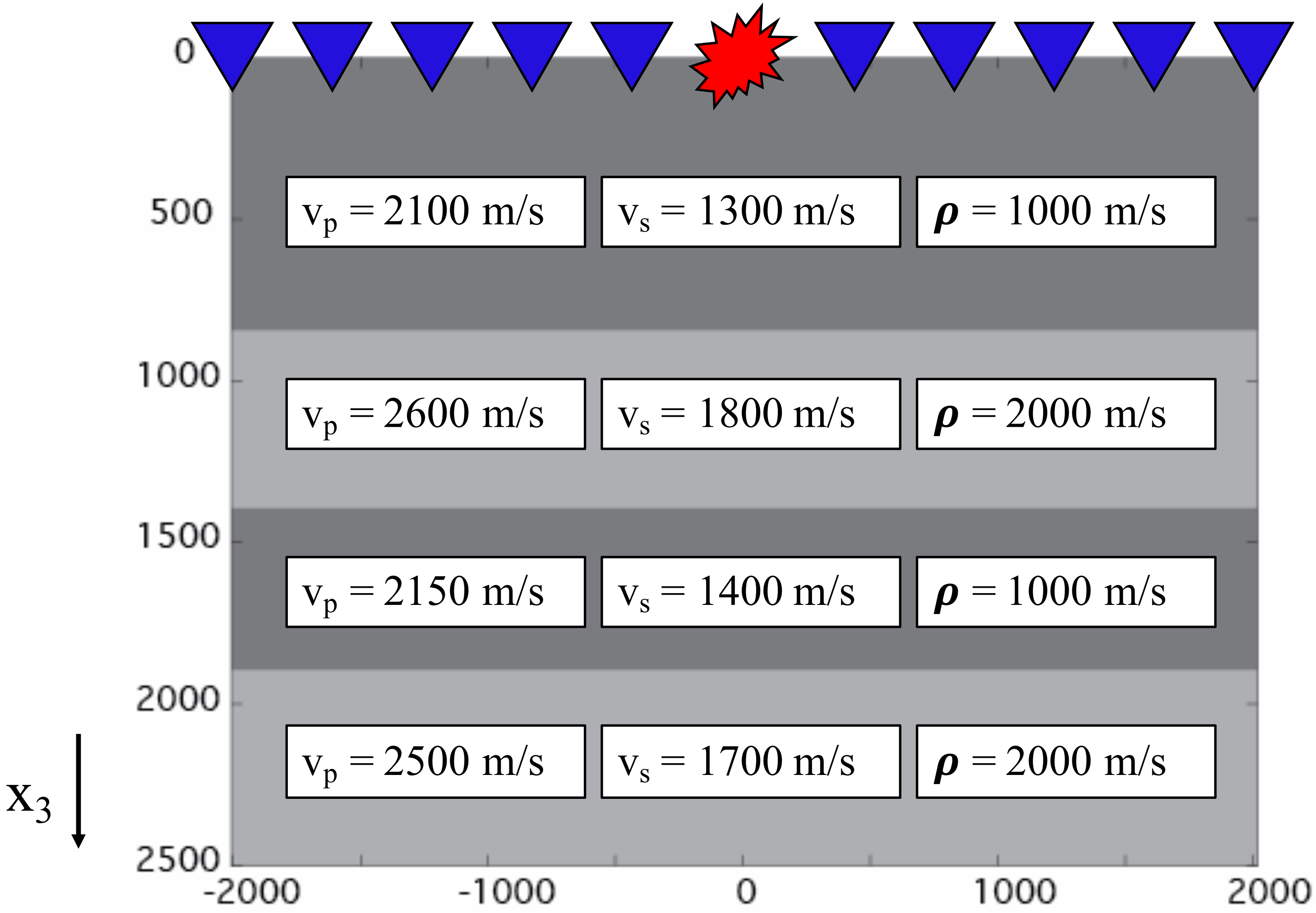
Reflection response

Introduction

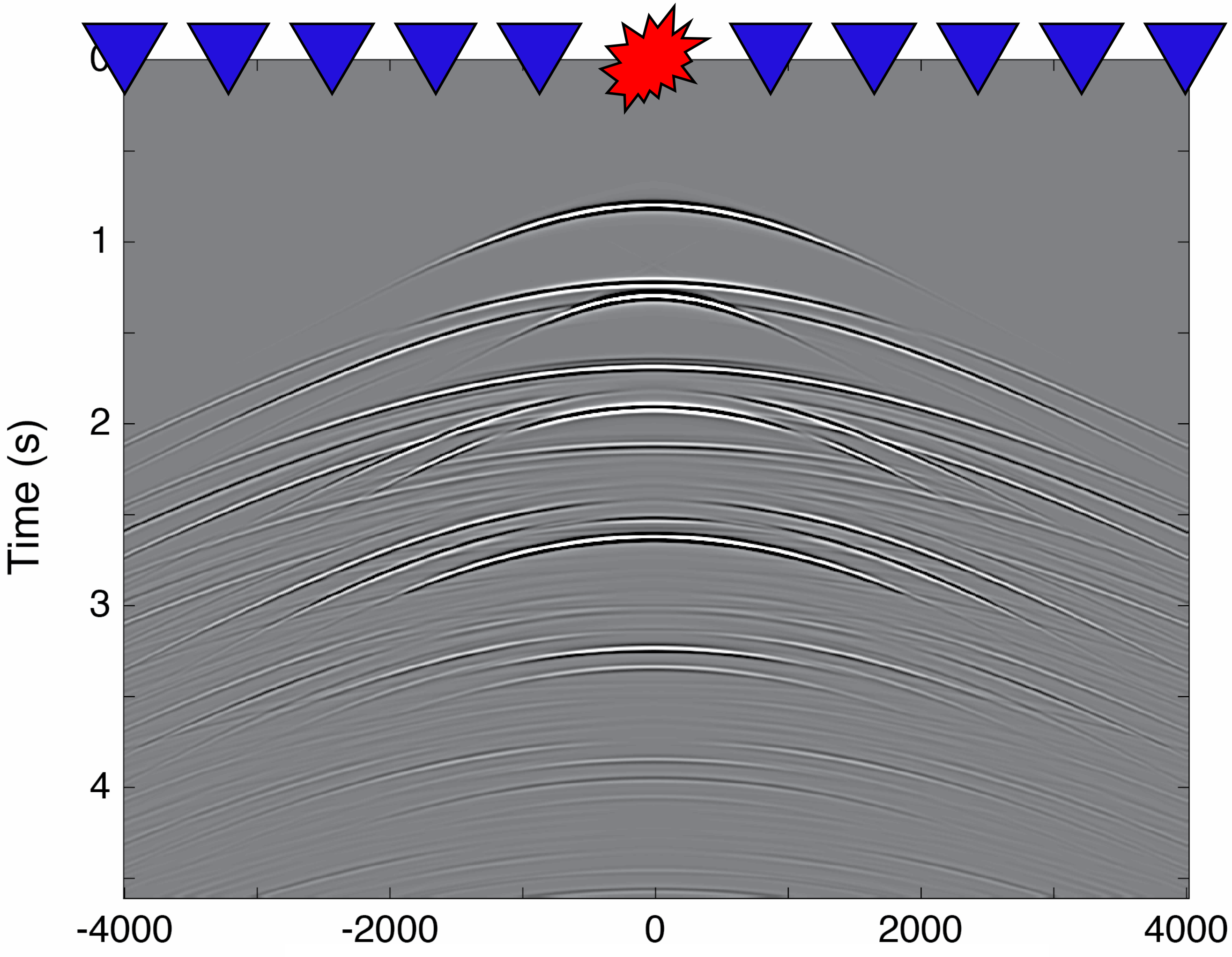


Reflection response

Introduction

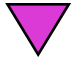


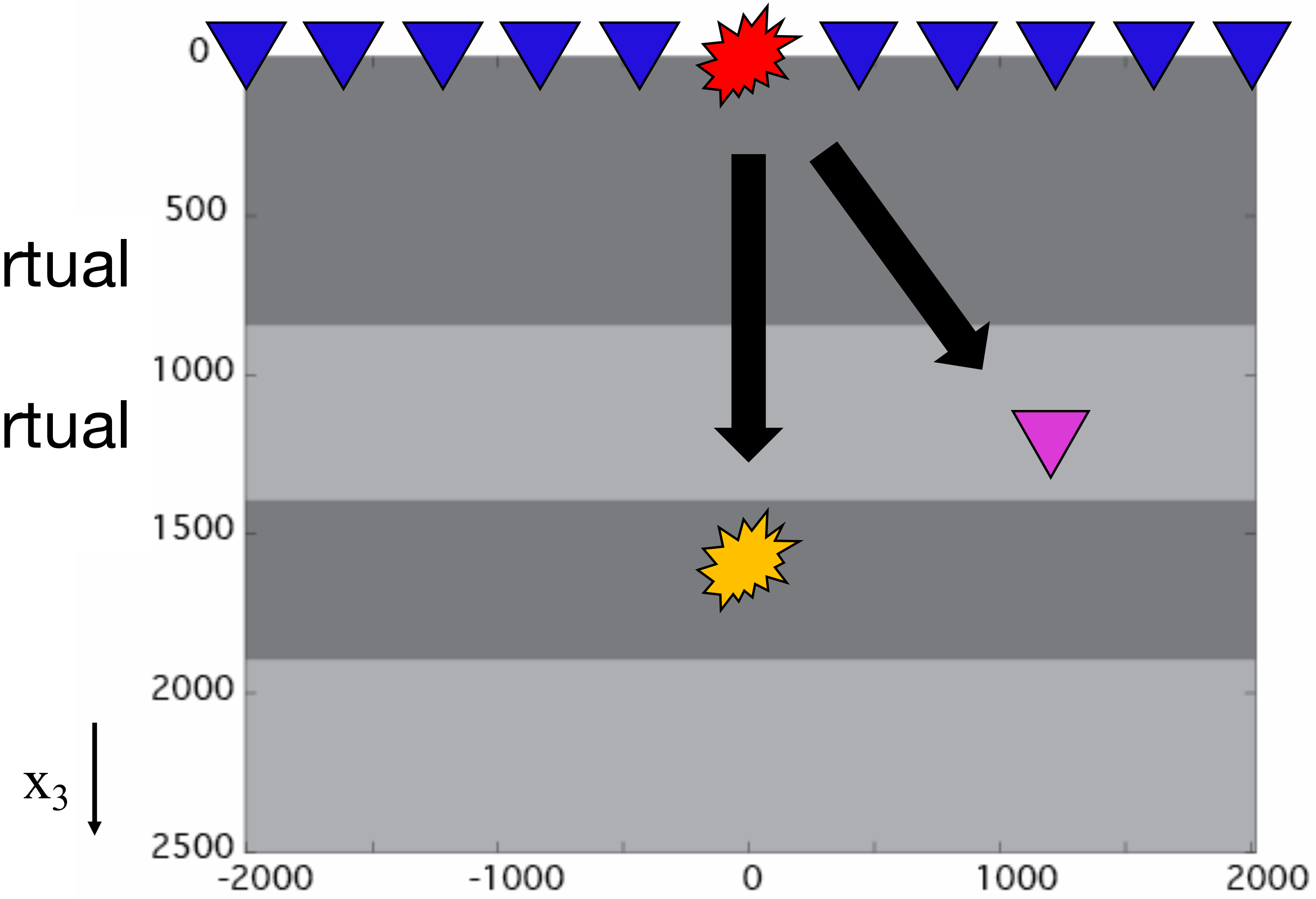
Introduction



Reflection response

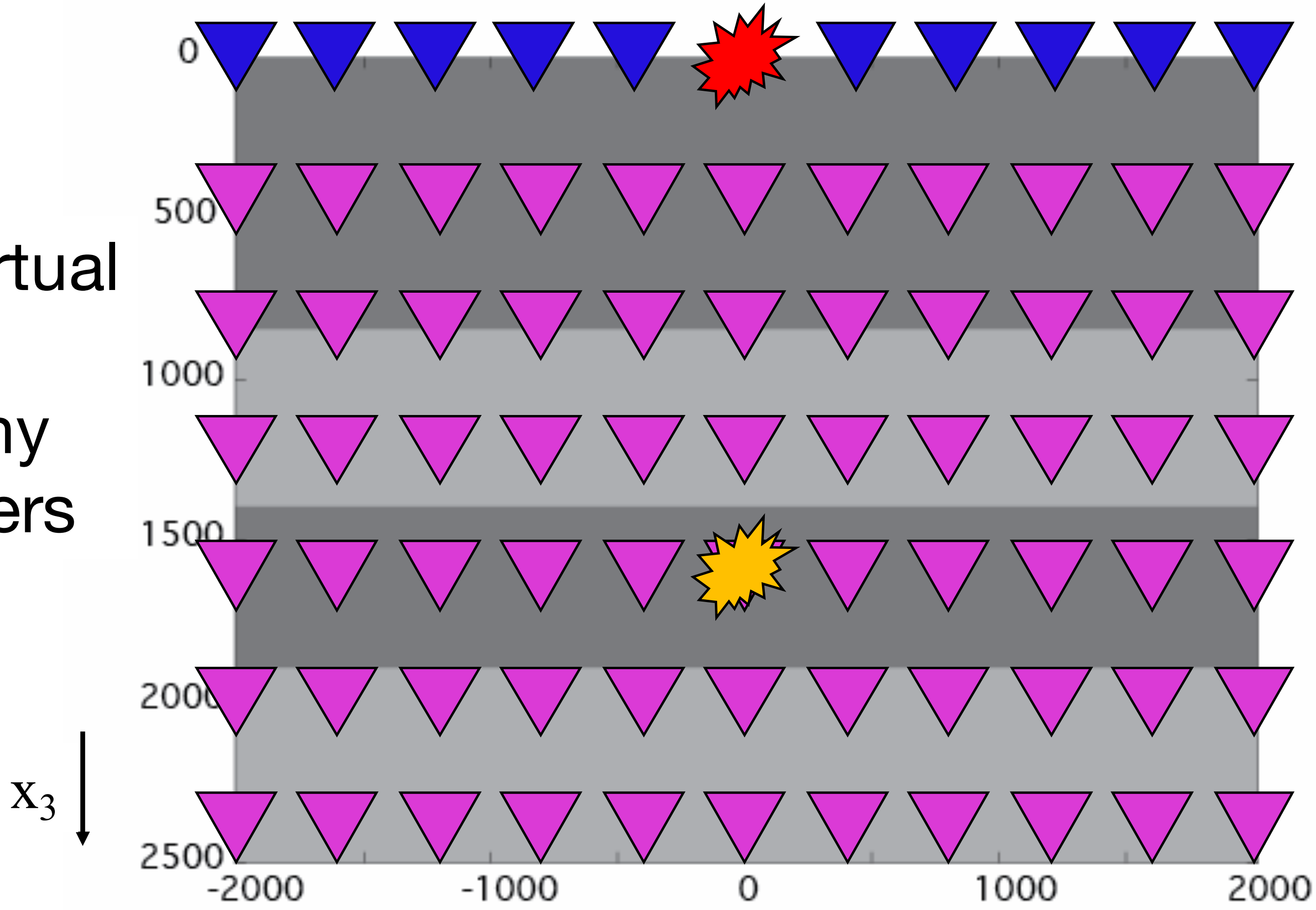
Introduction

- Create a virtual source
- Create a virtual receiver

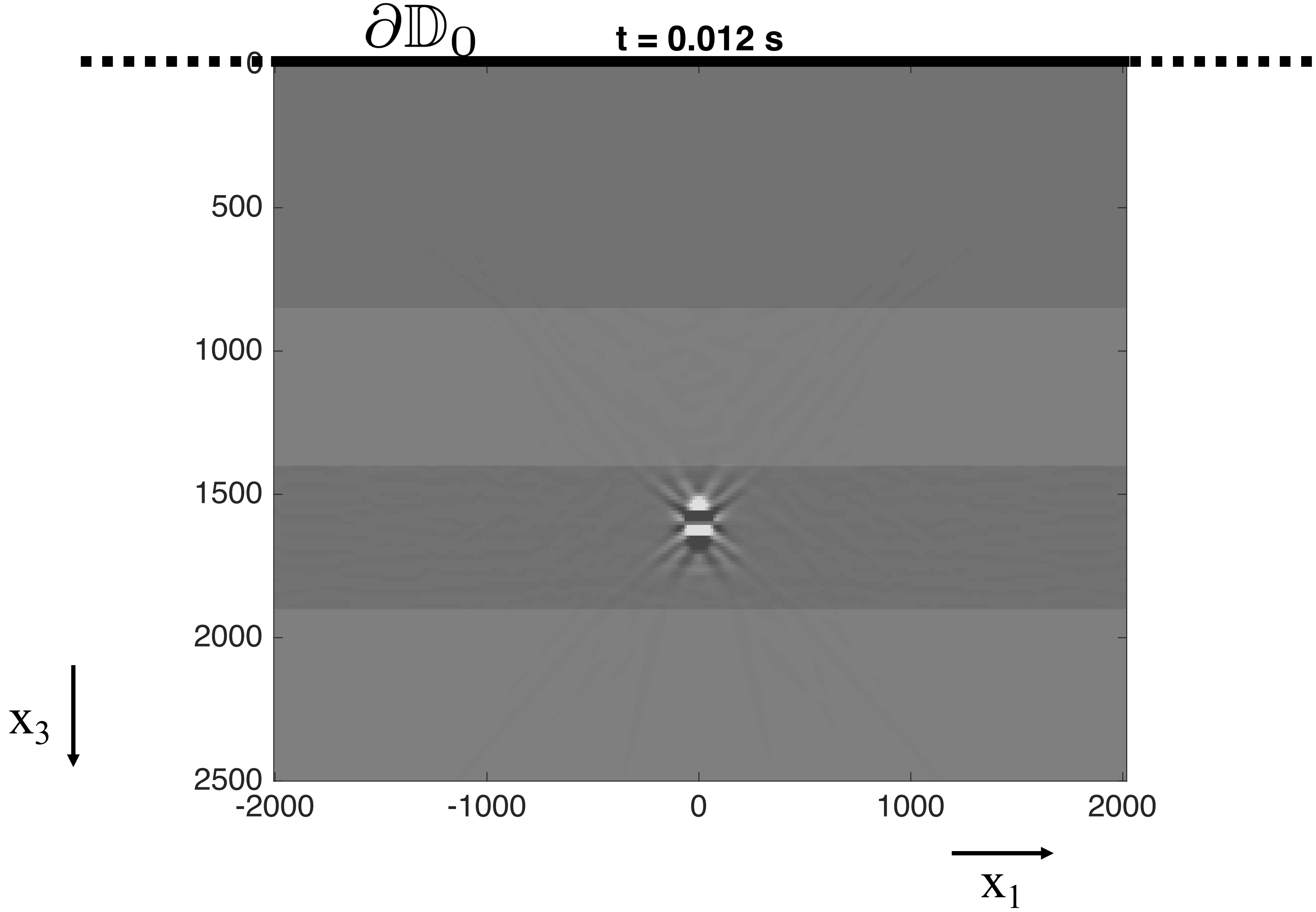


Introduction

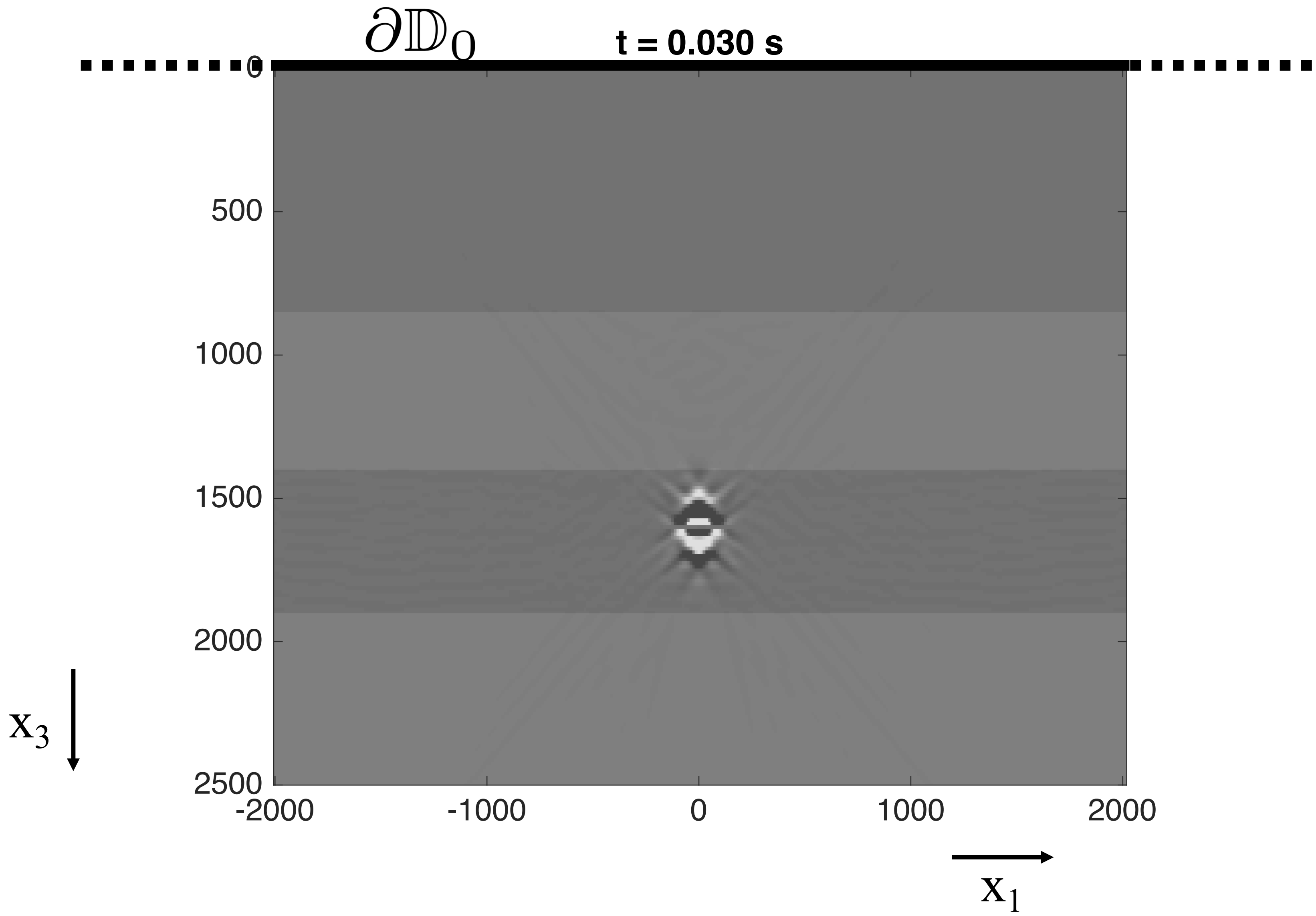
- Create a virtual source
- Create many virtual receivers



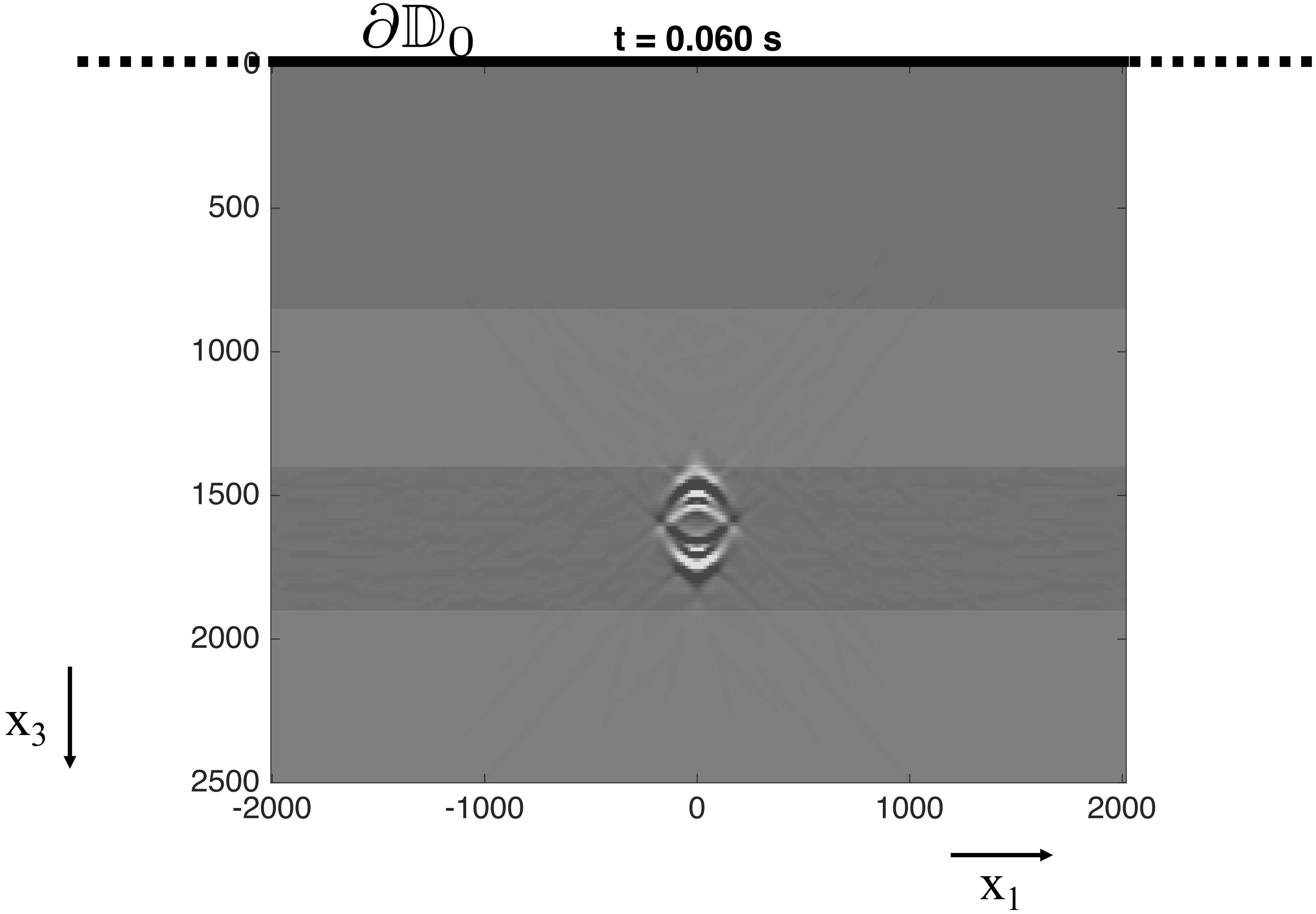
Introduction



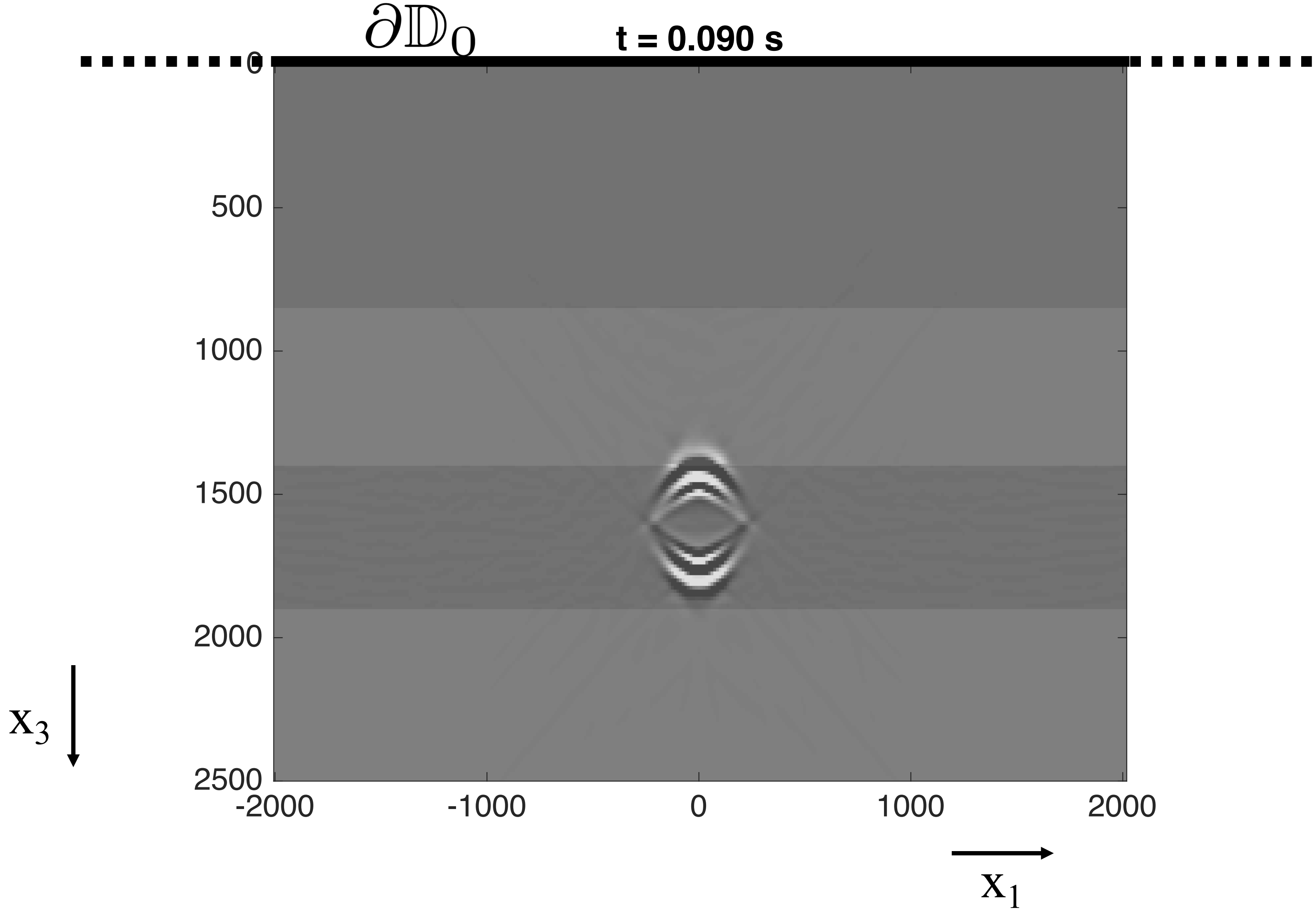
Introduction



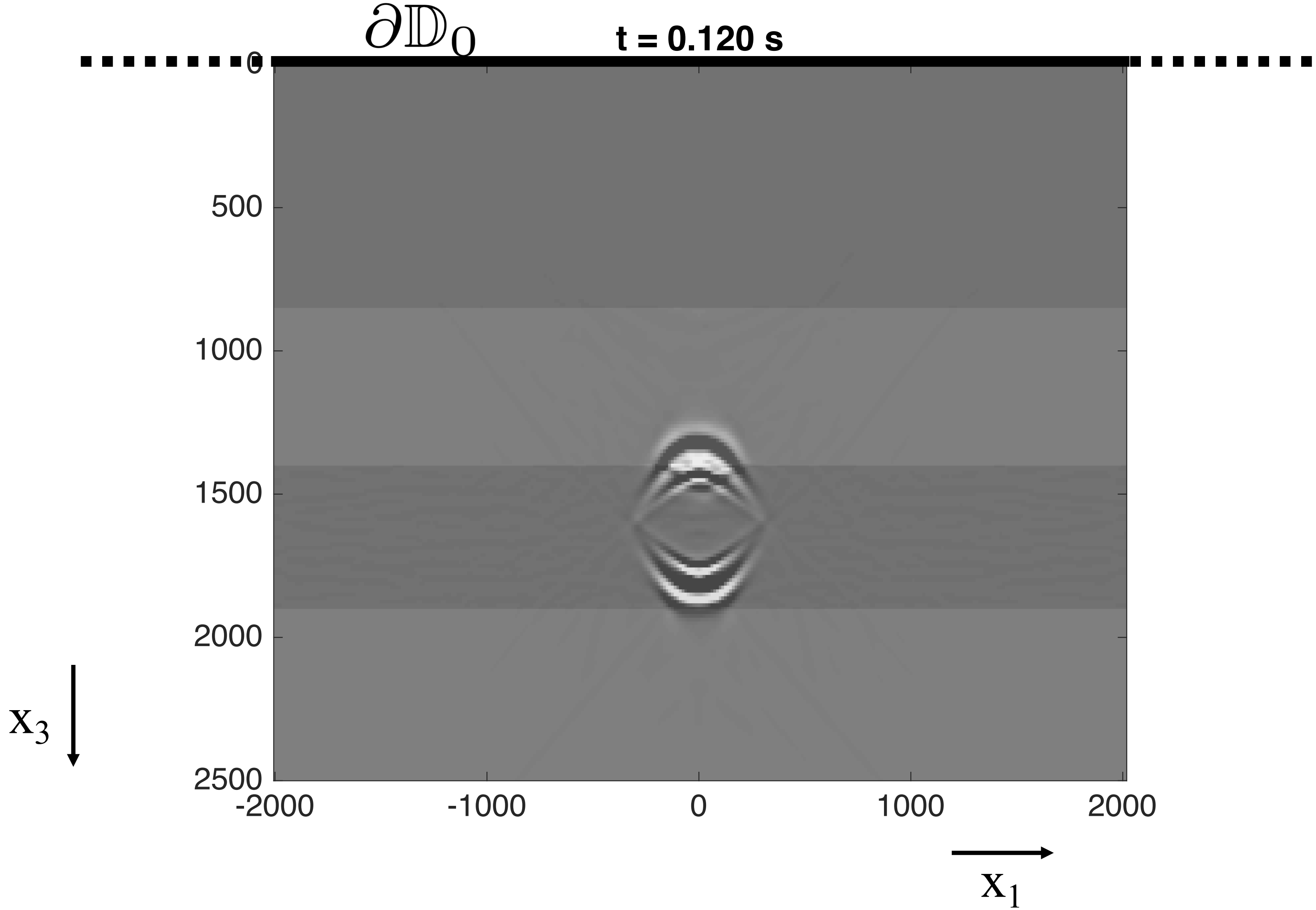
Introduction



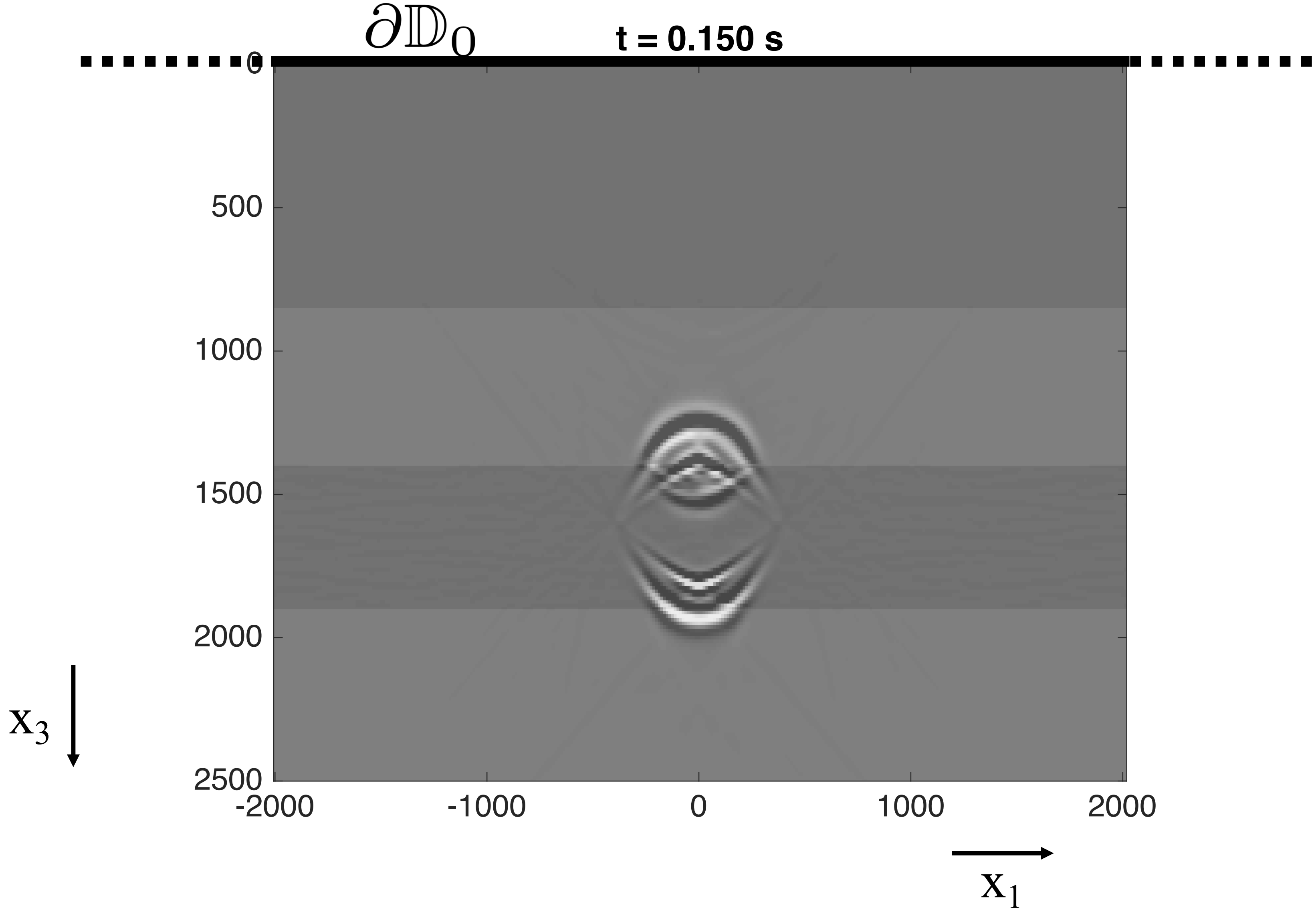
Introduction



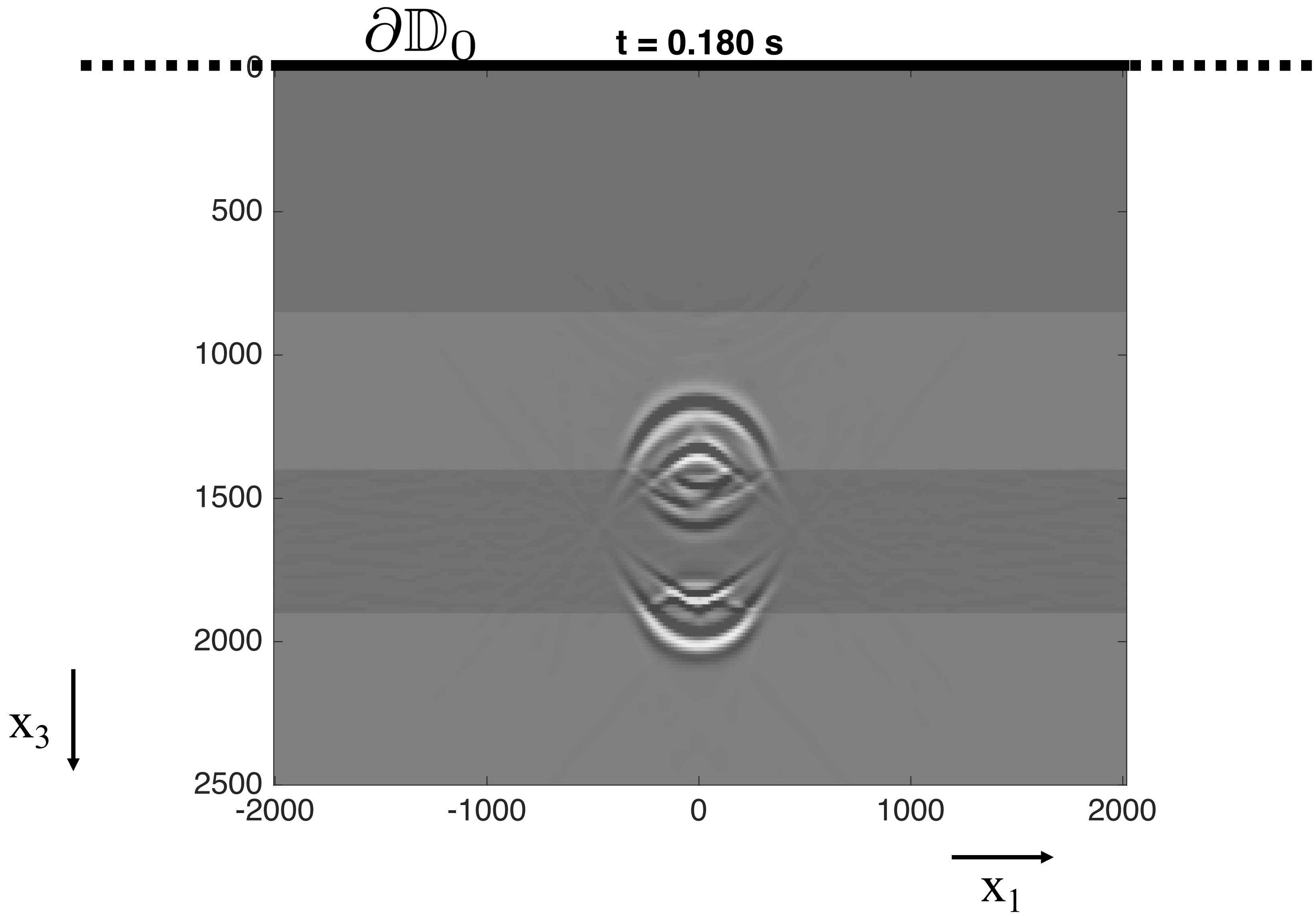
Introduction



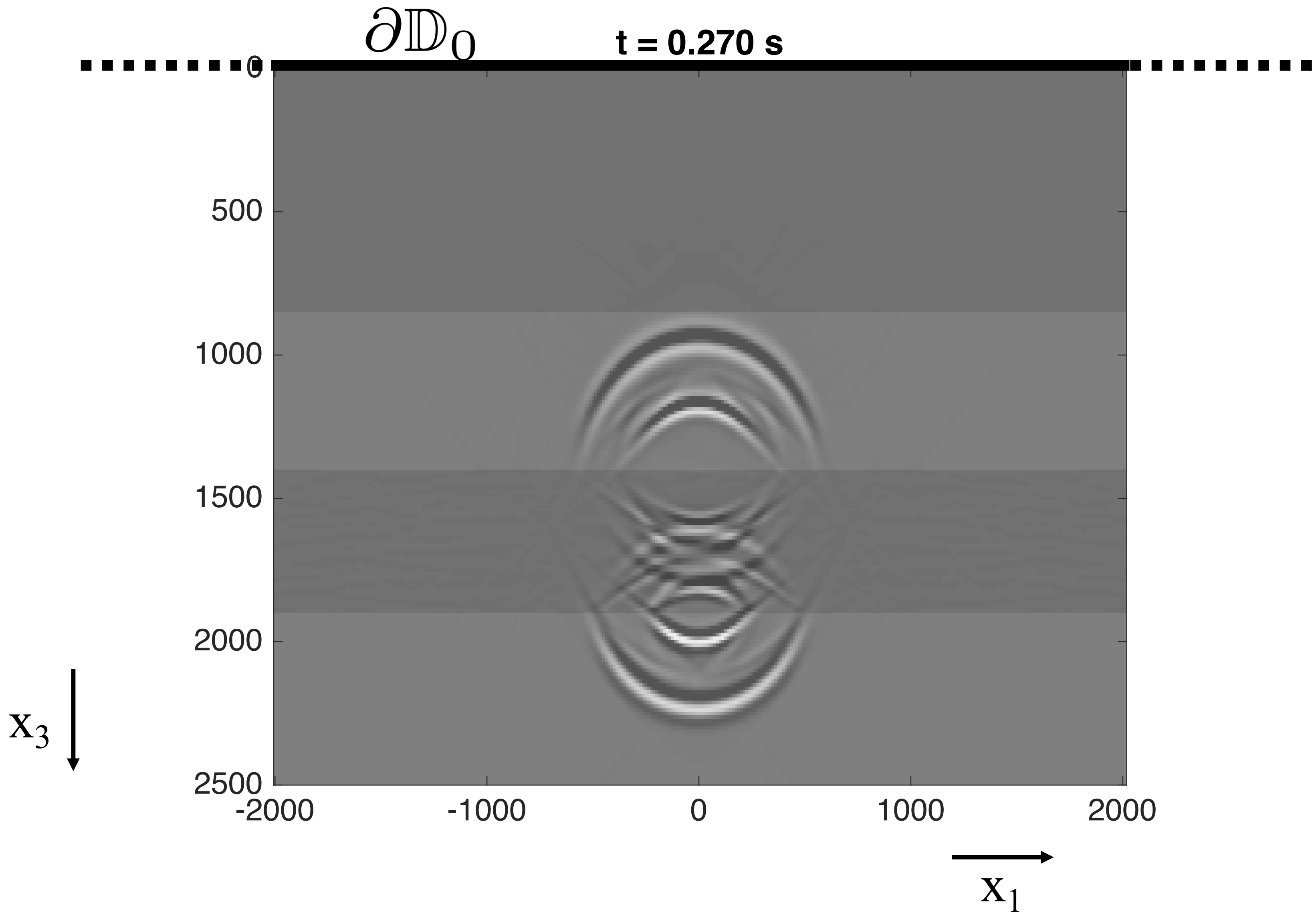
Introduction



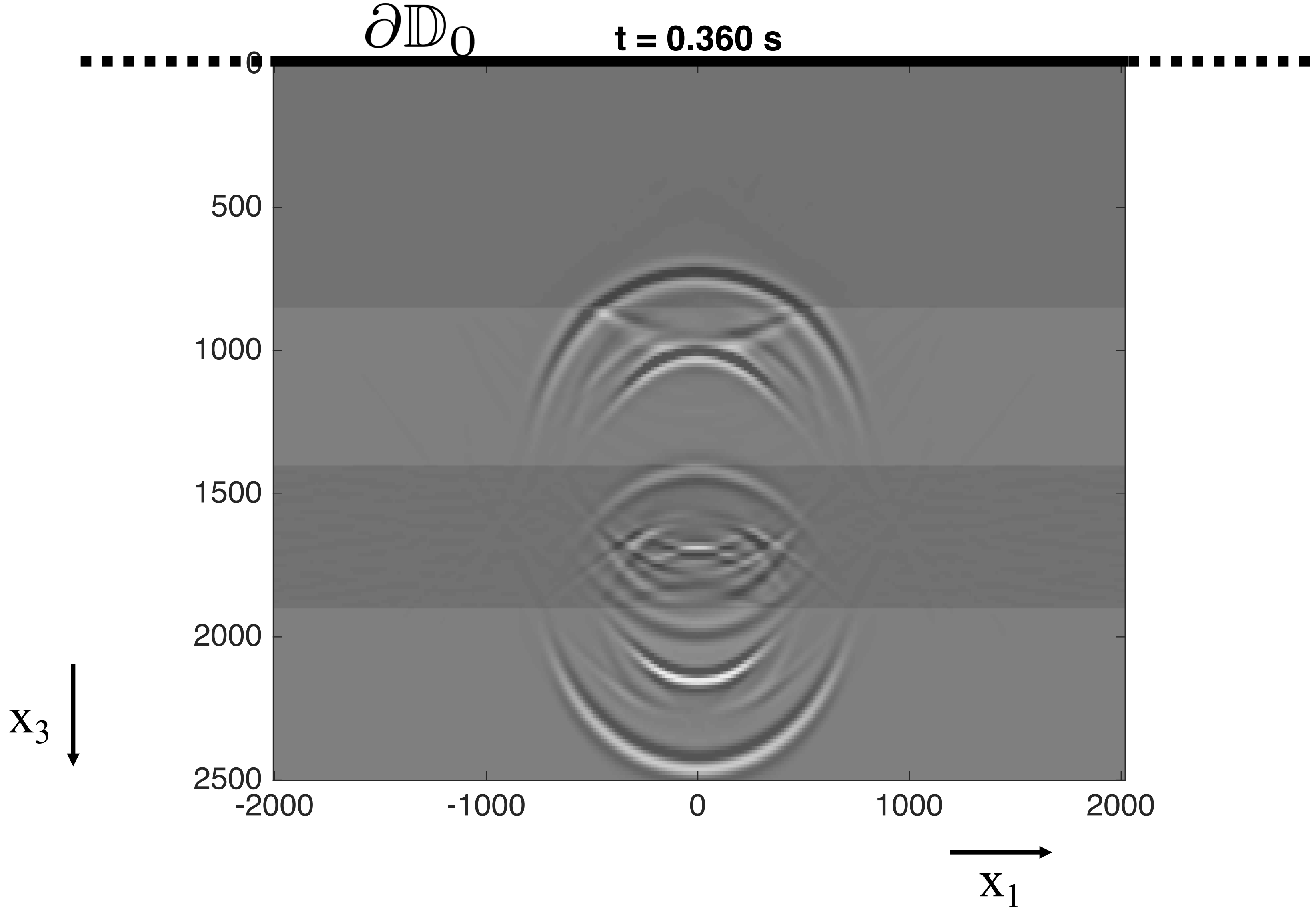
Introduction



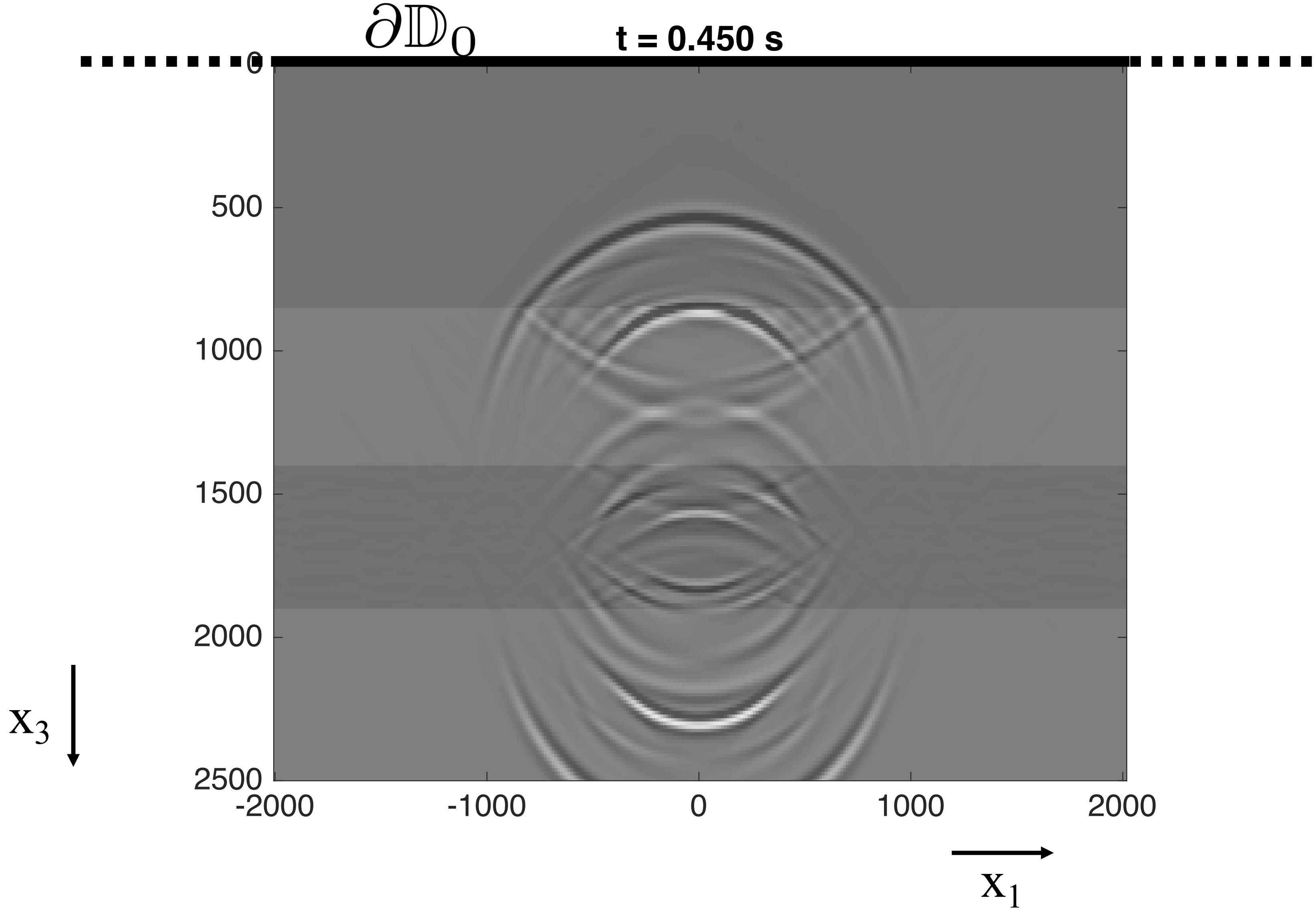
Introduction



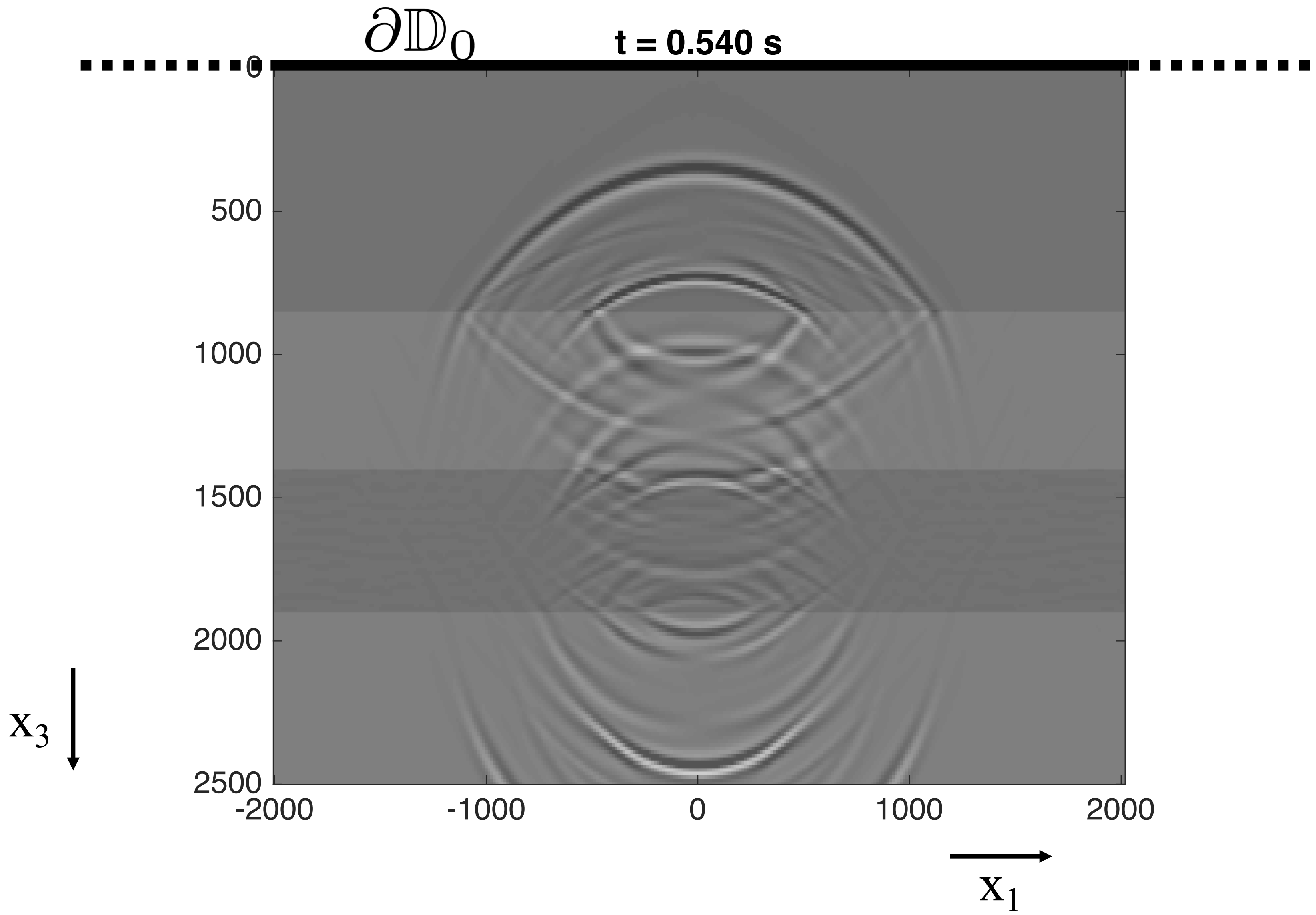
Introduction



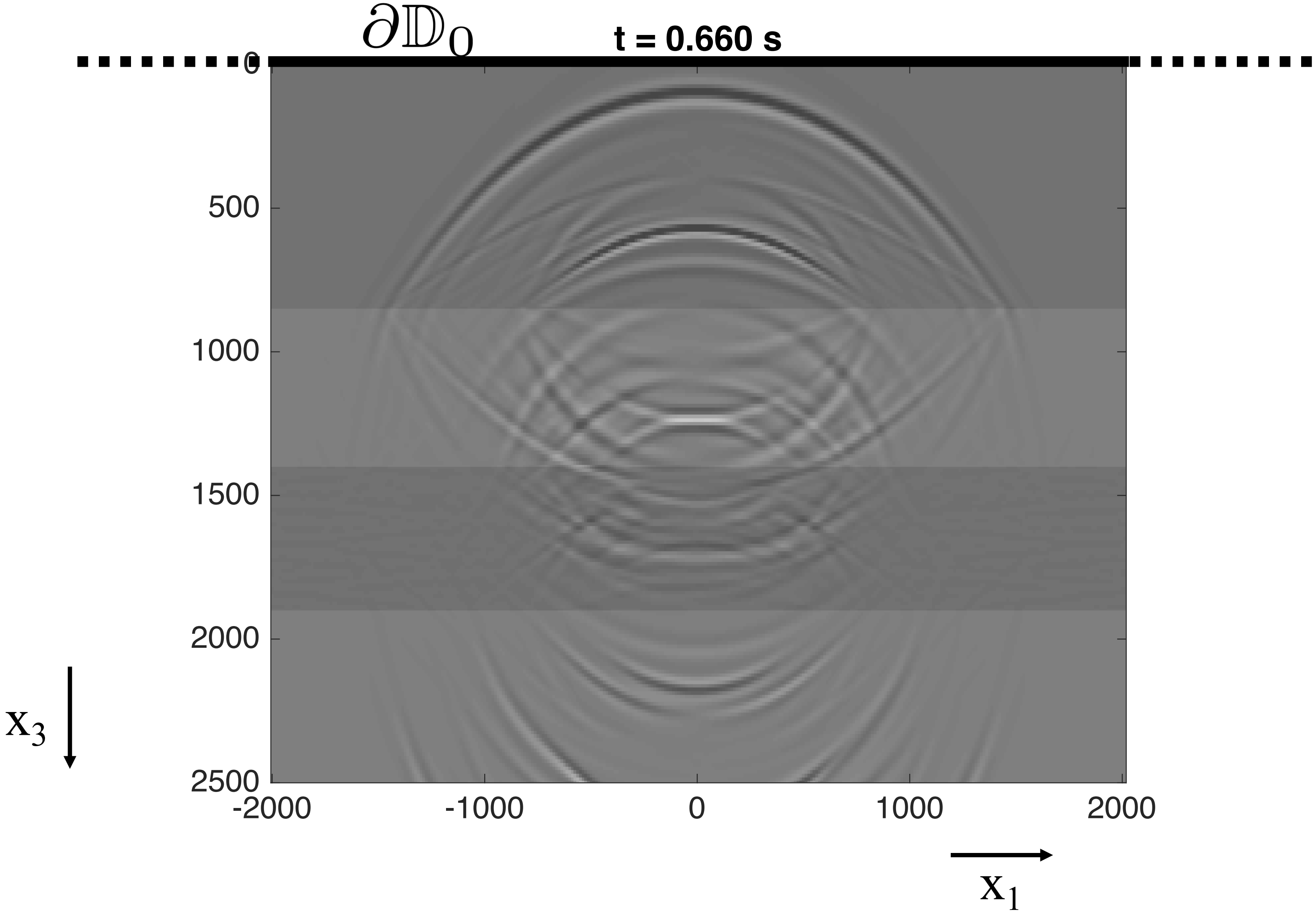
Introduction



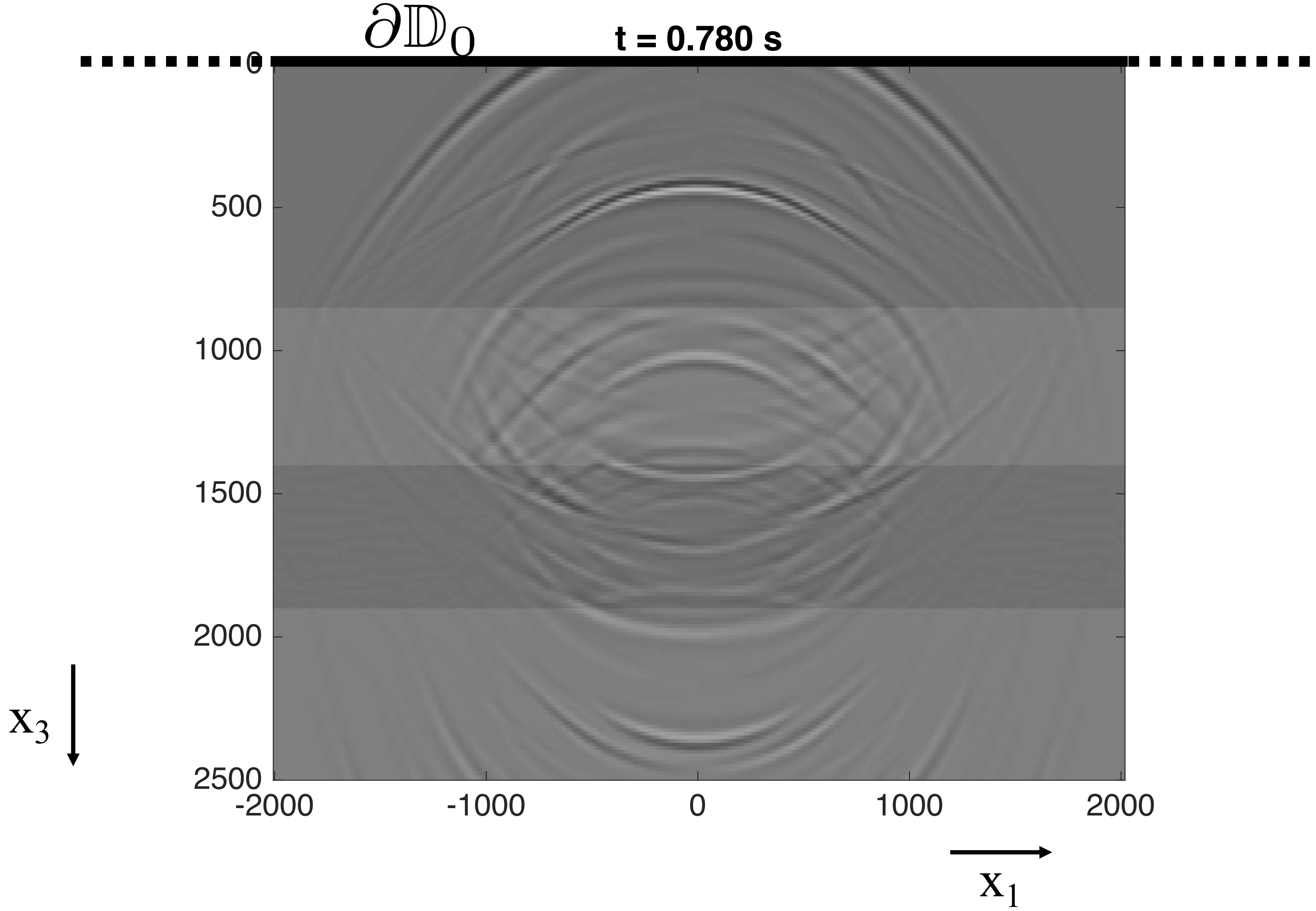
Introduction



Introduction



Introduction



Contents

1. Introduction
2. **Homogeneous Green's function**
3. Elastodynamic double-sided homogeneous Green's function representation
4. Elastodynamic single-sided homogeneous Green's function representation
5. Conclusions

TU Delft

WAVES

Homogeneous Green's function

- Acoustic wave equation

$$\nabla^2 G(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G(x, x_s, t) = -\delta(x - x_s)\delta(t)$$

Homogeneous Green's function

- Acoustic wave equation

$$\nabla^2 G(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G(x, x_s, t) = -\delta(x - x_s)\delta(t)$$

- Homogeneous acoustic wave equation

$$\nabla^2 G_h(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G_h(x, x_s, t) = 0$$

TU Delft

WAVES

Homogeneous Green's function

- Acoustic wave equation

$$\nabla^2 G(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G(x, x_s, t) = -\delta(x - x_s)\delta(t)$$

- Homogeneous acoustic wave equation

$$\nabla^2 G_h(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G_h(x, x_s, t) = 0$$

- Homogenous Green's function

$$G_h(x, x_s, t) = G(x, x_s, t) - G(x, x_s, -t)$$

TU Delft

WAVES

Homogeneous Green's function

- Acoustic wave equation

$$\nabla^2 G(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G(x, x_s, t) = -\delta(x - x_s)\delta(t)$$

- Homogeneous acoustic wave equation

$$\nabla^2 G_h(x, x_s, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} G_h(x, x_s, t) = 0$$

- Homogenous Green's function

$$G_h(x, x_s, t) = G(x, x_s, t) - G(x, x_s, -t)$$

$$G_h(x, x_s, \omega) = G(x, x_s, \omega) - G^*(x, x_s, \omega)$$

Homogeneous Green's function

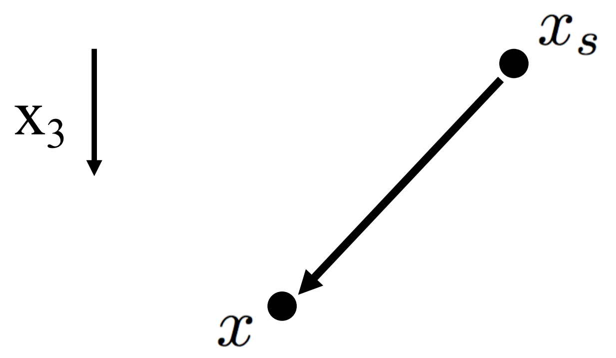
- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix} (x, x_s, \omega)$$

Homogeneous Green's function

- One-way wavefields

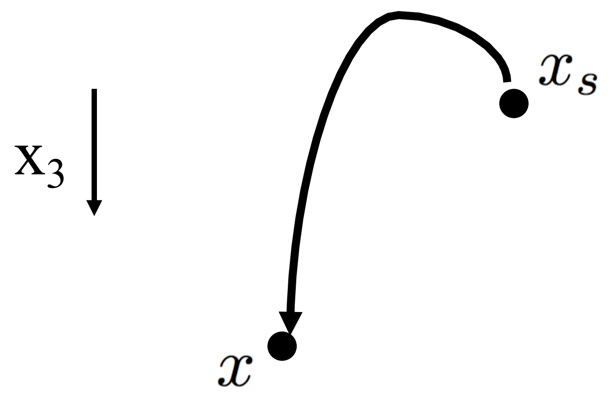
$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix} (x, x_s, \omega)$$



Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & \boxed{G^{+,-}} \\ G^{-,+} & G^{-,-} \end{pmatrix}(x, x_s, \omega)$$



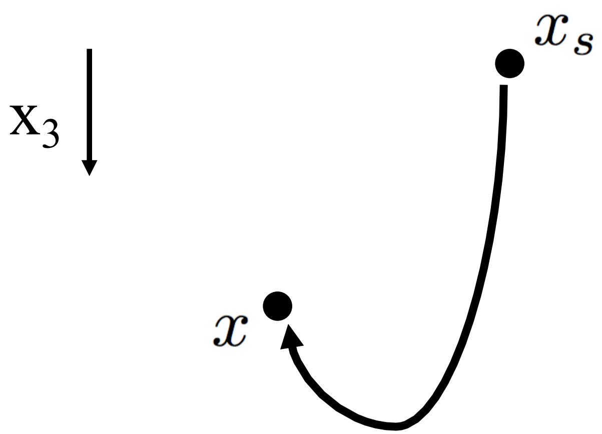
TU Delft

WAVES

Homogeneous Green's function

- One-way wavefields

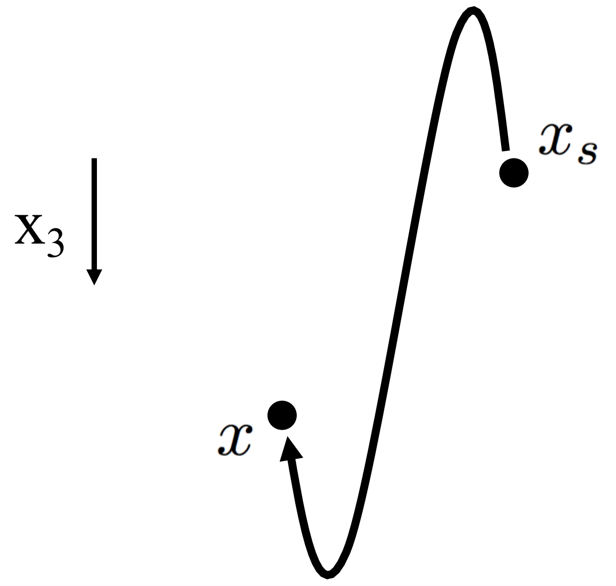
$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix}(x, x_s, \omega)$$



Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & \boxed{G^{-,-}} \end{pmatrix}(x, x_s, \omega)$$



Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix} (x, x_s, \omega)$$

- P- / S-Wave potentials

$$G^{+,+}(x, x_s, \omega) = \begin{pmatrix} G^{+,+}_{p,p} & G^{+,+}_{p,s} \\ G^{+,+}_{s,p} & G^{+,+}_{s,s} \end{pmatrix} (x, x_s, \omega)$$

TU Delft

WAVES

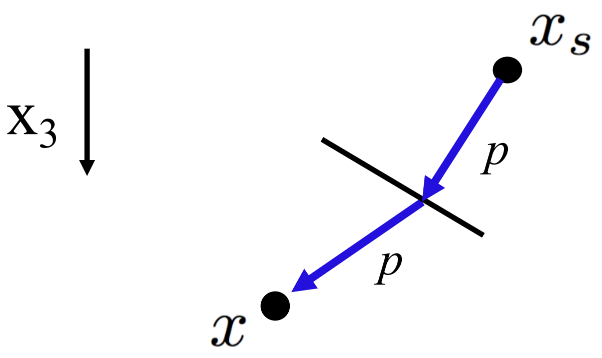
Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix}(x, x_s, \omega)$$

- P- / S-Wave potentials

$$G^{+,+}(x, x_s, \omega) = \begin{pmatrix} G^{+,+}_{p,p} & G^{+,+}_{p,s} \\ G^{+,+}_{s,p} & G^{+,+}_{s,s} \end{pmatrix}(x, x_s, \omega)$$



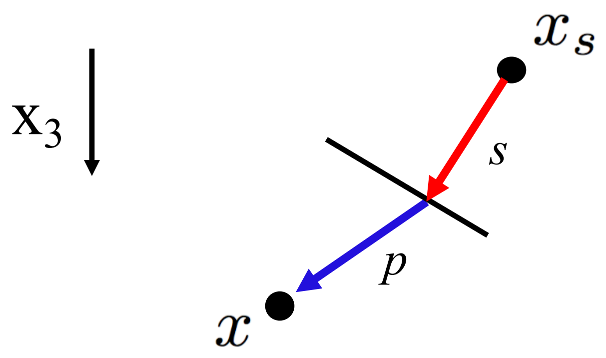
Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix}(x, x_s, \omega)$$

- P- / S-Wave potentials

$$G^{+,+}(x, x_s, \omega) = \begin{pmatrix} G^{+,+}_{p,p} & \boxed{G^{+,+}_{p,s}} \\ G^{+,+}_{s,p} & G^{+,+}_{s,s} \end{pmatrix}(x, x_s, \omega)$$



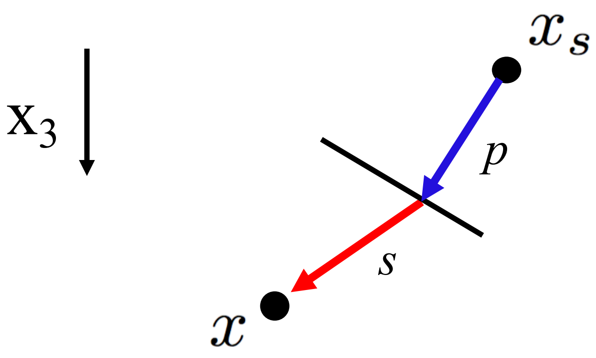
Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix} (x, x_s, \omega)$$

- P- / S-Wave potentials

$$G^{+,+}(x, x_s, \omega) = \begin{pmatrix} G^{+,+}_{p,p} & G^{+,+}_{p,s} \\ \boxed{G^{+,+}_{s,p}} & G^{+,+}_{s,s} \end{pmatrix} (x, x_s, \omega)$$



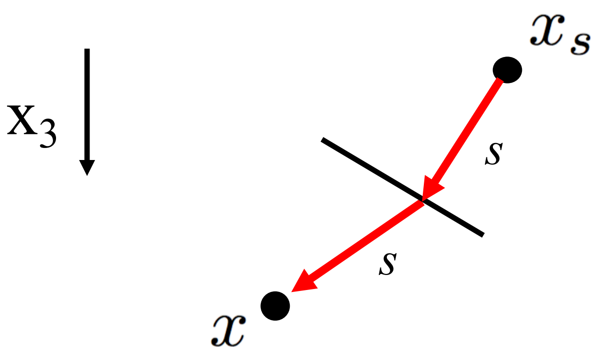
Homogeneous Green's function

- One-way wavefields

$$\mathbf{G}(x, x_s, \omega) = \begin{pmatrix} G^{+,+} & G^{+,-} \\ G^{-,+} & G^{-,-} \end{pmatrix} (x, x_s, \omega)$$

- P- / S-Wave potentials

$$G^{+,+}(x, x_s, \omega) = \begin{pmatrix} G^{+,+}_{p,p} & G^{+,+}_{p,s} \\ G^{+,+}_{s,p} & G^{+,+}_{s,s} \end{pmatrix} (x, x_s, \omega)$$



Homogeneous Green's function

- One-way wave equation

$$\partial_3 \mathbf{G}(x, x_s, \omega) - \mathcal{B}\mathbf{G}(x, x_s, \omega) = \mathbf{I}\delta(x - x_s)$$

TU Delft

WAVES

Homogeneous Green's function

- One-way wave equation

$$\partial_3 \mathbf{G}(x, x_s, \omega) - \mathcal{B}\mathbf{G}(x, x_s, \omega) = \mathbf{I}\delta(x - x_s)$$

- Time-reversed one-way wave equation

$$\partial_3 \{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} - \mathcal{B}\{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} = \mathbf{I}\delta(x - x_s)$$

TU Delft

WAVES

Homogeneous Green's function

- One-way wave equation

$$\partial_3 \mathbf{G}(x, x_s, \omega) - \mathcal{B}\mathbf{G}(x, x_s, \omega) = \mathbf{I}\delta(x - x_s)$$

- Time-reversed one-way wave equation

$$\partial_3 \{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} - \mathcal{B}\{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} = \mathbf{I}\delta(x - x_s)$$

- Homogeneous Green's function, definition:

$$\partial_3 \mathbf{G}_h(x, x_s, \omega) - \mathcal{B}\mathbf{G}_h(x, x_s, \omega) = \mathbf{0}$$

Homogeneous Green's function

- One-way wave equation

$$\partial_3 \mathbf{G}(x, x_s, \omega) - \mathcal{B}\mathbf{G}(x, x_s, \omega) = \mathbf{I}\delta(x - x_s)$$

- Time-reversed one-way wave equation

$$\partial_3 \{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} - \mathcal{B}\{\mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}\} = \mathbf{I}\delta(x - x_s)$$

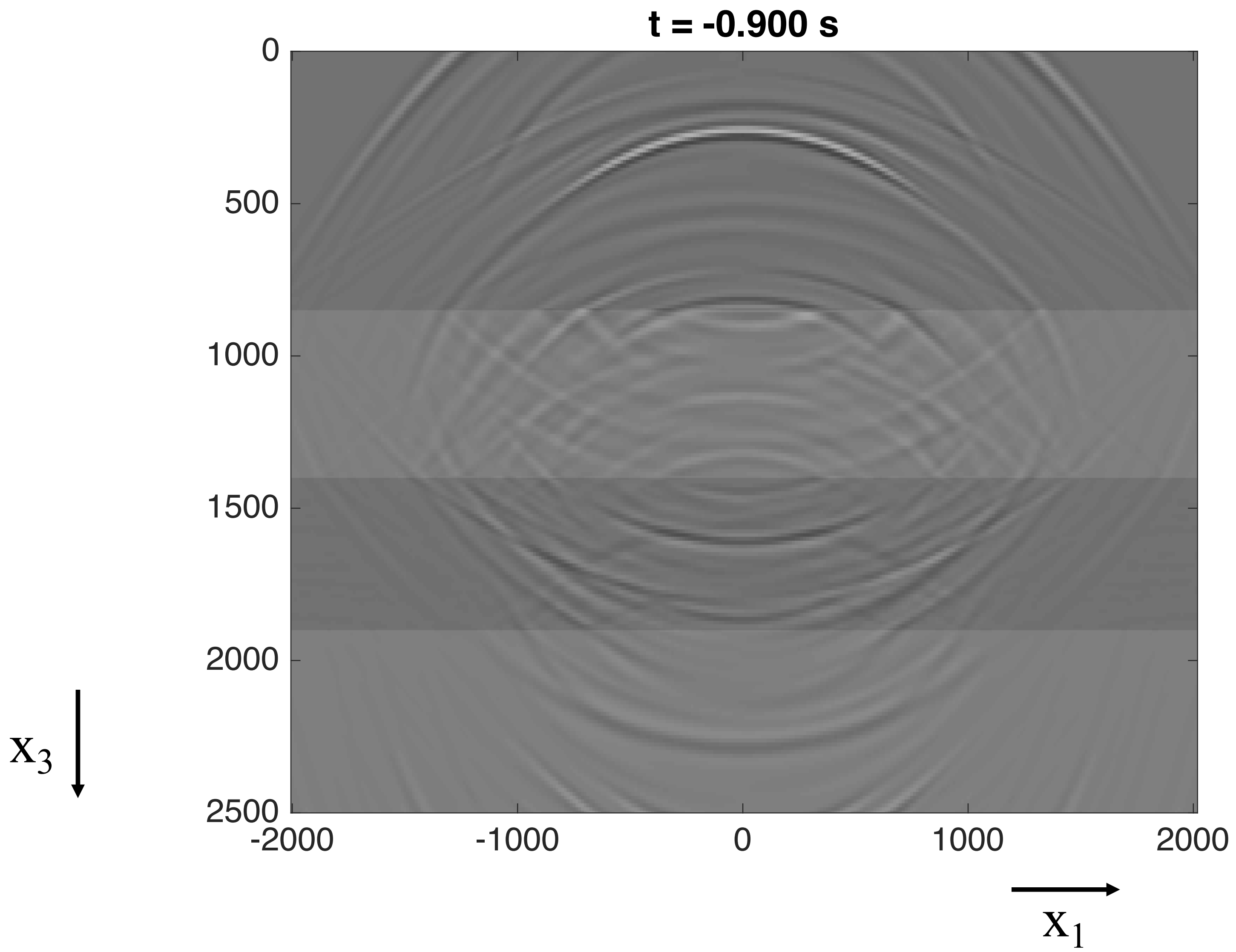
- Homogeneous Green's function, definition:

$$\partial_3 \mathbf{G}_h(x, x_s, \omega) - \mathcal{B}\mathbf{G}_h(x, x_s, \omega) = \mathbf{0}$$

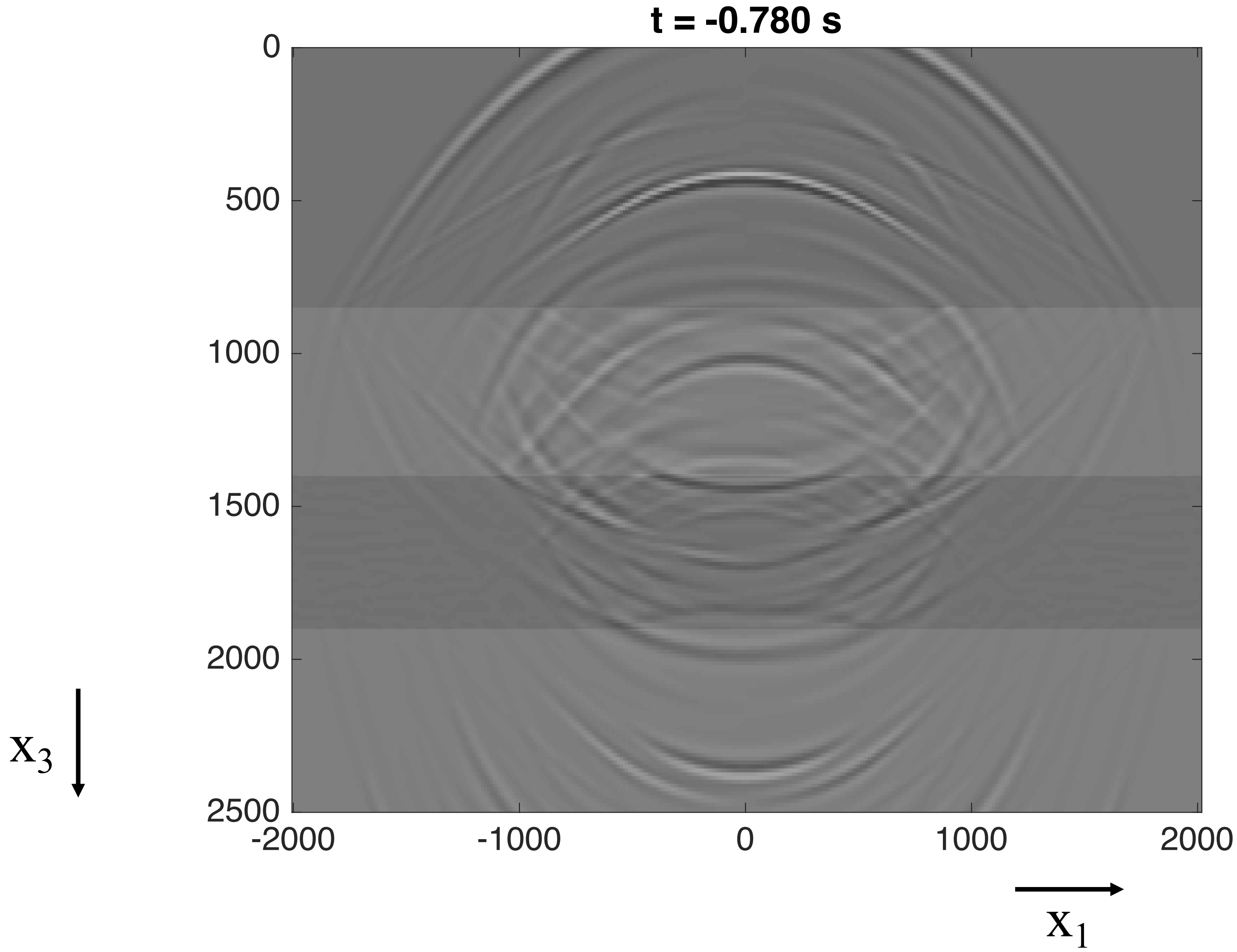
- Homogeneous Green's function

$$\mathbf{G}_h(x, x_s, \omega) = \mathbf{G}(x, x_s, \omega) - \mathbf{K}\mathbf{G}^*(x, x_s, \omega)\mathbf{K}$$

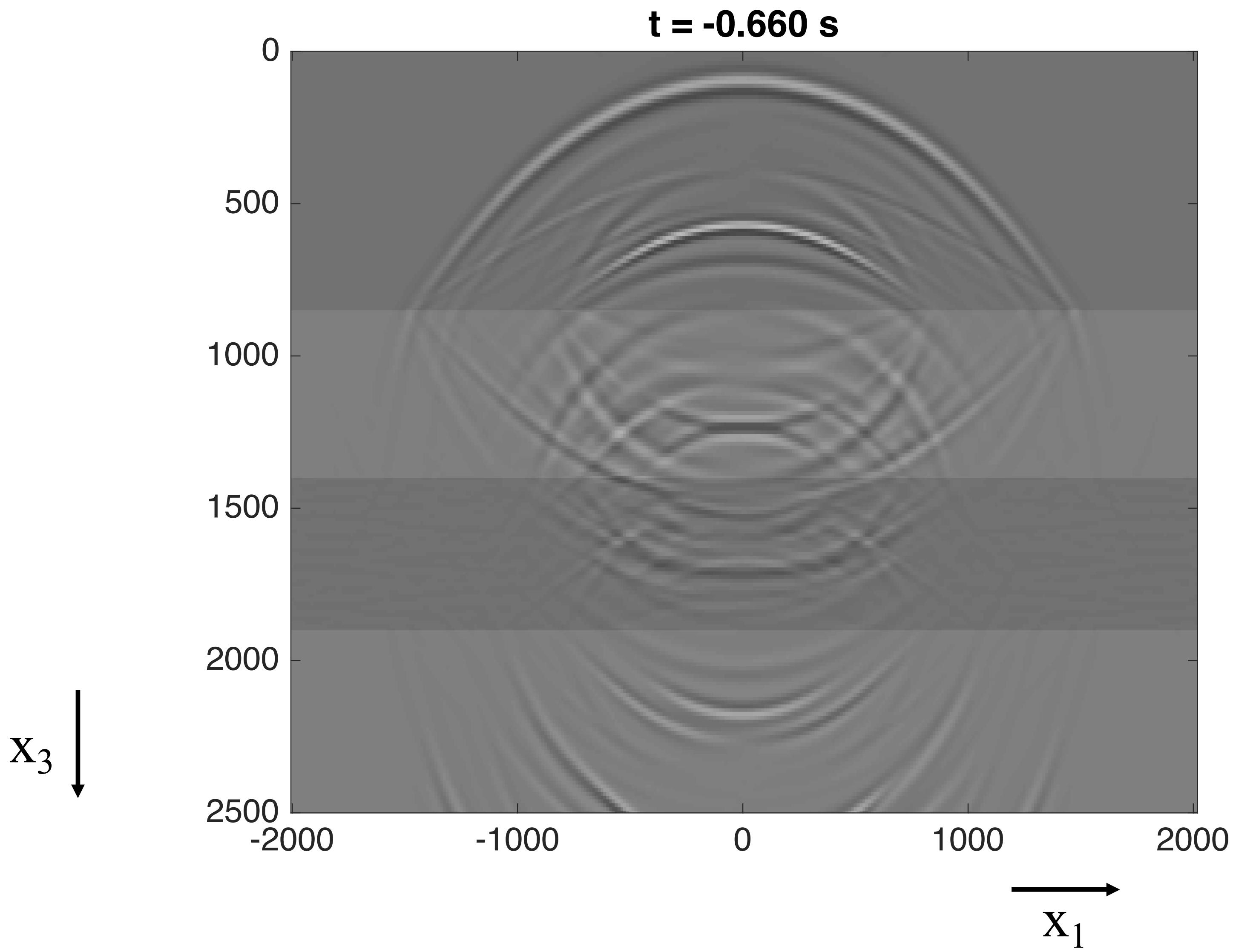
Homogeneous Green's function



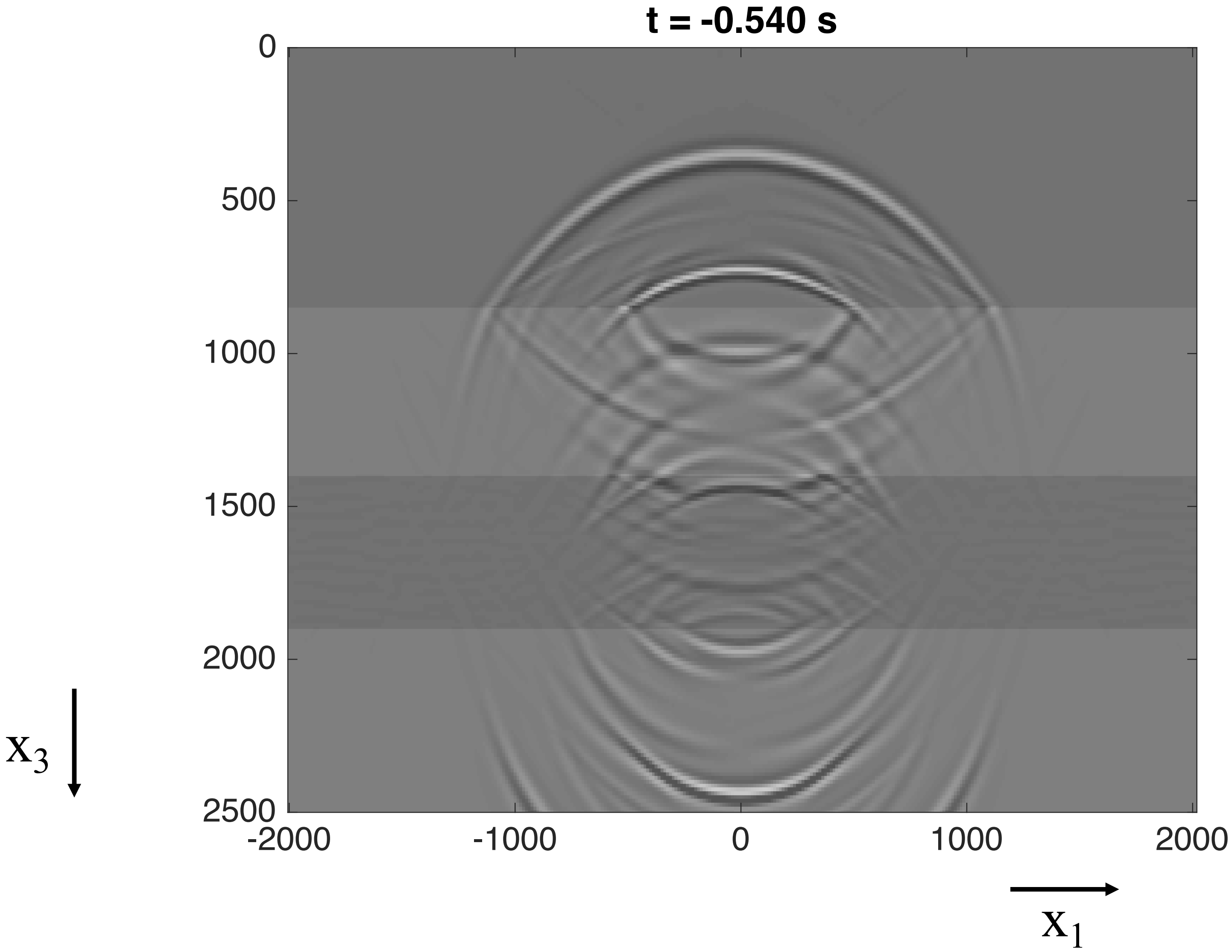
Homogeneous Green's function



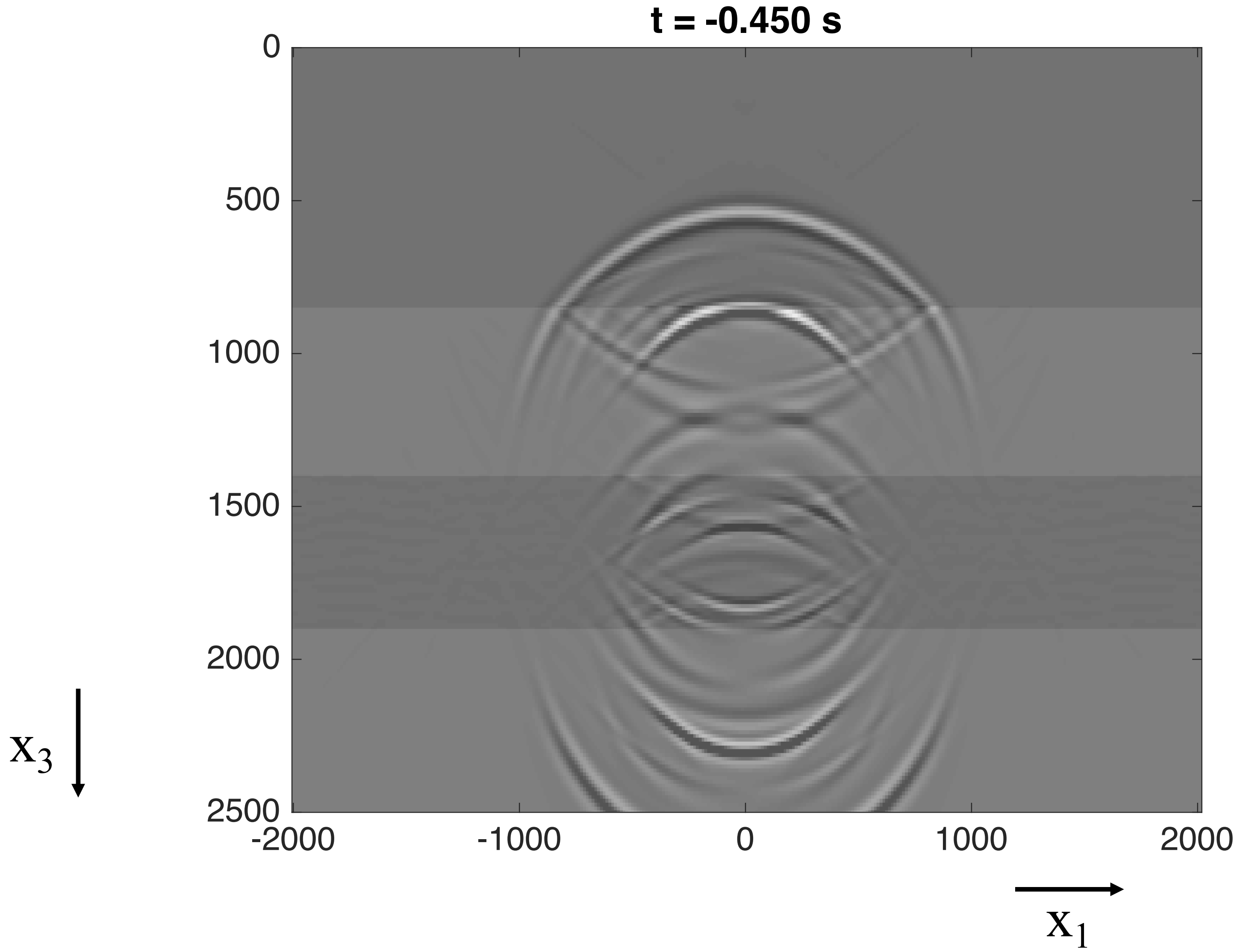
Homogeneous Green's function



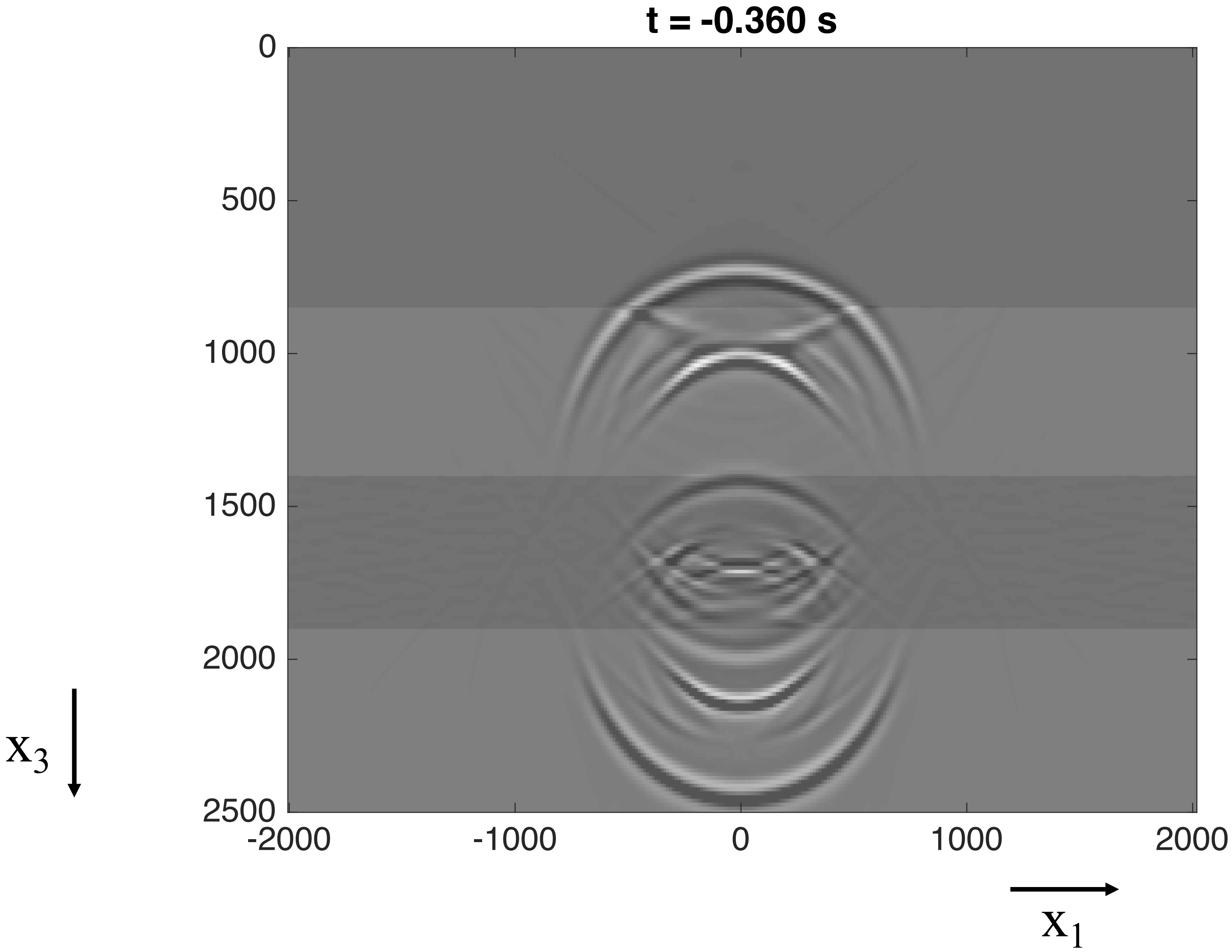
Homogeneous Green's function



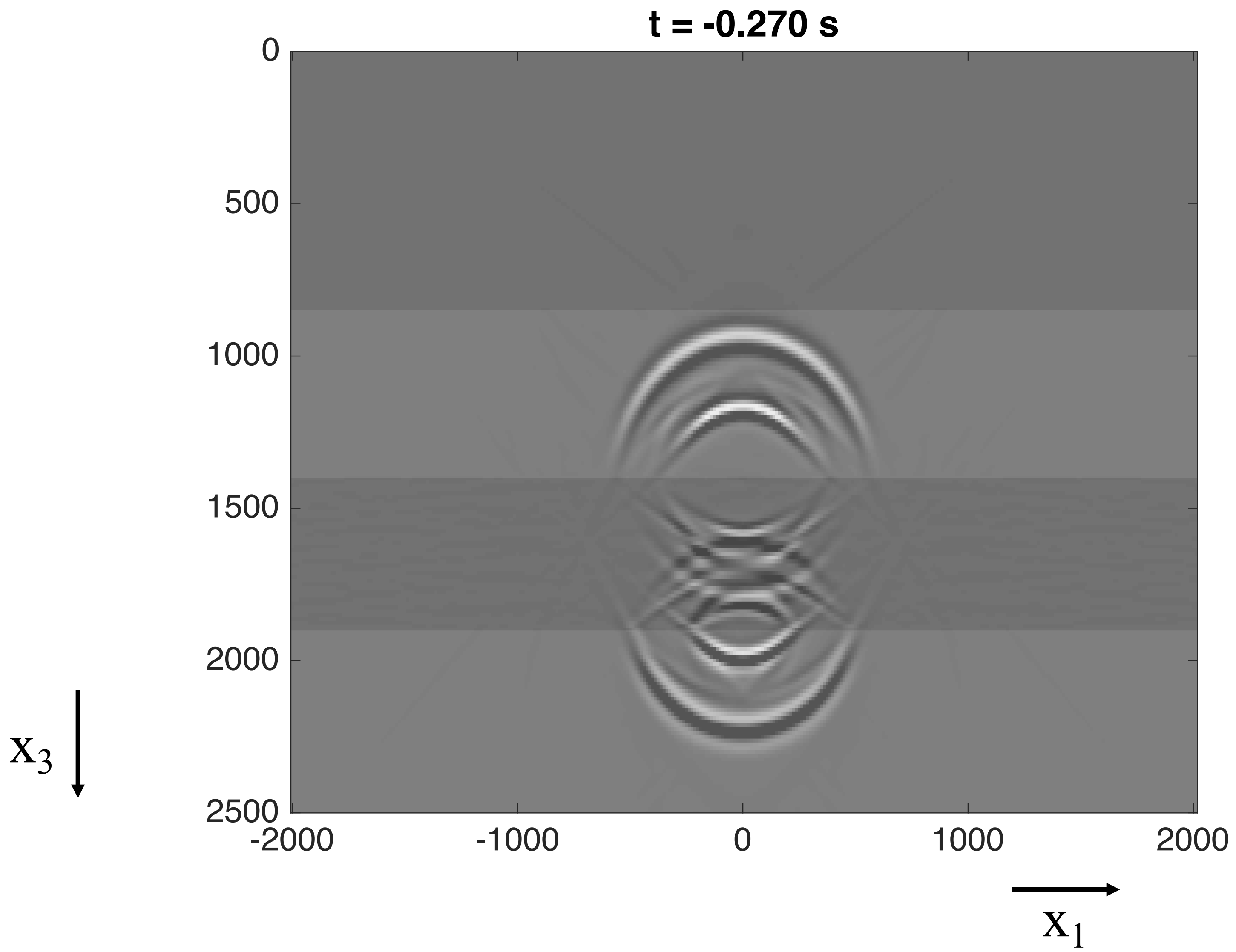
Homogeneous Green's function



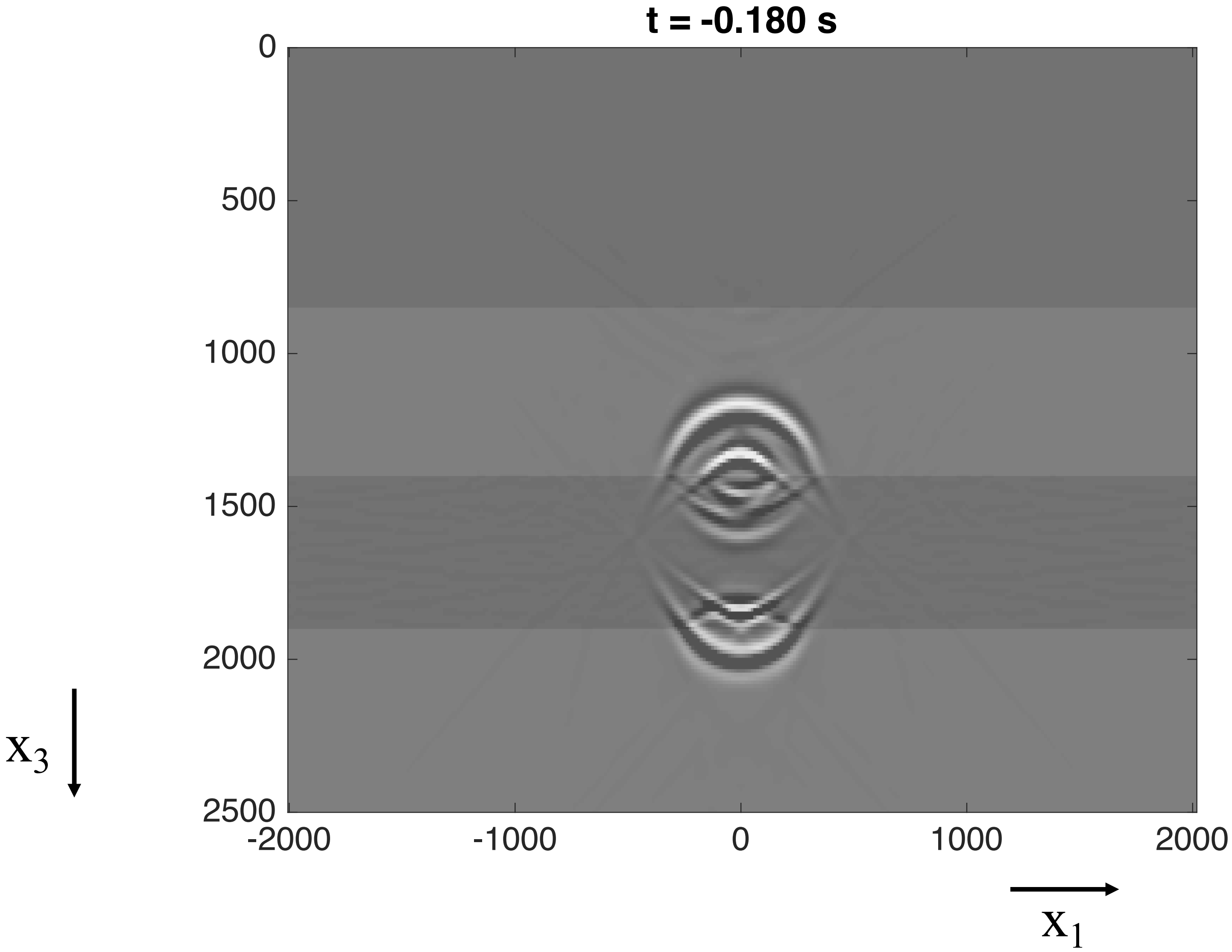
Homogeneous Green's function



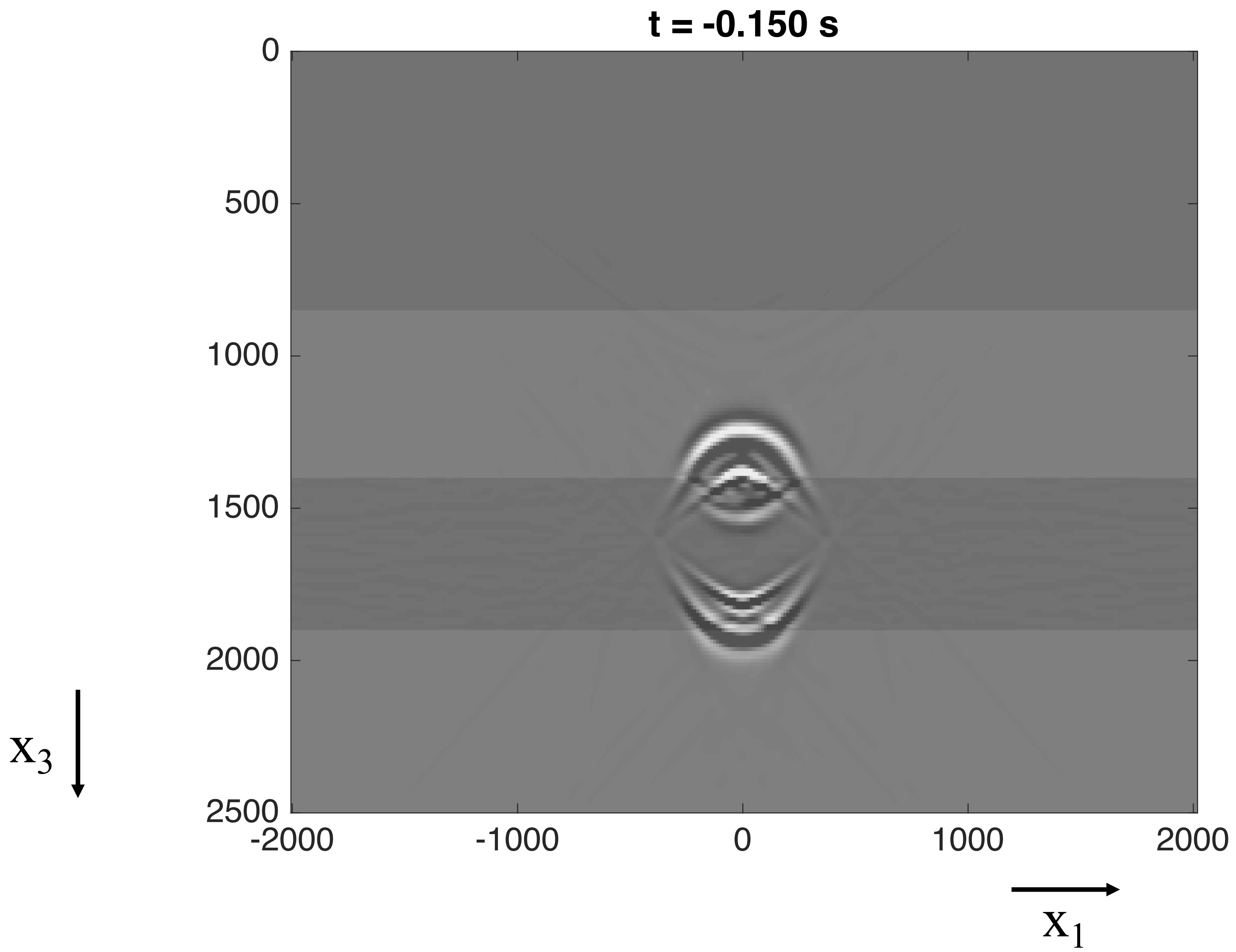
Homogeneous Green's function



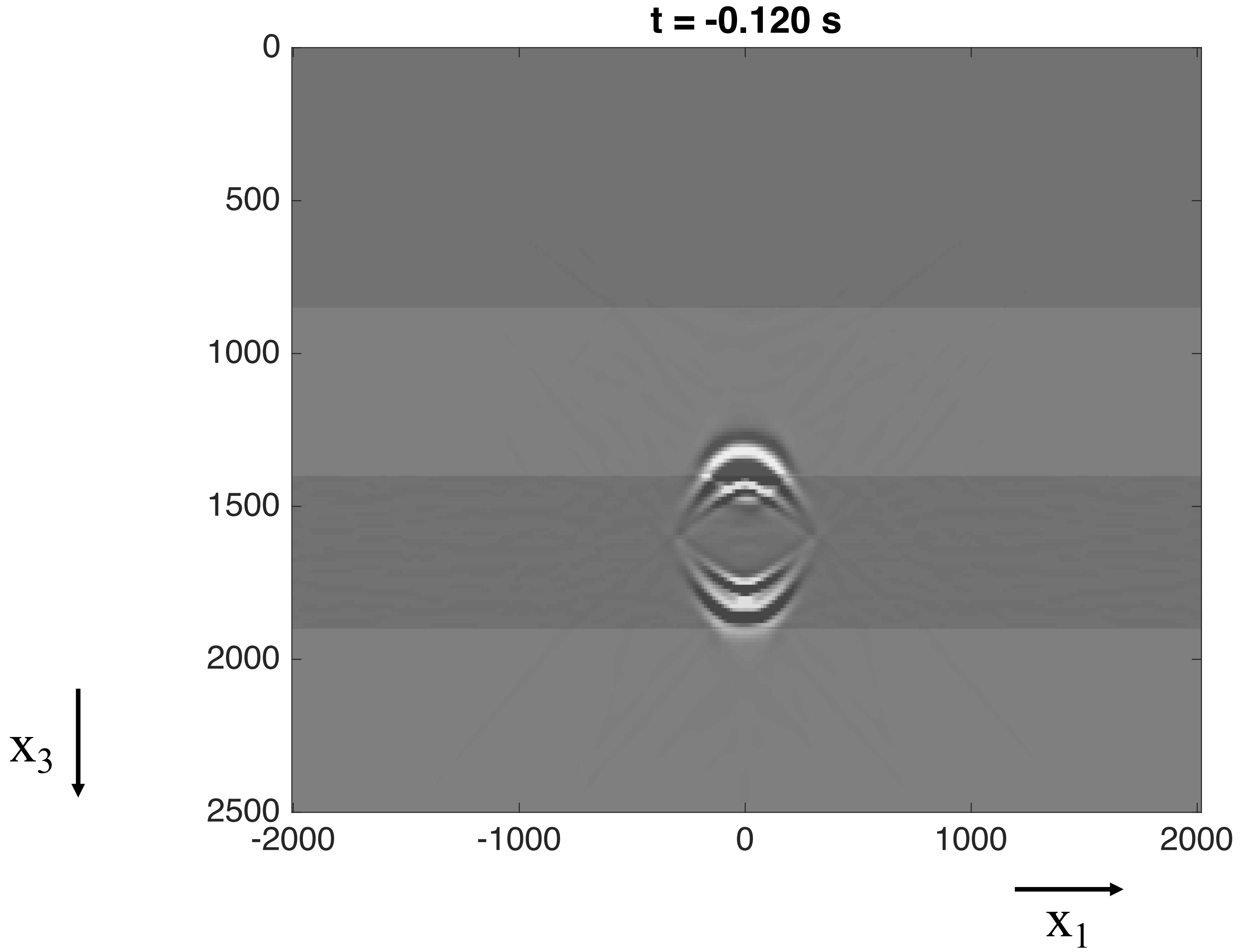
Homogeneous Green's function



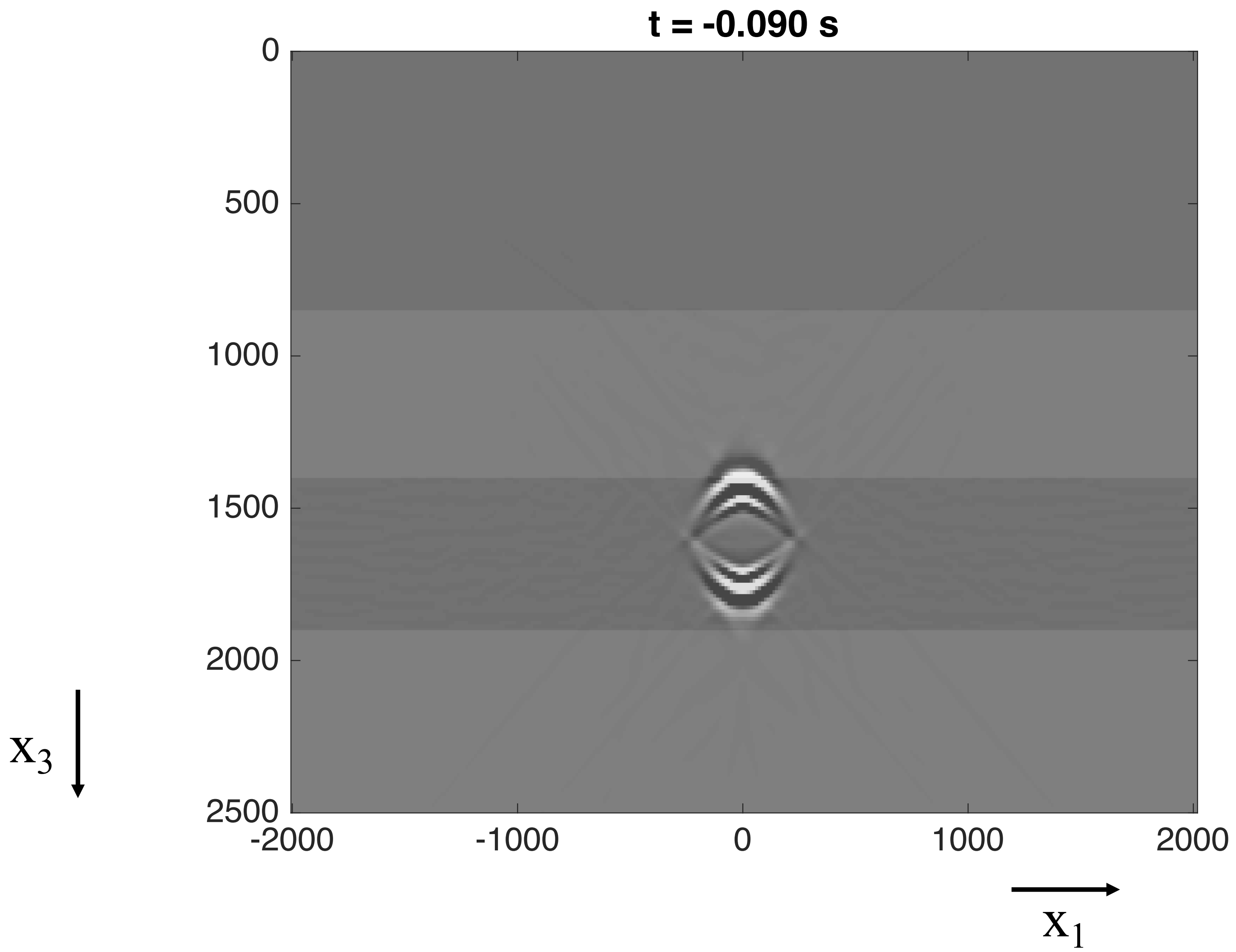
Homogeneous Green's function



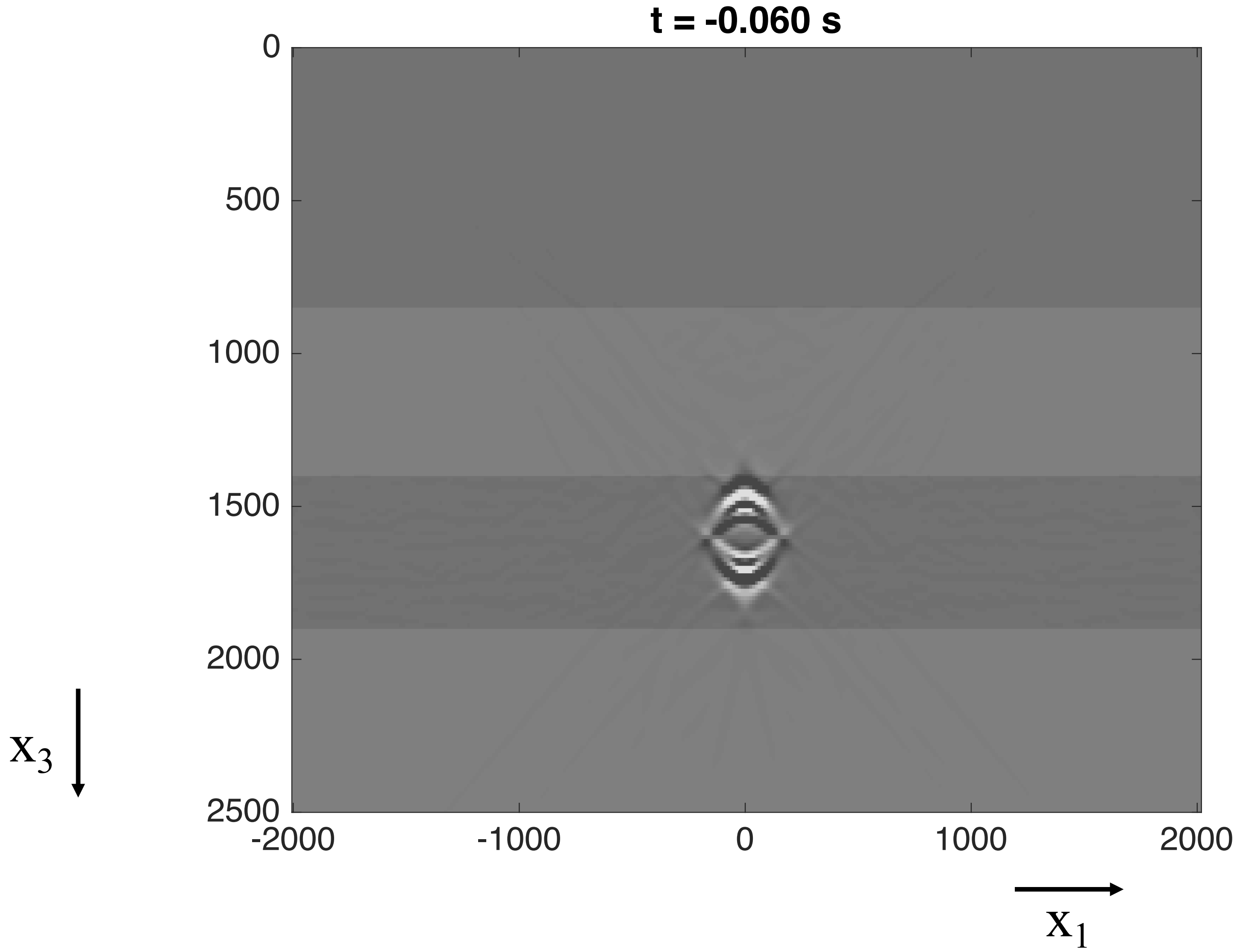
Homogeneous Green's function



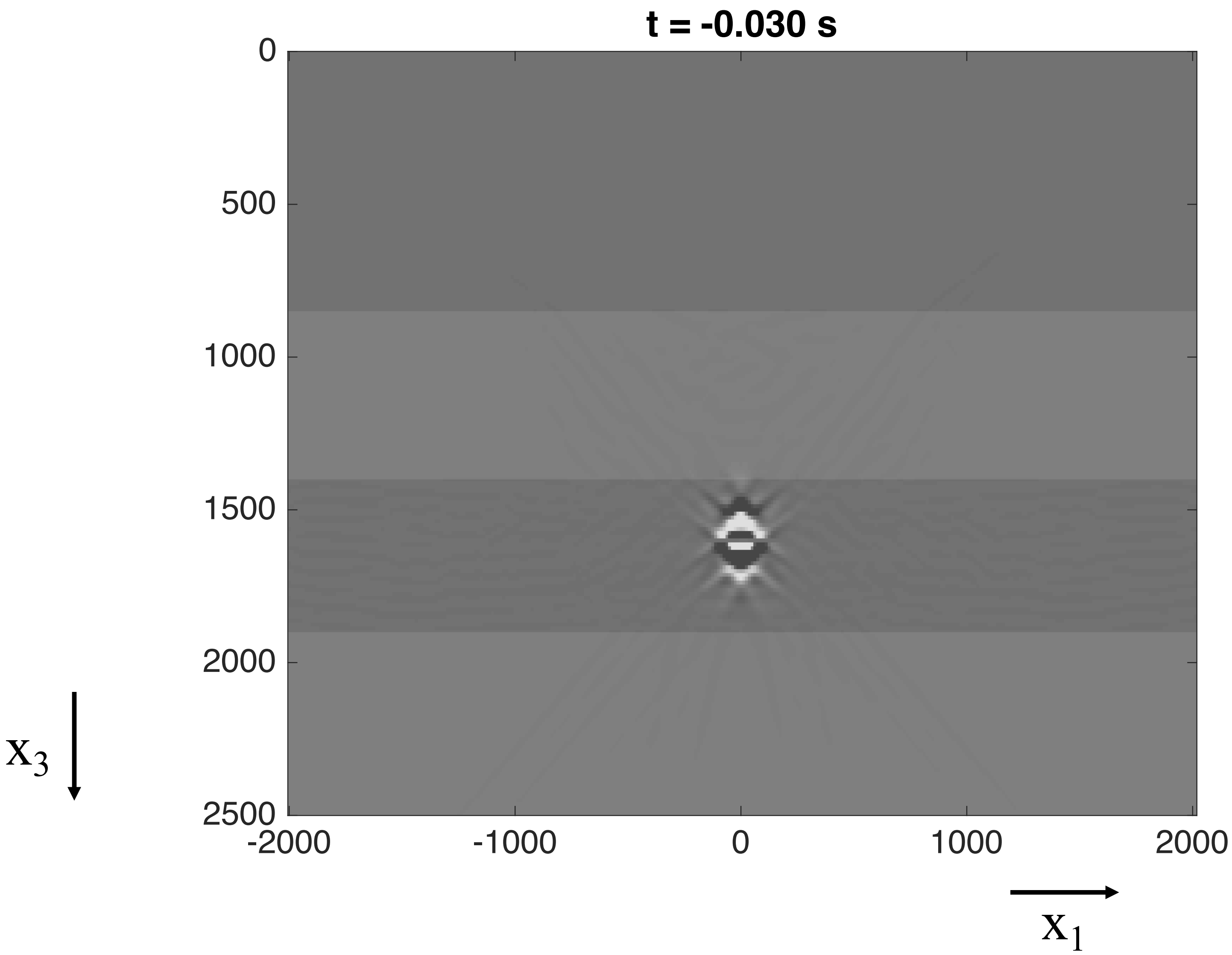
Homogeneous Green's function



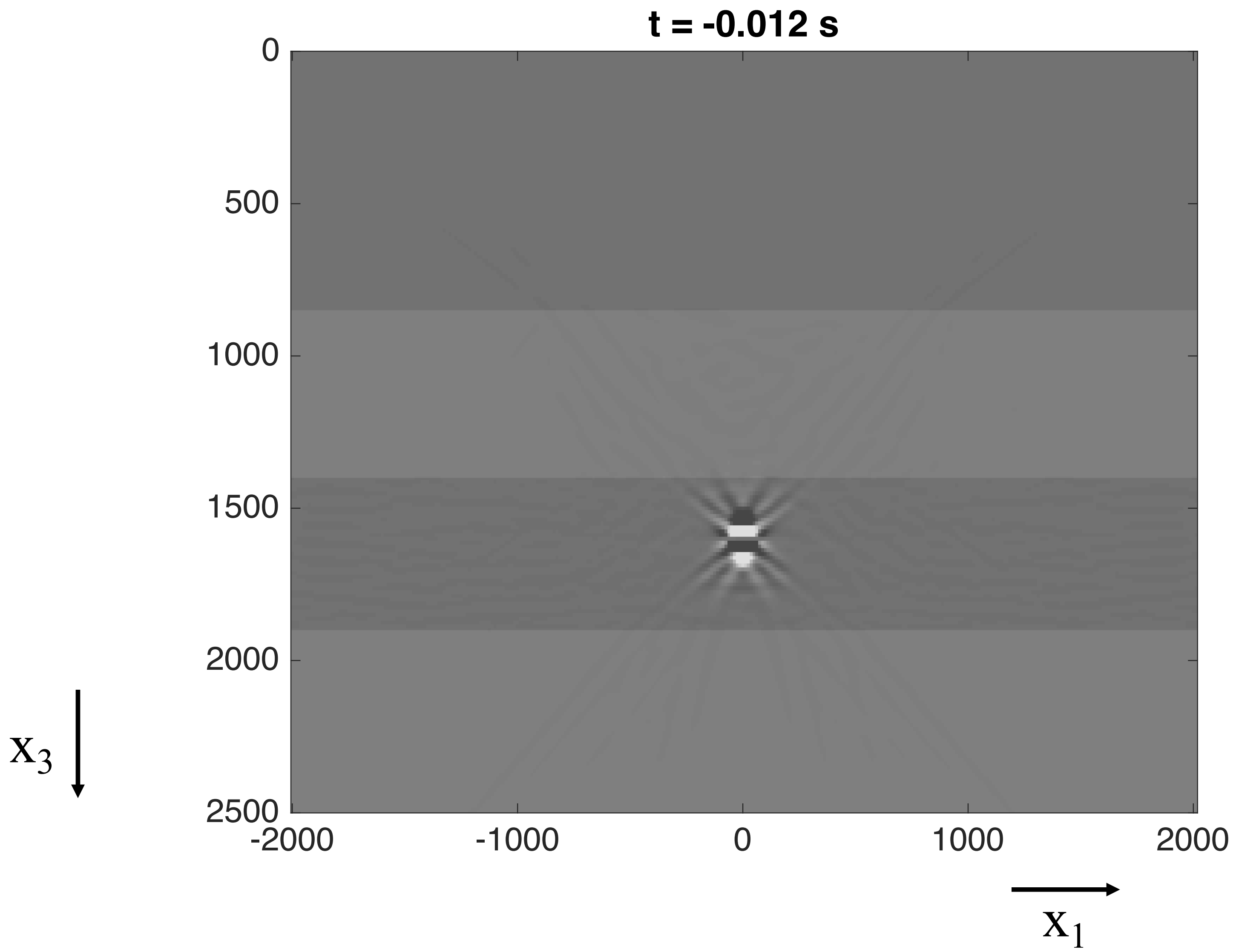
Homogeneous Green's function



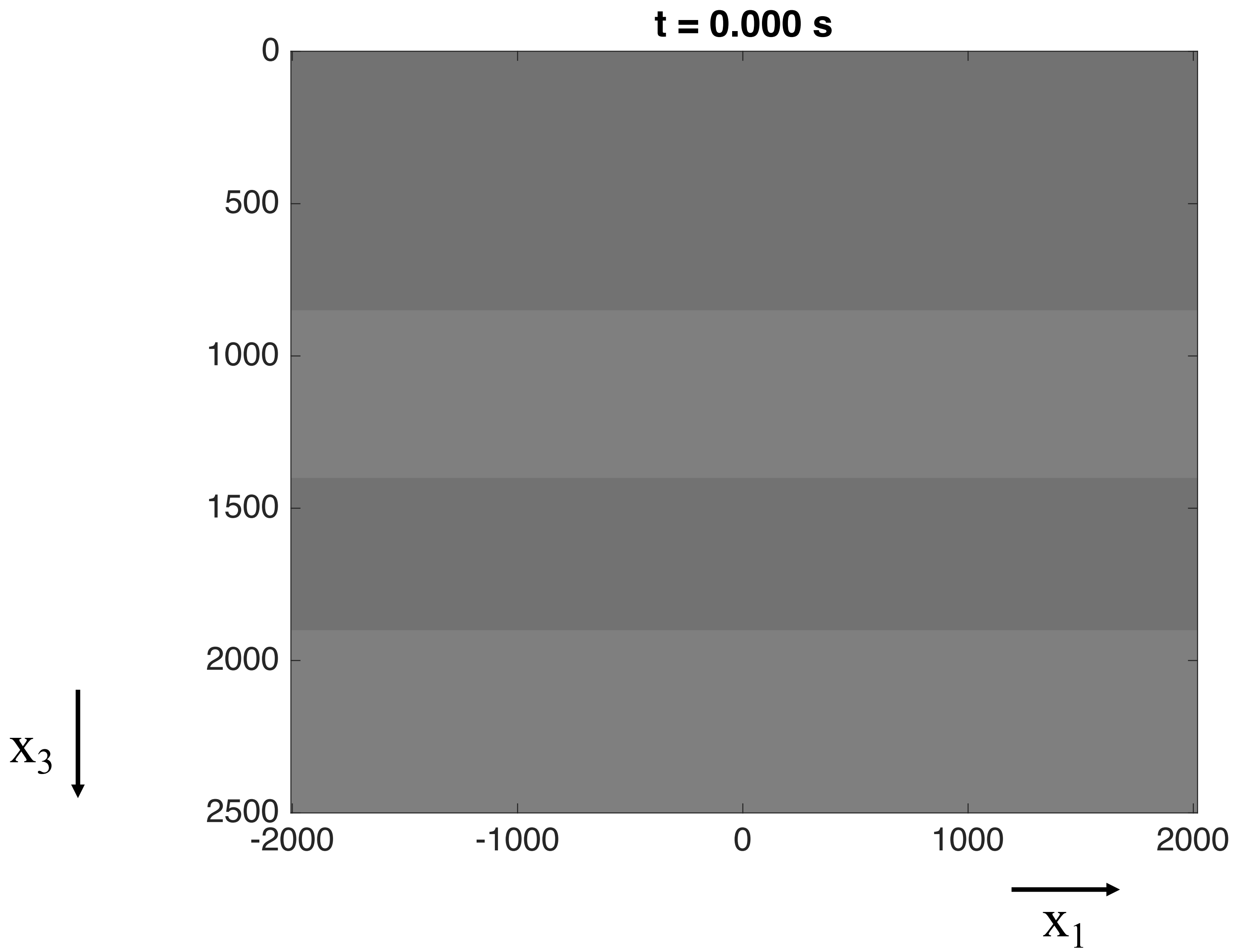
Homogeneous Green's function



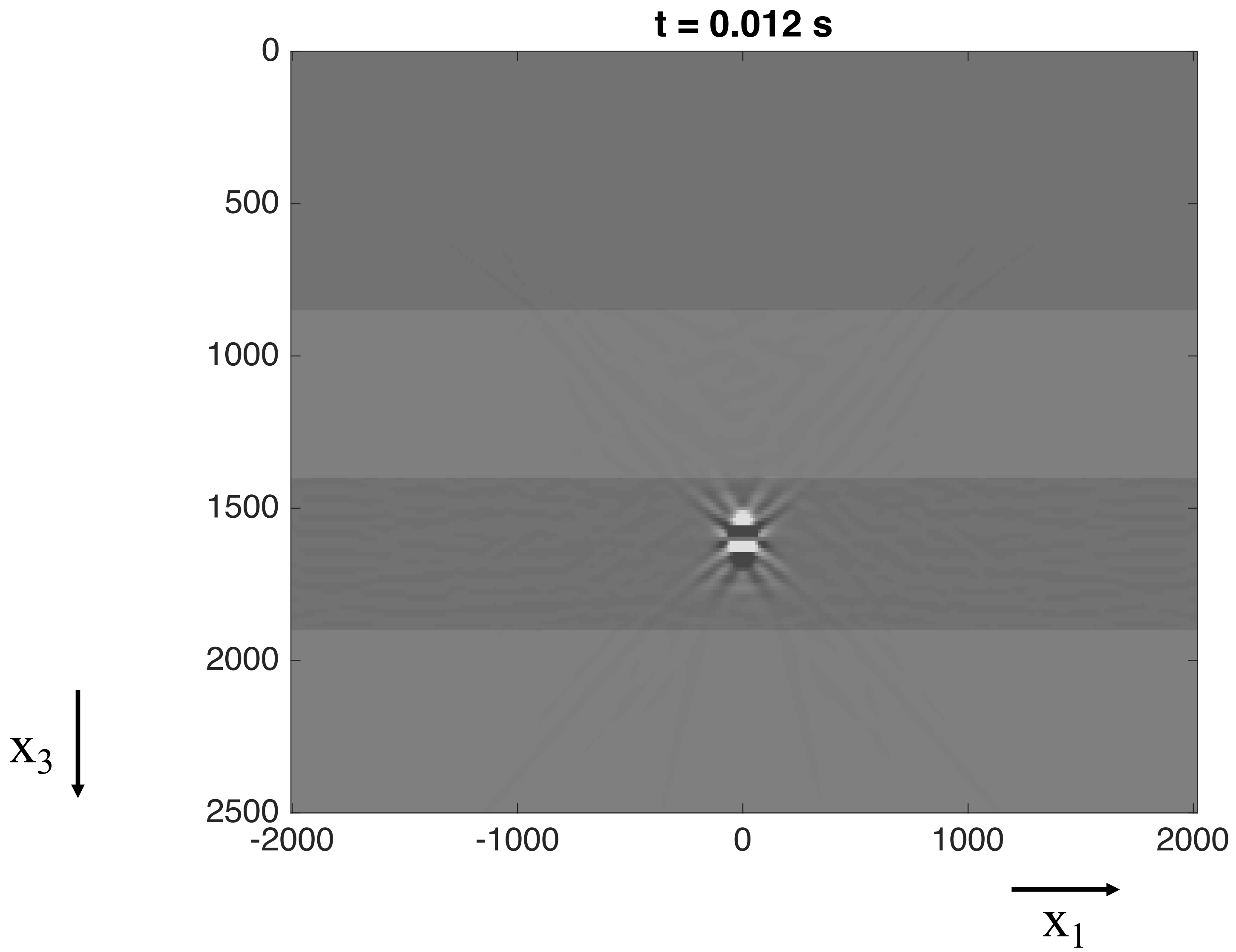
Homogeneous Green's function



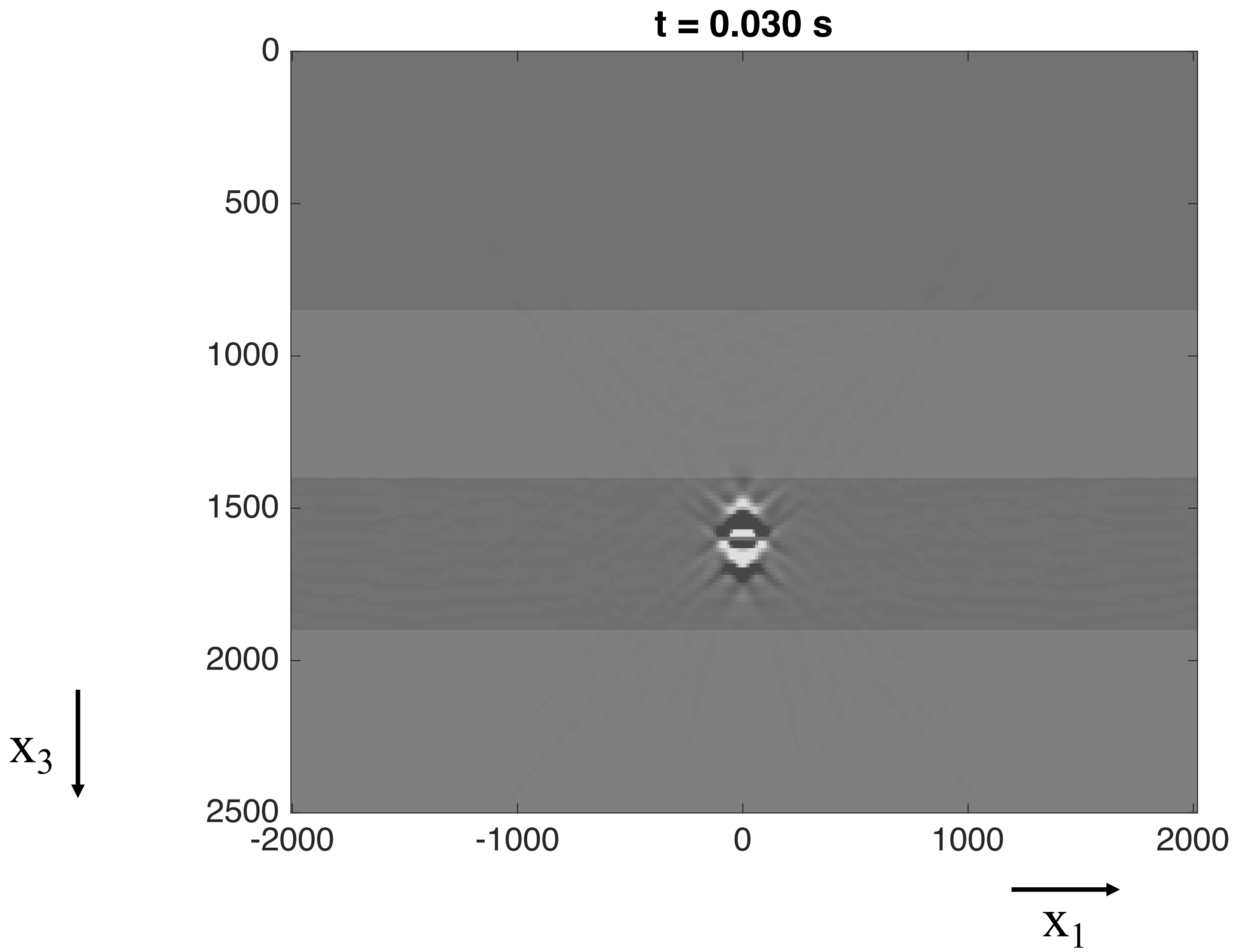
Homogeneous Green's function



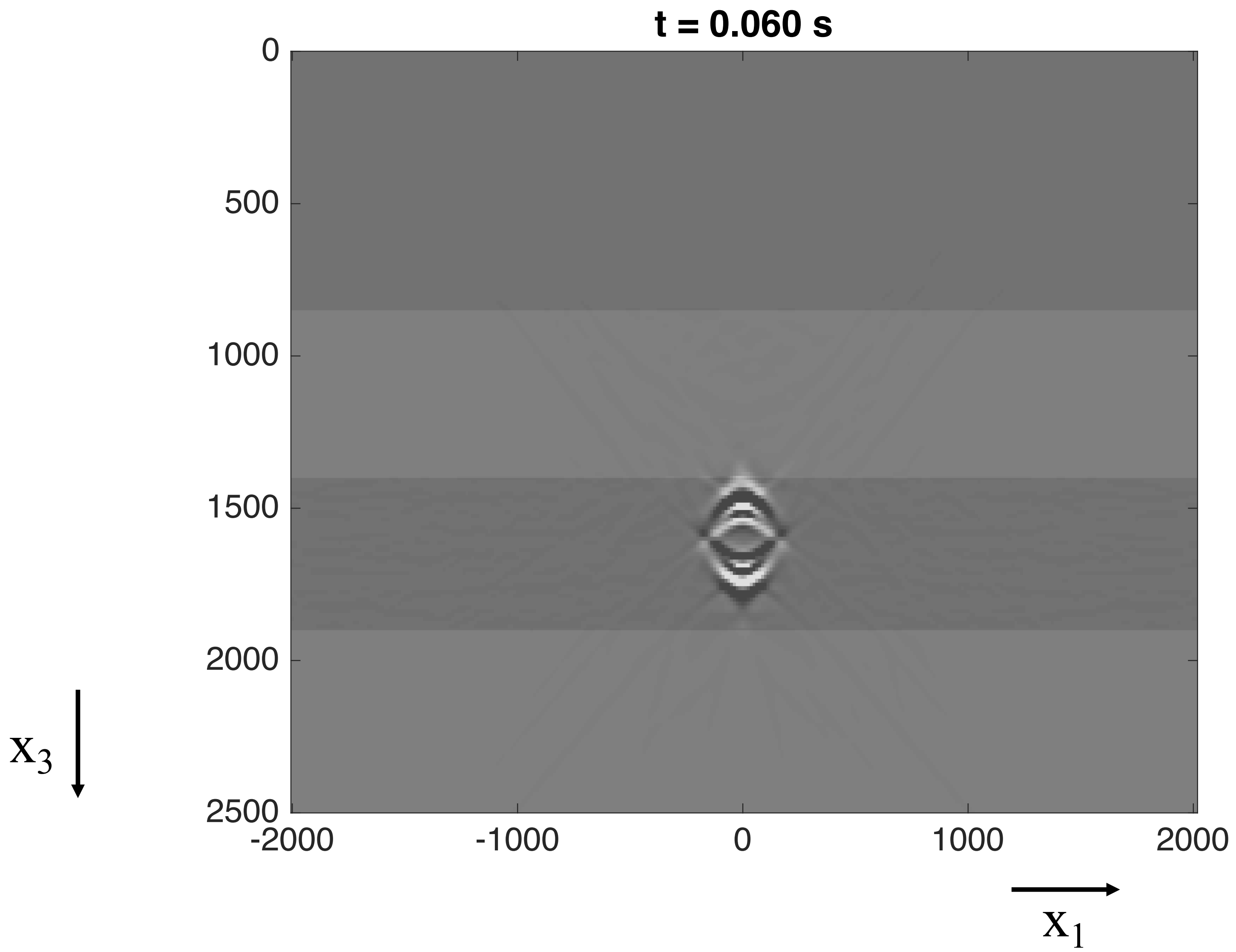
Homogeneous Green's function



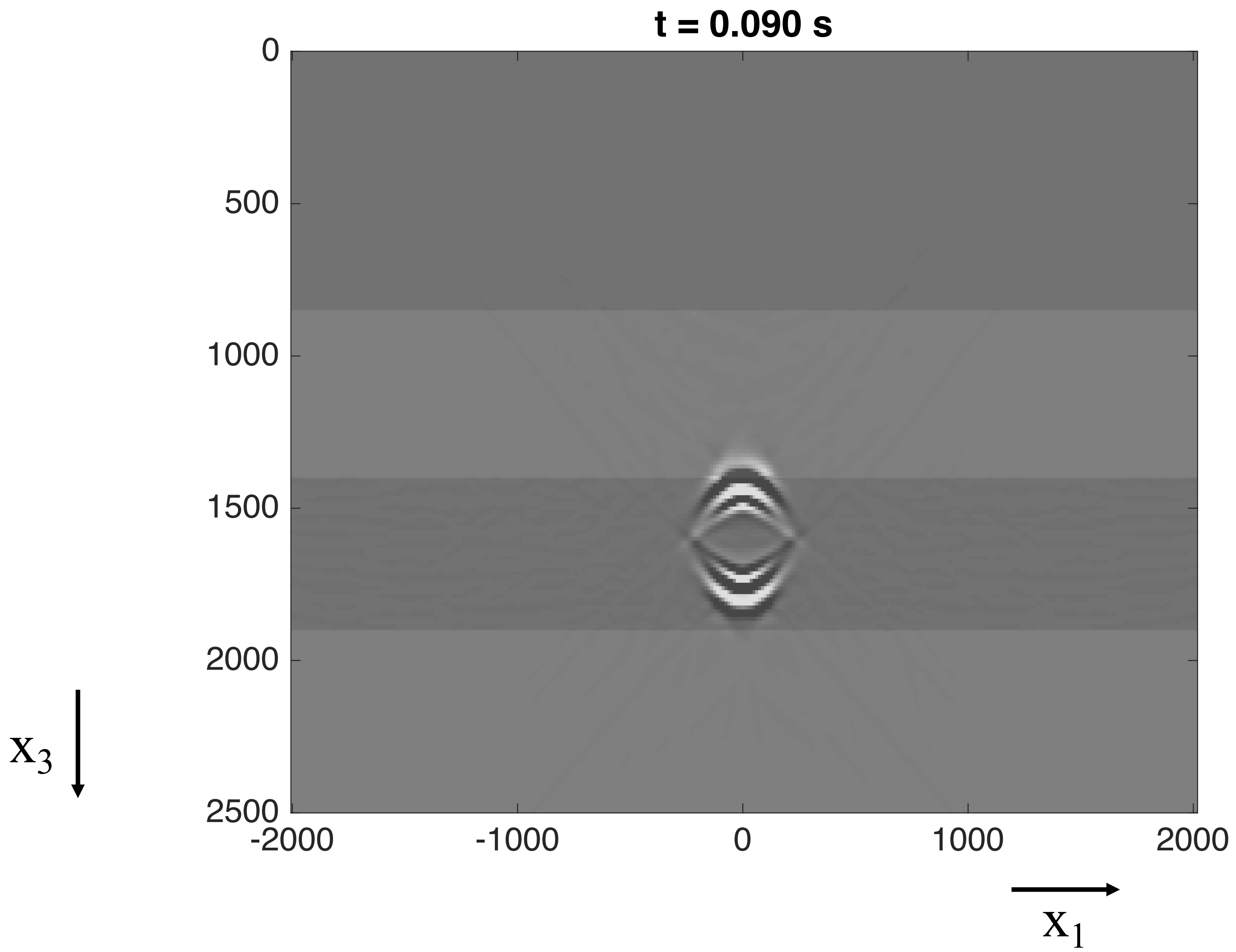
Homogeneous Green's function



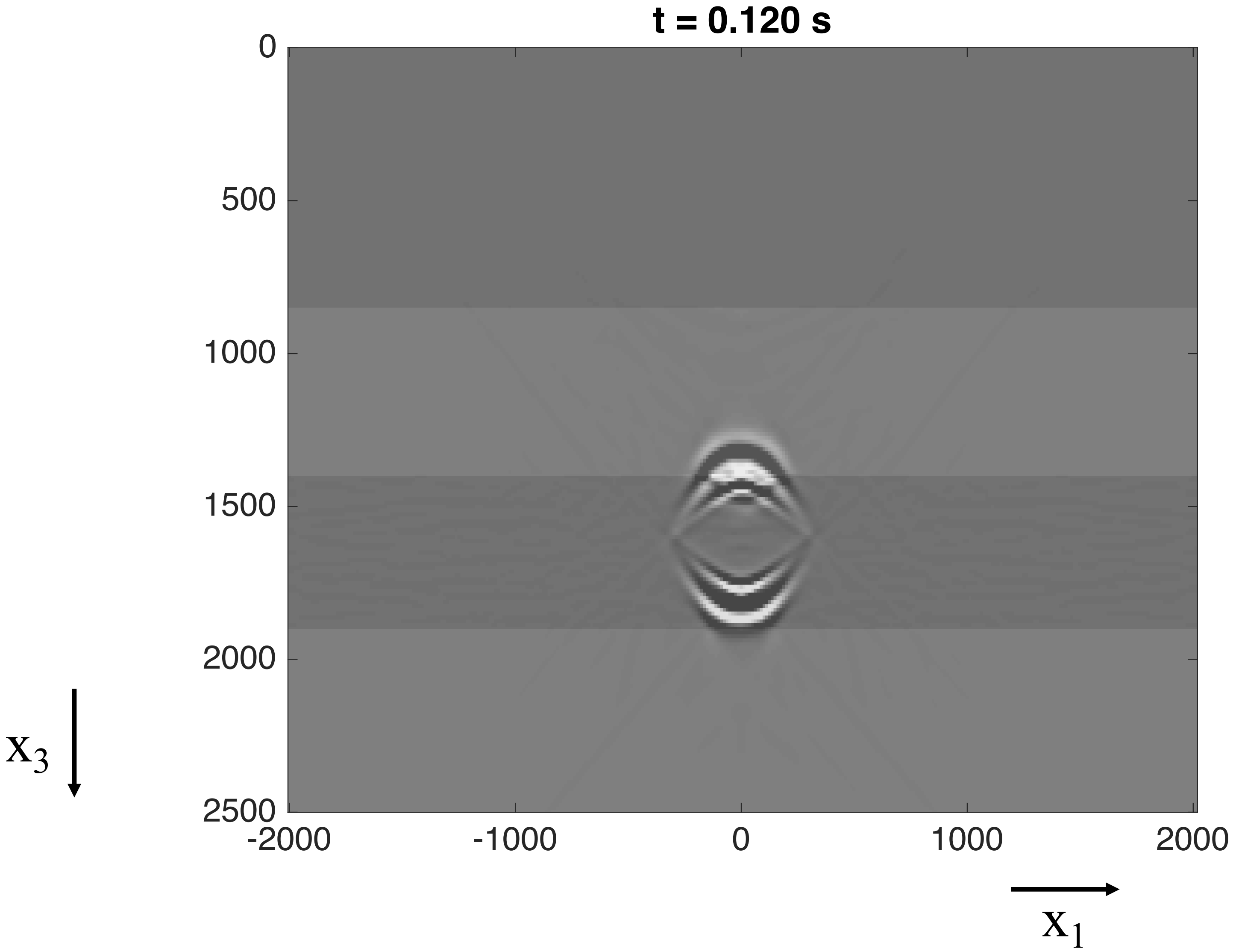
Homogeneous Green's function



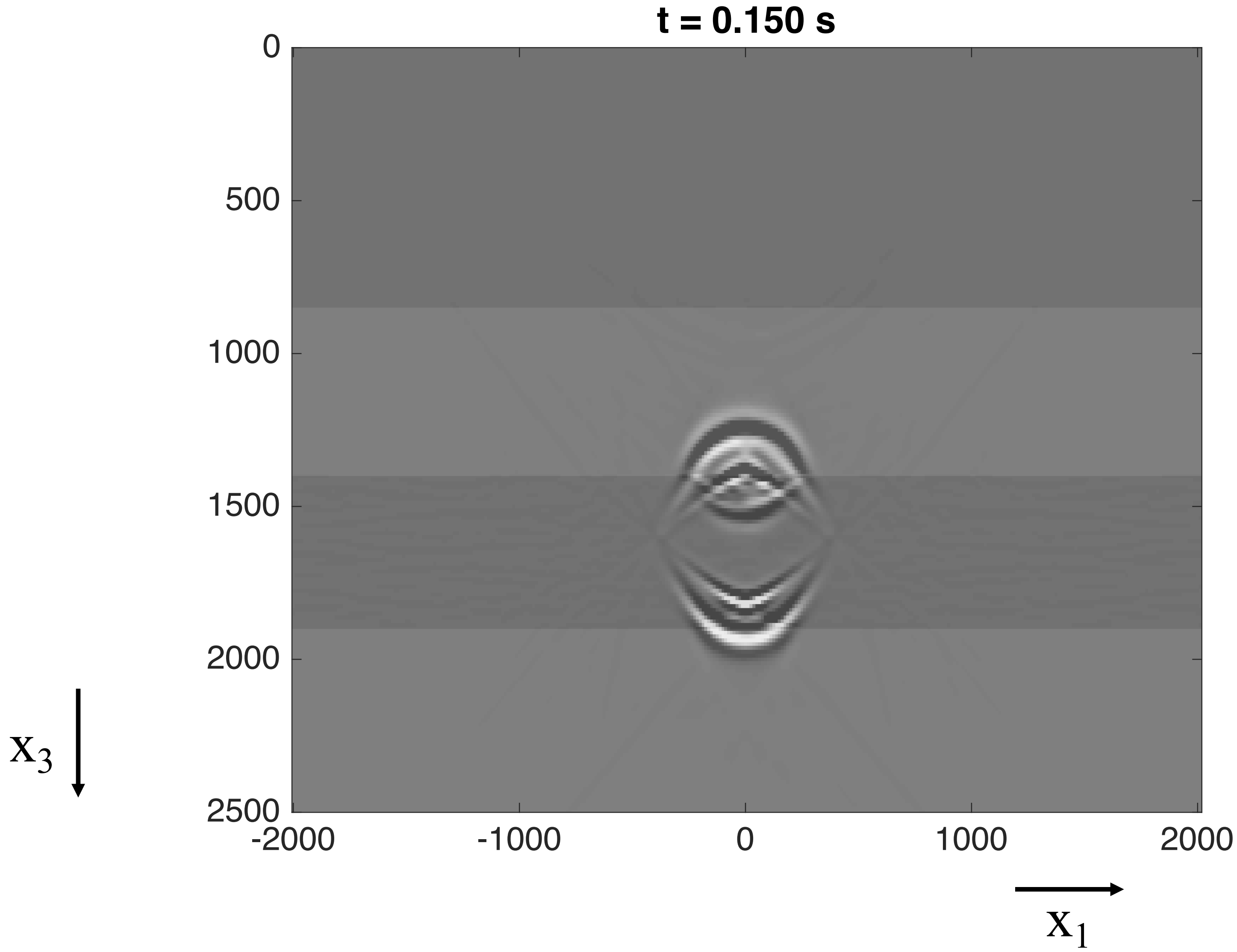
Homogeneous Green's function



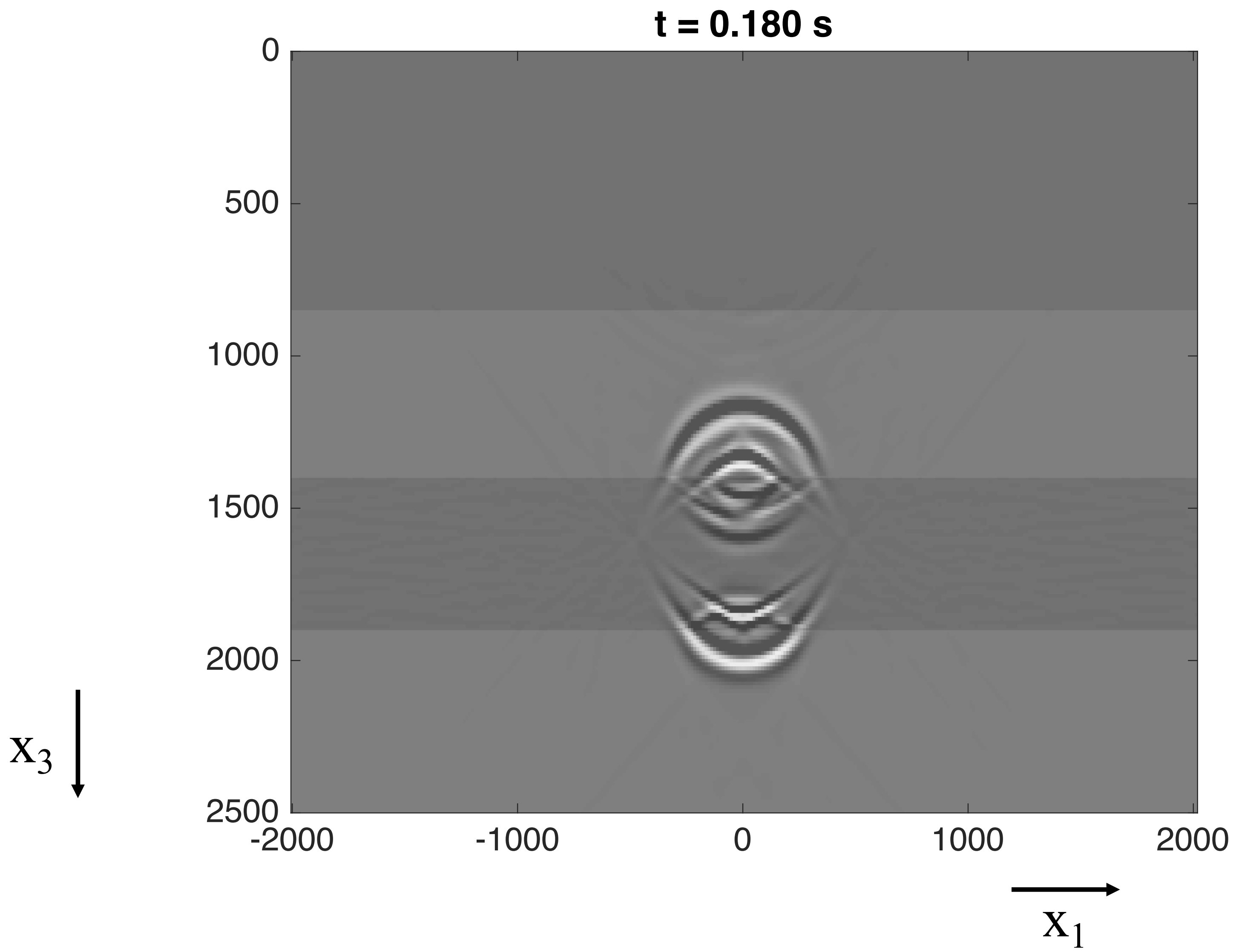
Homogeneous Green's function



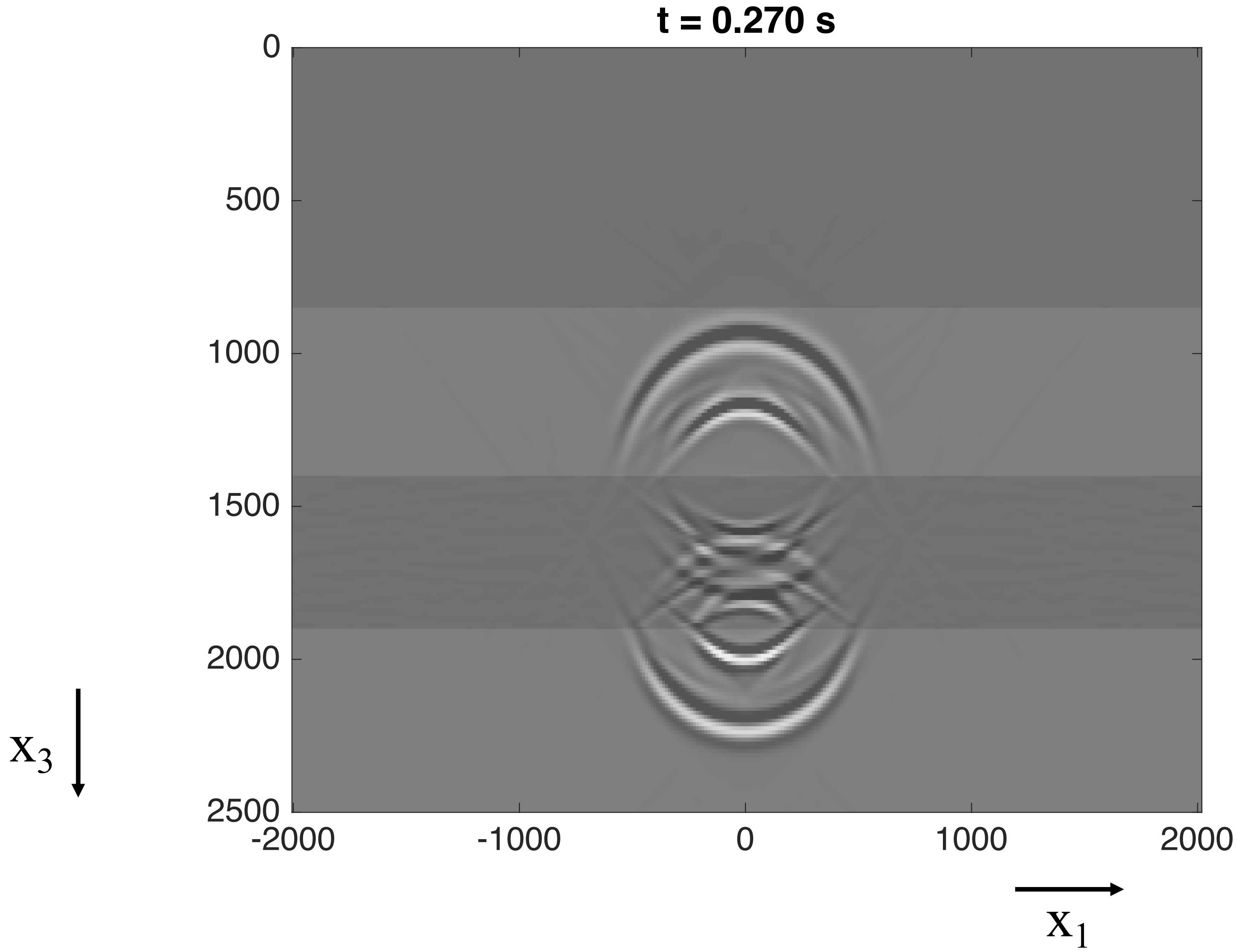
Homogeneous Green’s function



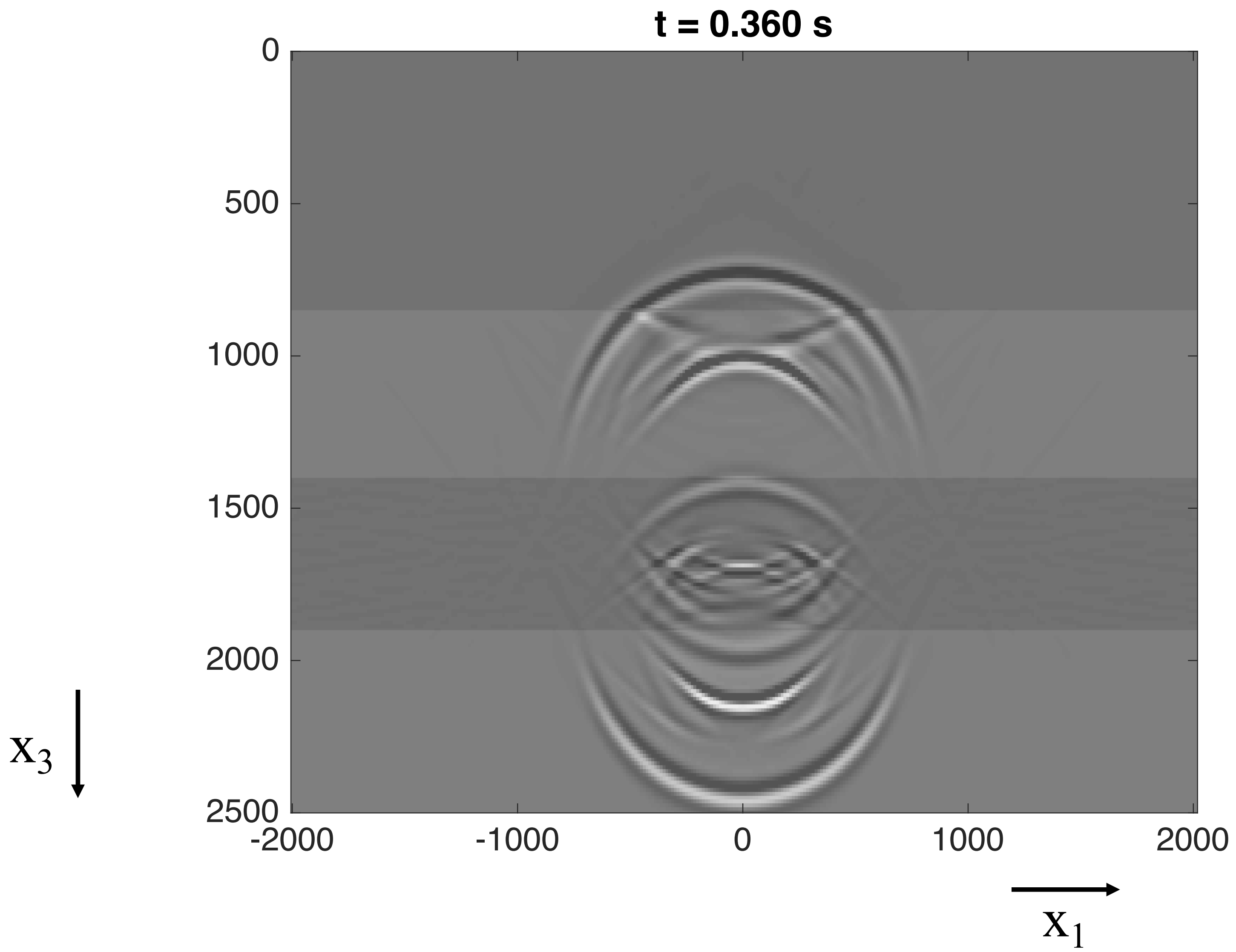
Homogeneous Green's function



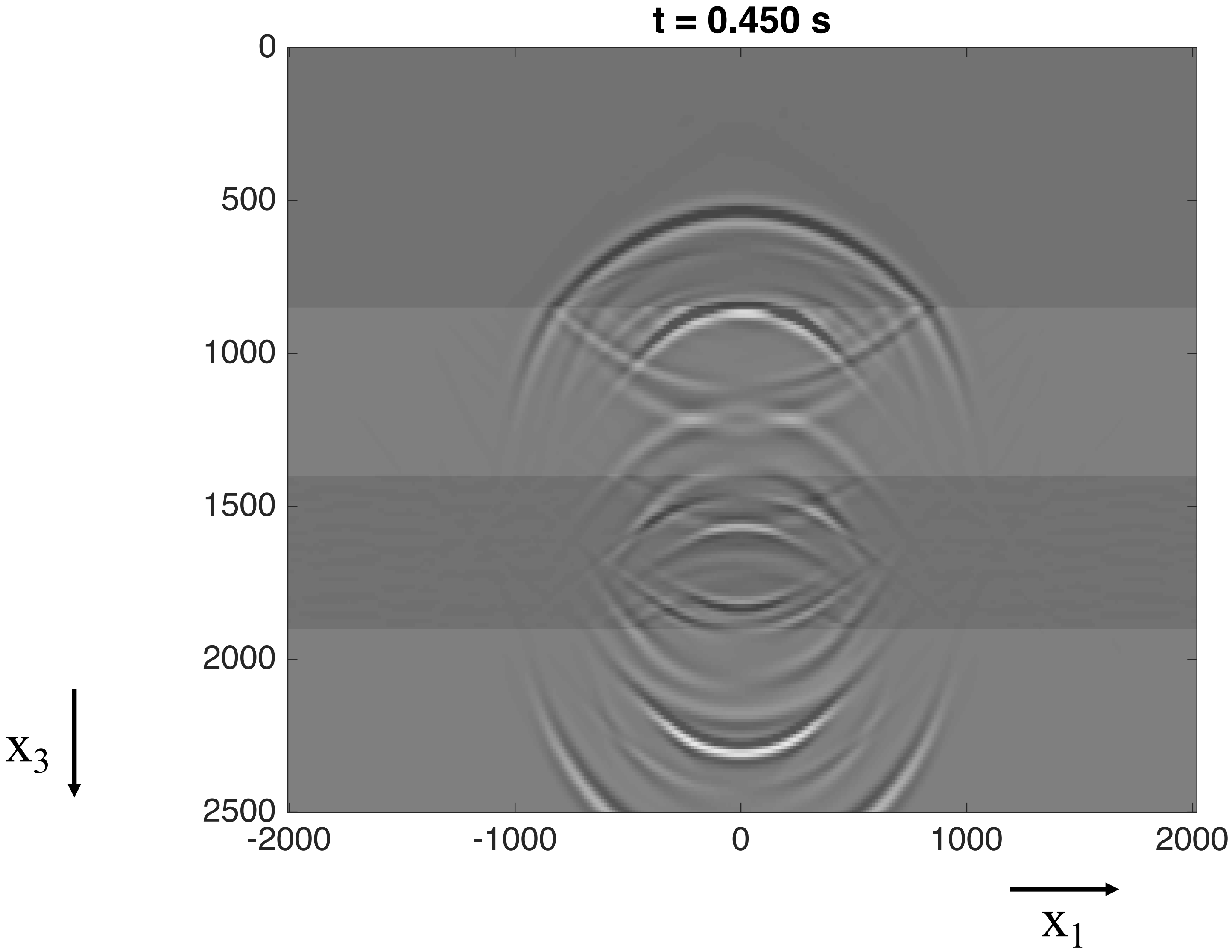
Homogeneous Green's function



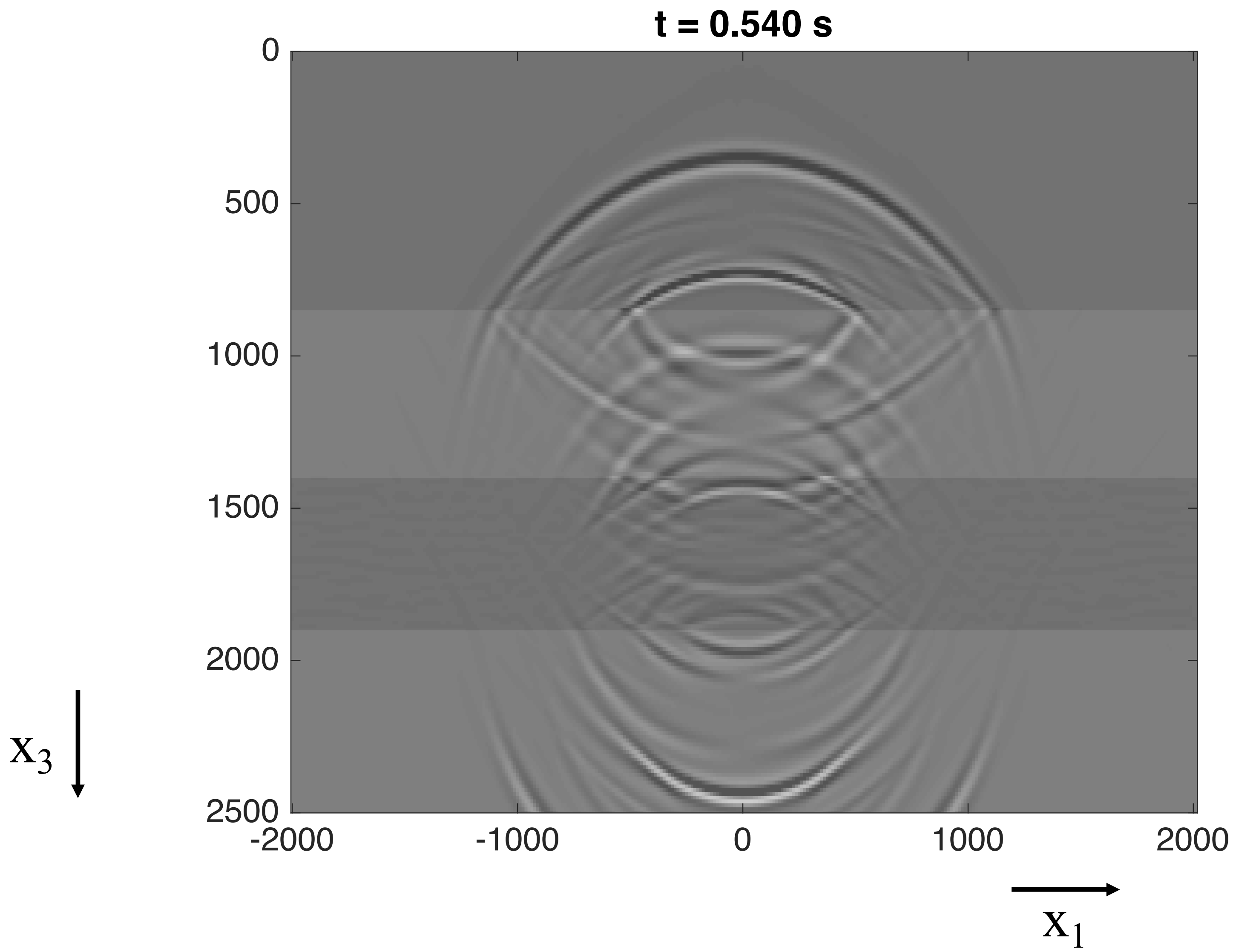
Homogeneous Green's function



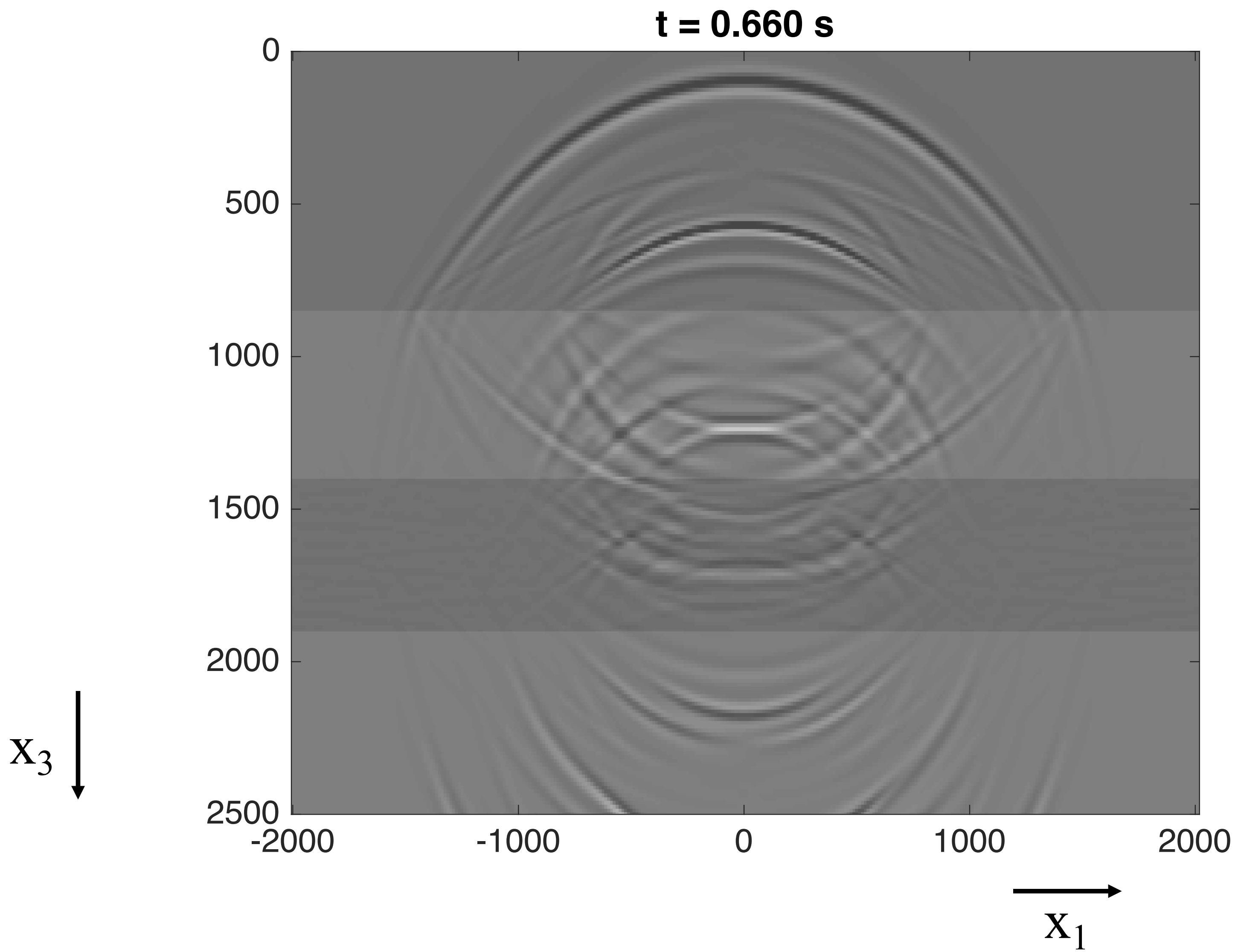
Homogeneous Green's function



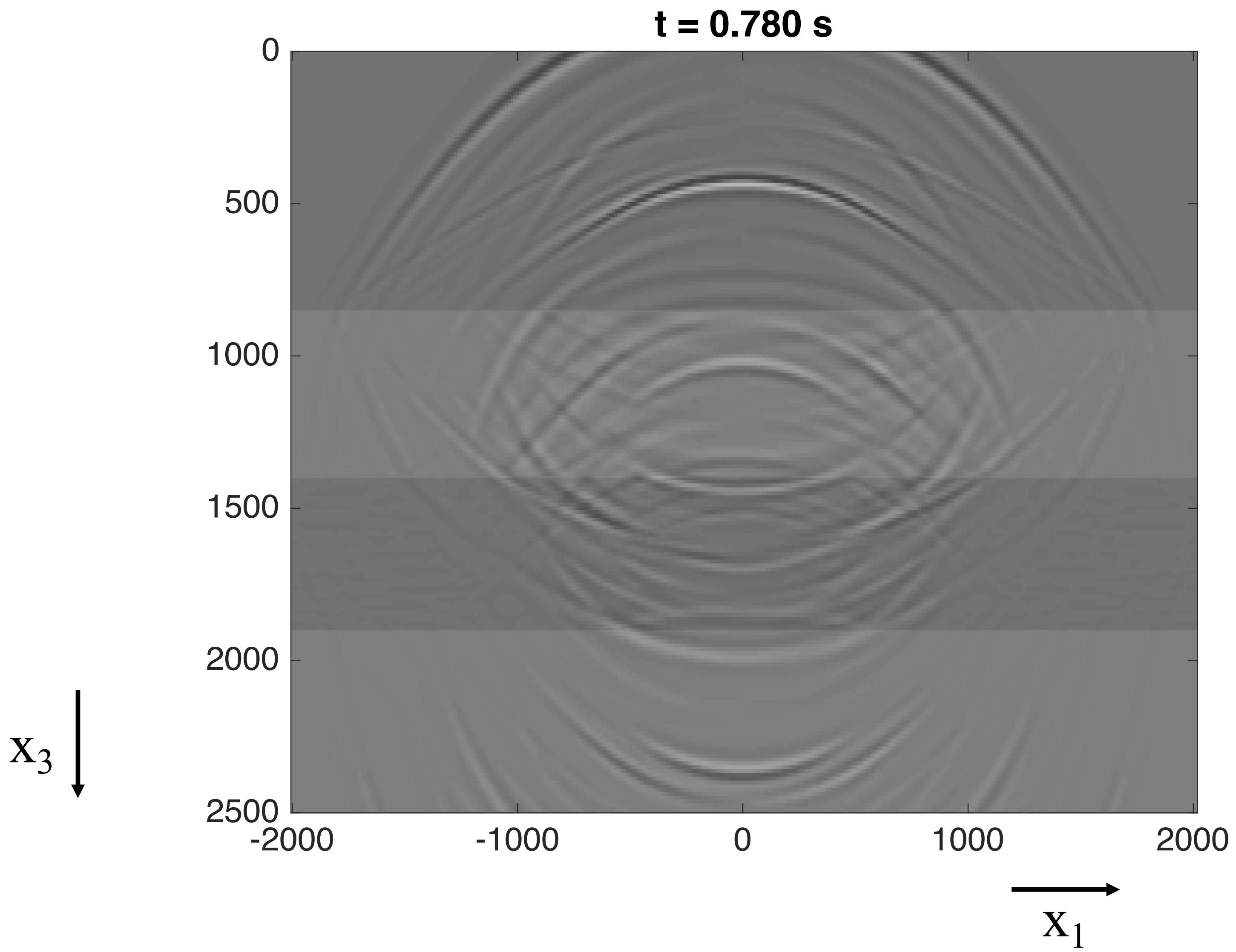
Homogeneous Green's function



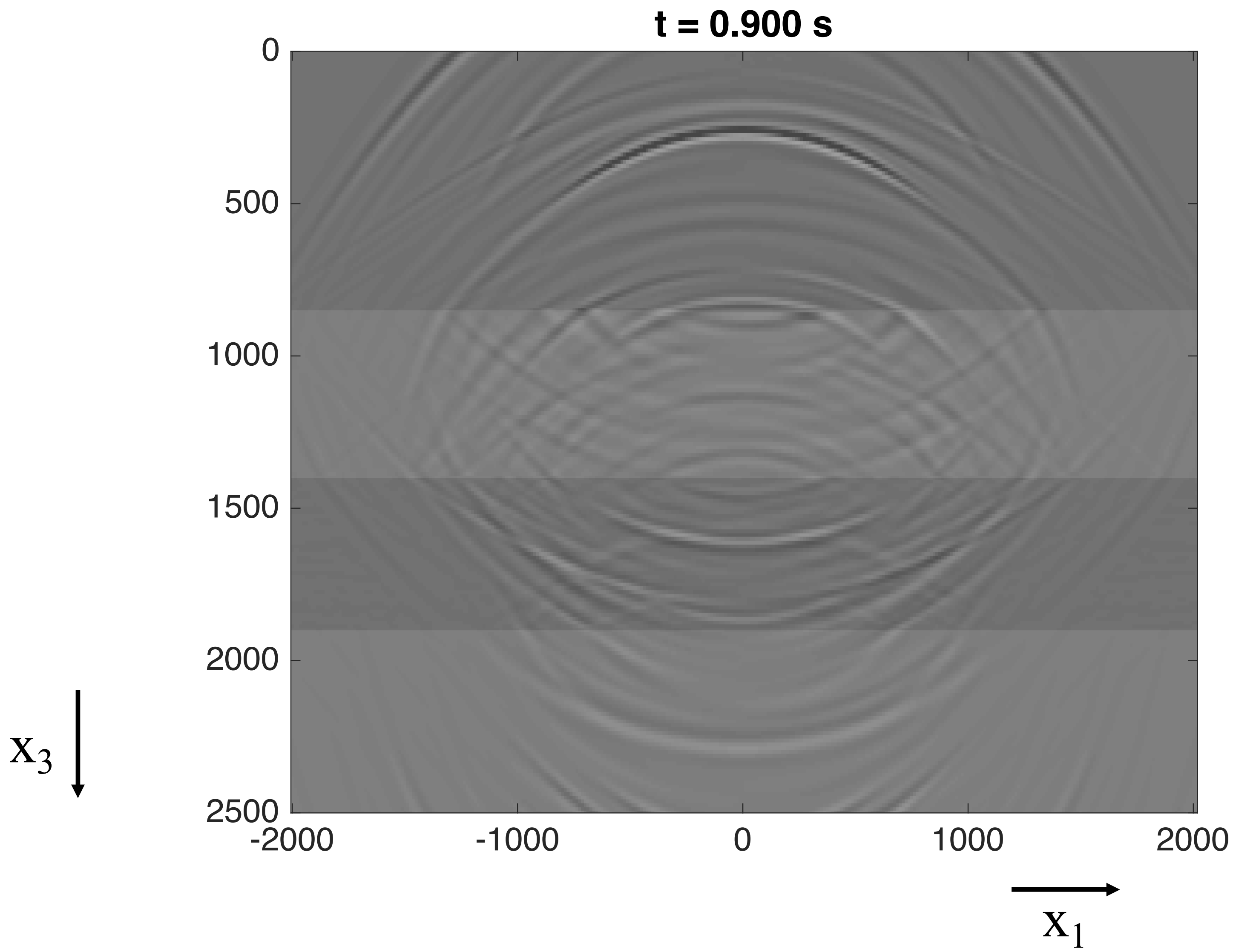
Homogeneous Green's function



Homogeneous Green's function



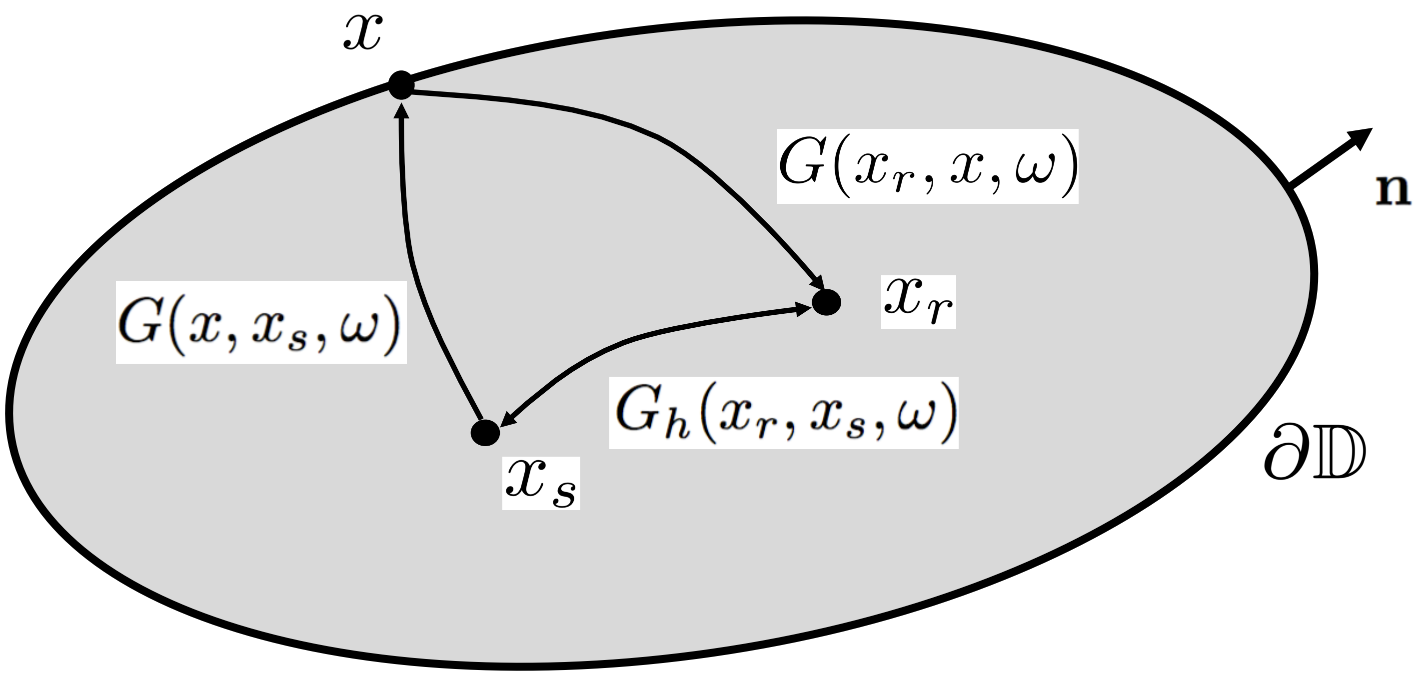
Homogeneous Green's function



Contents

1. Introduction
2. Homogeneous Green's function
3. **Elastodynamic double-sided homogeneous Green's function representation**
4. Elastodynamic single-sided homogeneous Green's function representation
5. Conclusions

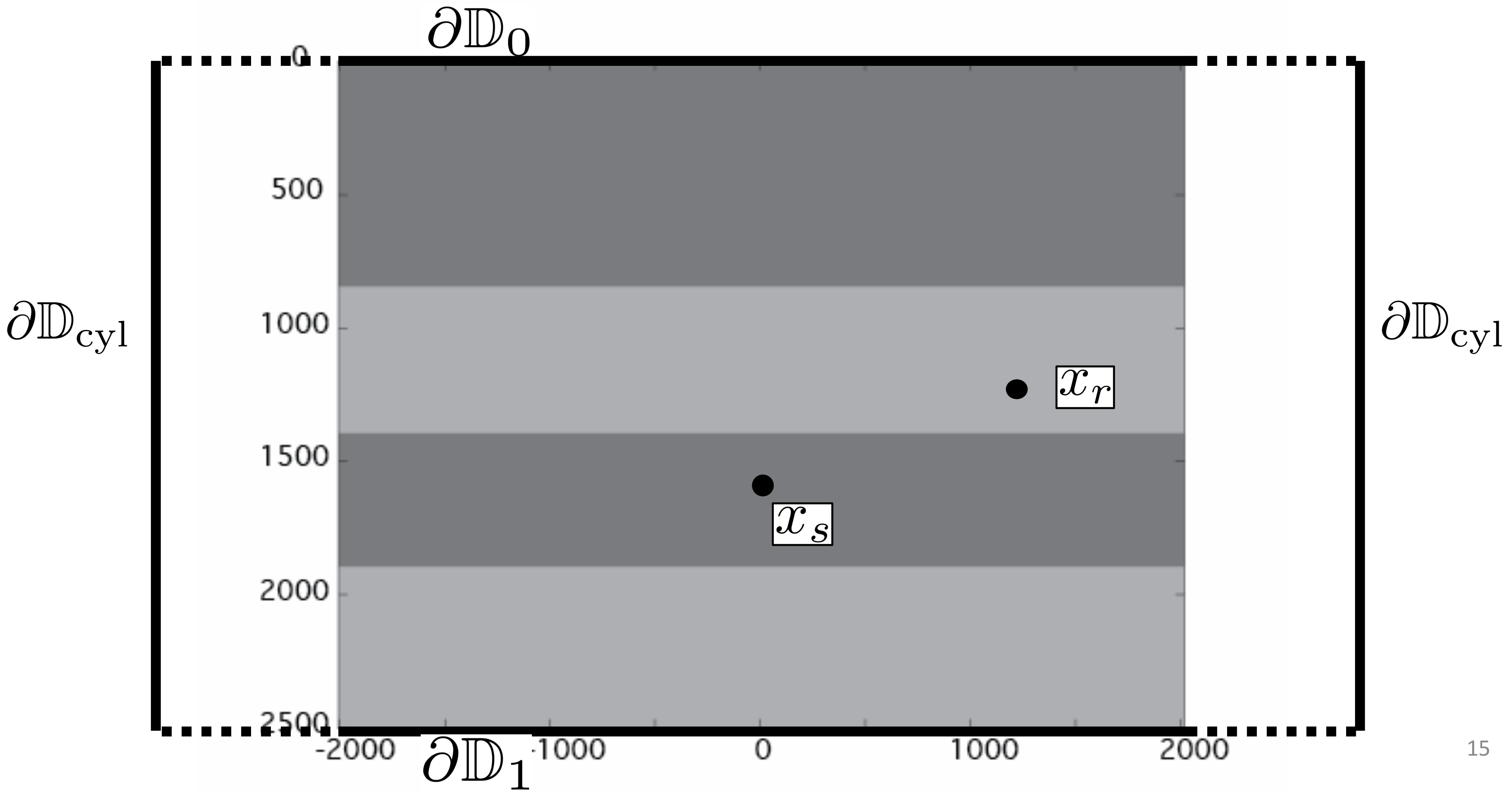
Acoustic homogeneous Green's function representation



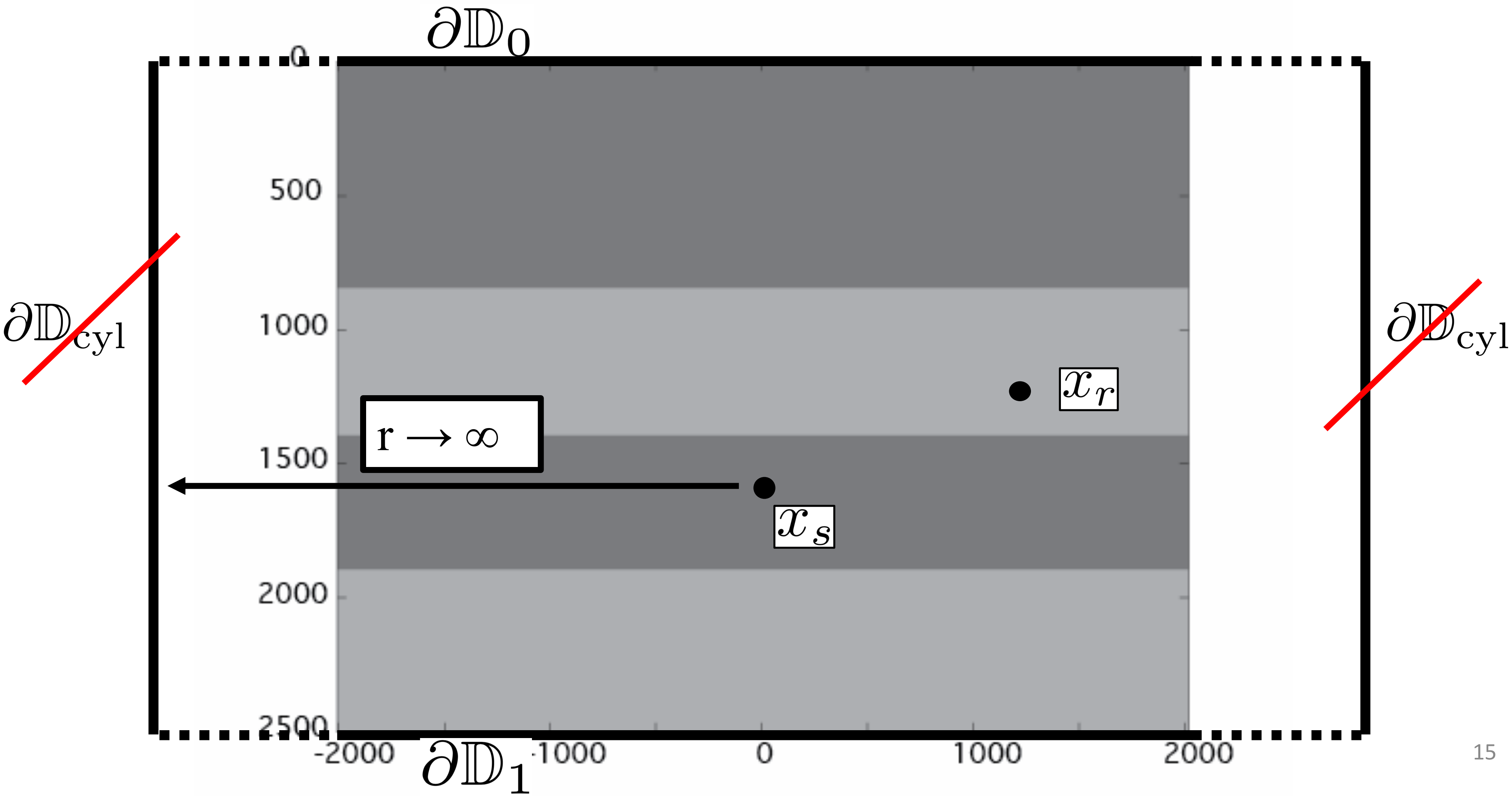
$$G_h(x_r, x_s, \omega) = \int_{\partial\mathbb{D}} \{G^*(x_r, x, \omega)\nabla G(x, x_s, \omega) - \nabla G^*(x_r, x, \omega) G(x, x_s, \omega)\} \cdot \mathbf{n}\, \mathrm{d}^2 x$$

(Porter, Robert P. "Diffraction-Limited, Scalar Image Formation with Holograms of Arbitrary Shape " (1970) & Oristaglio, Michael L. "An inverse scattering formula that uses all the data." *Inverse Problems* 5.6 (1989): 1097.)

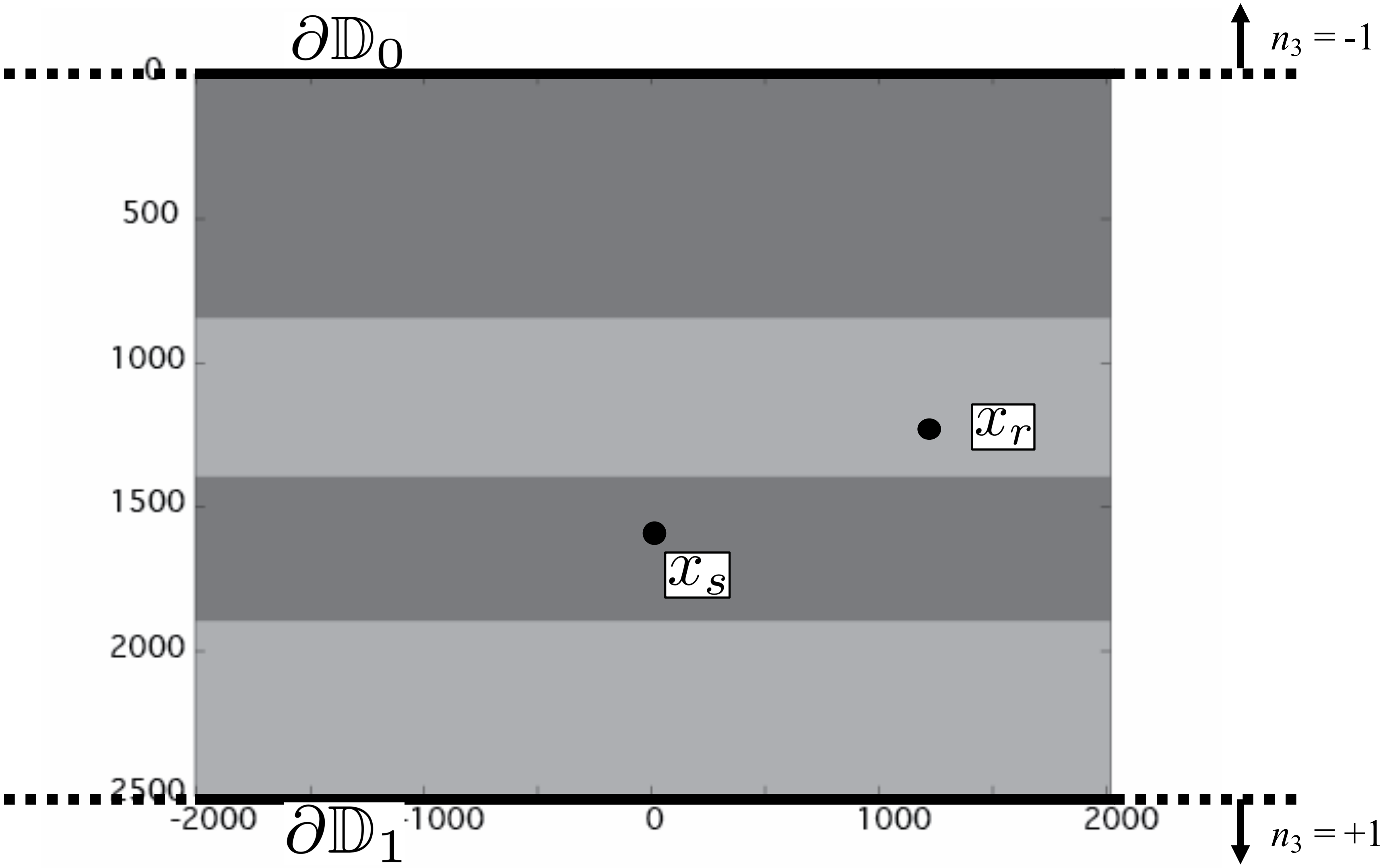
Elastodynamic double-sided homogeneous Green's function representation



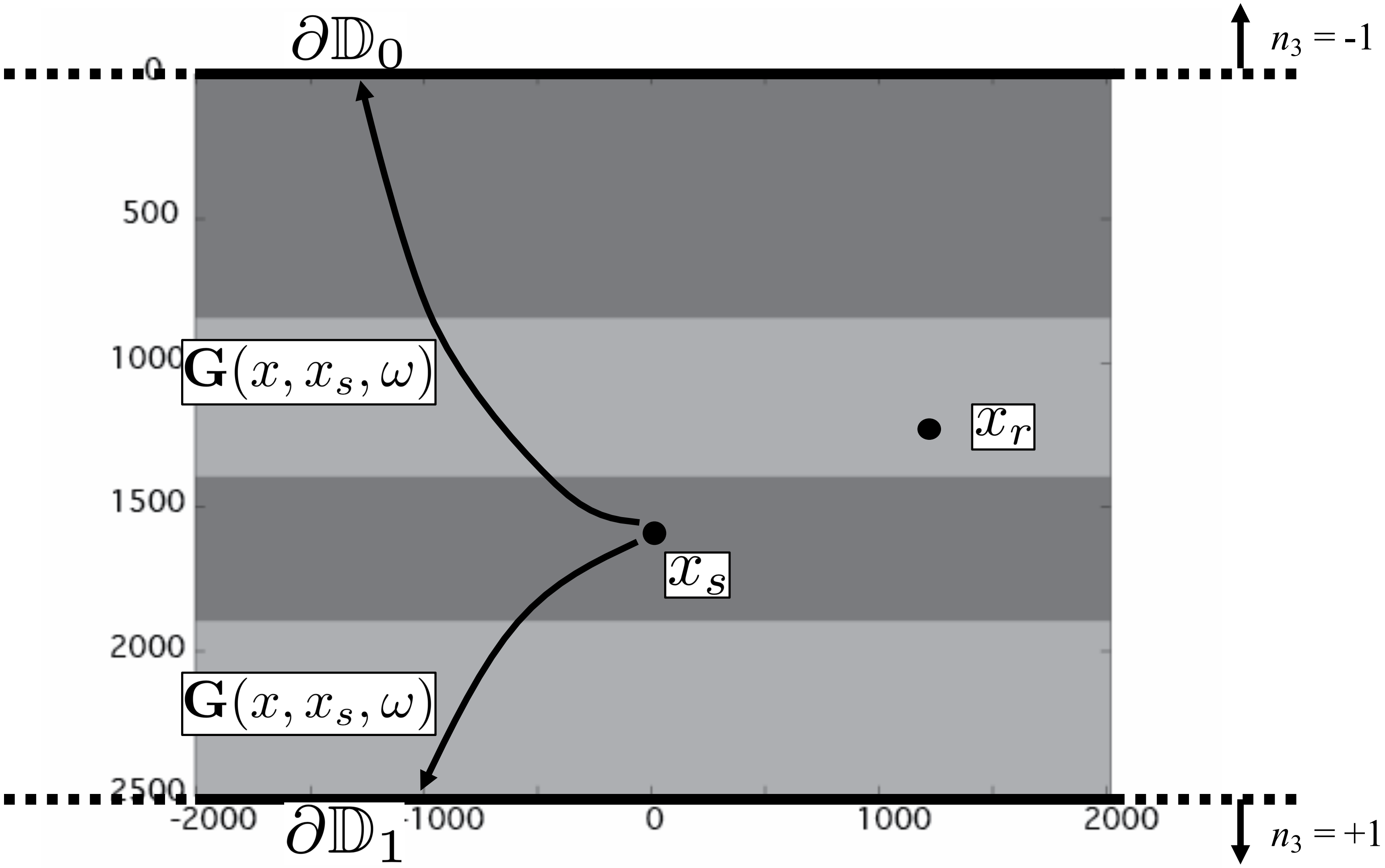
Elastodynamic double-sided homogeneous Green's function representation



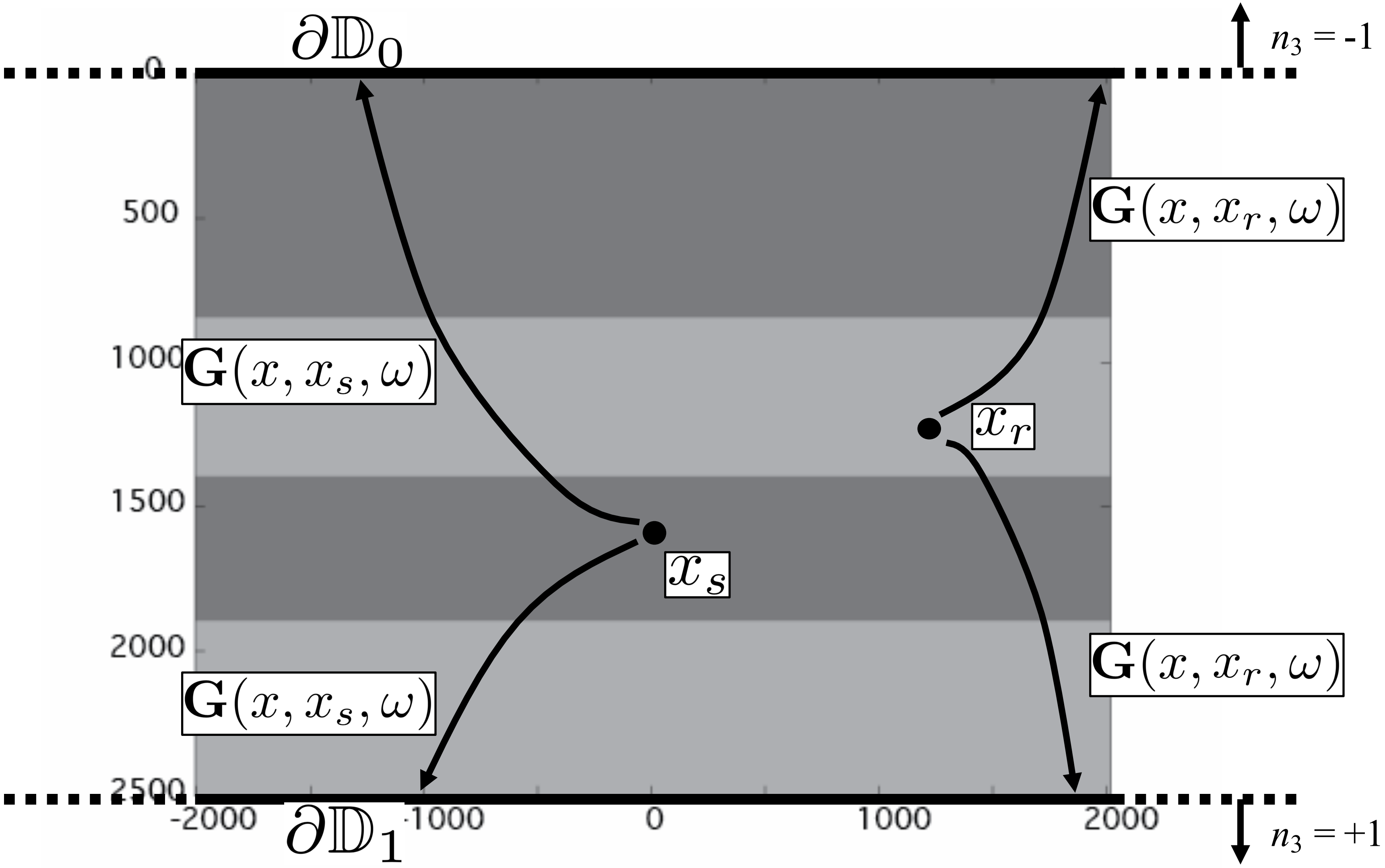
Elastodynamic double-sided homogeneous Green's function representation



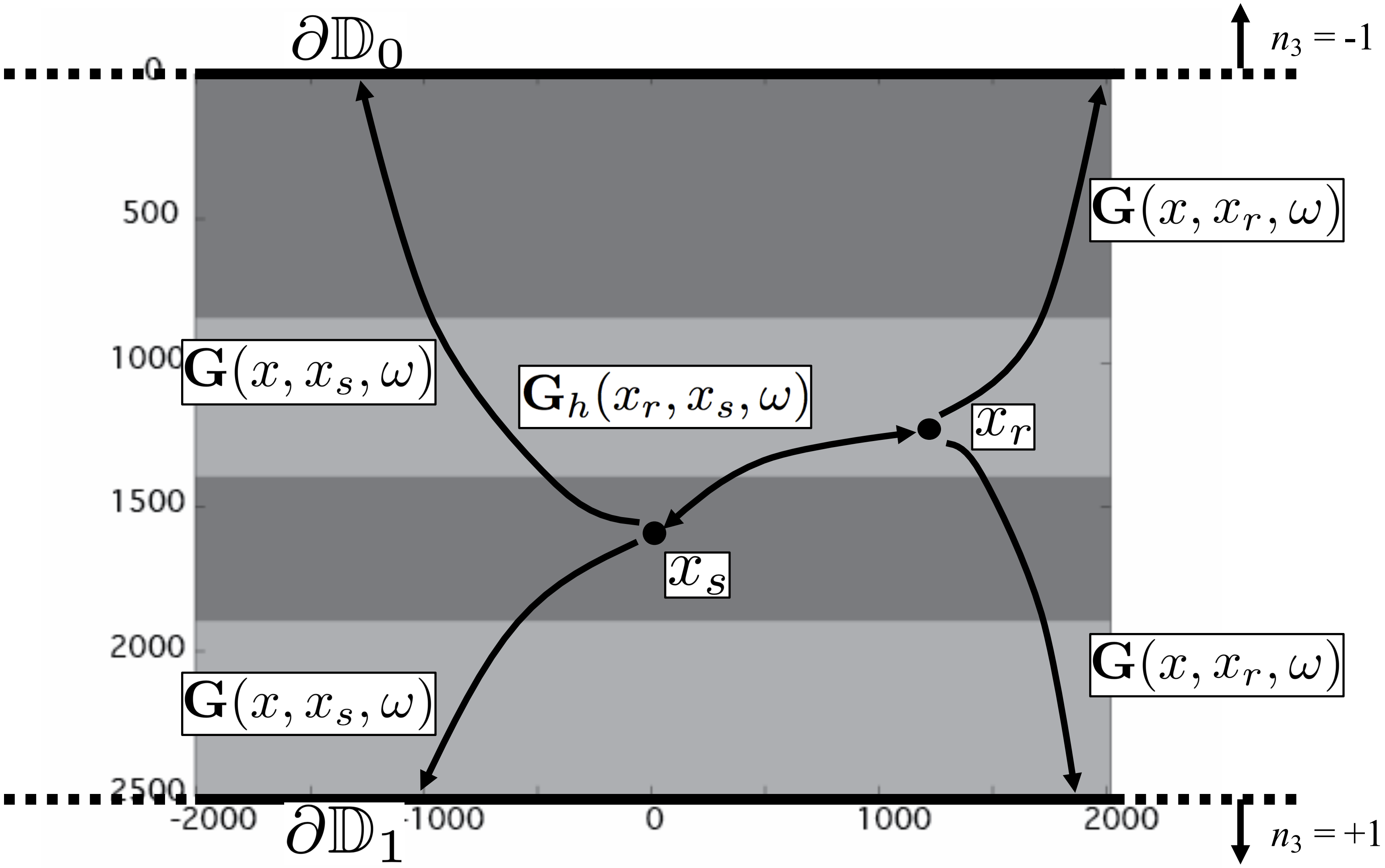
Elastodynamic double-sided homogeneous Green's function representation



Elastodynamic double-sided homogeneous Green's function representation

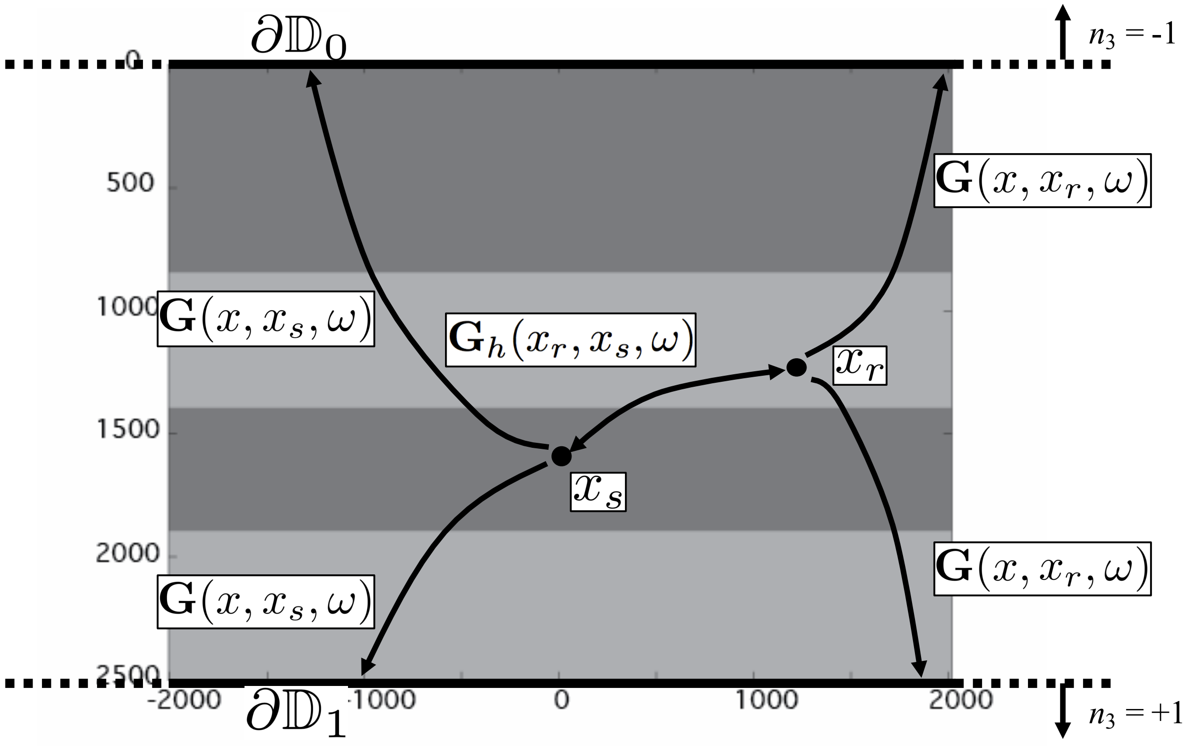


Elastodynamic double-sided homogeneous Green's function representation



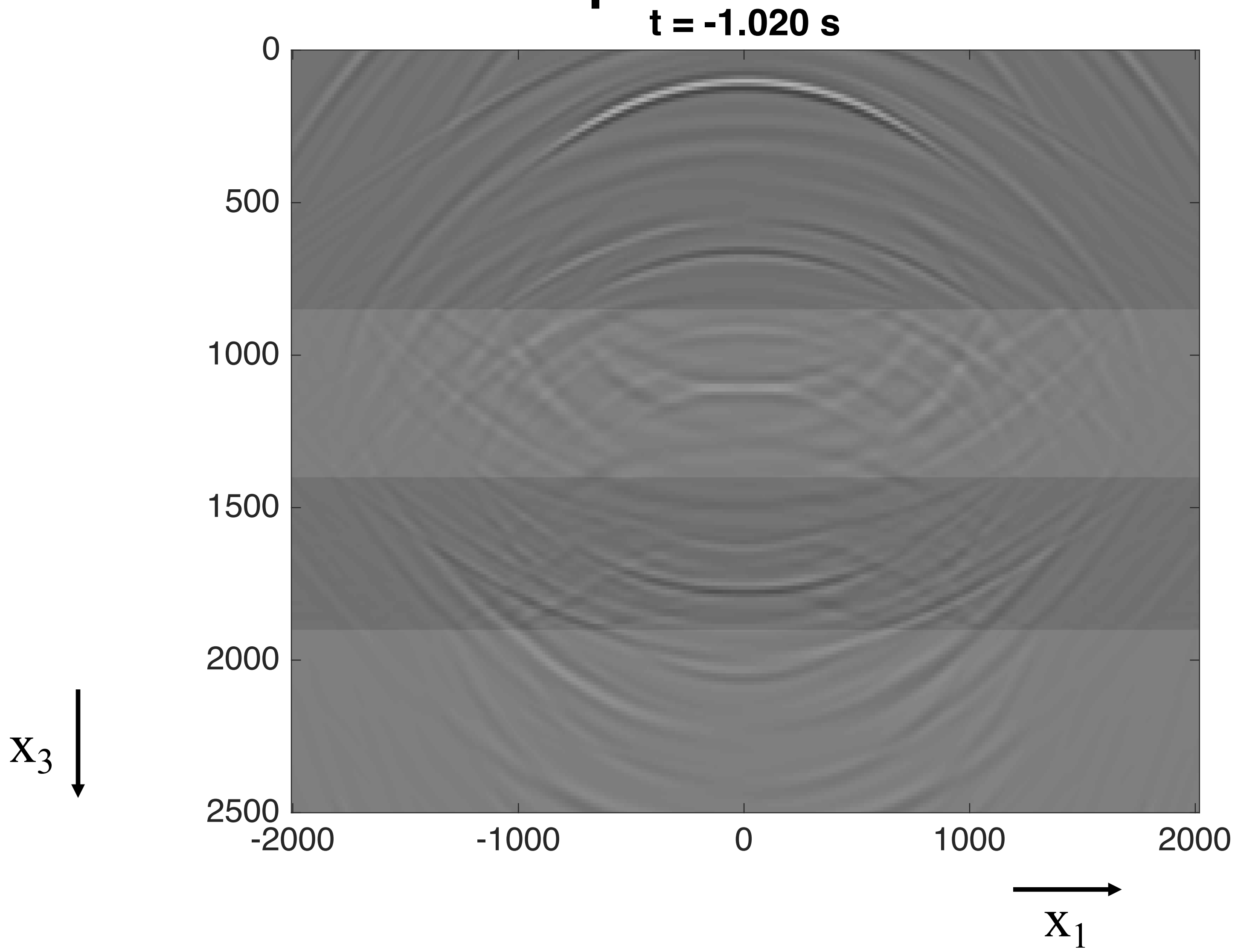
Elastodynamic double-sided homogeneous Green's function representation

$$\mathbf{G}_h(x_r, x_s, \omega) = \int_{\partial\mathbb{D}_0 \cup \partial\mathbb{D}_1} \mathbf{J}\mathbf{G}^\dagger(x, x_r, \omega)\mathbf{J}\mathbf{G}(x, x_s, \omega) n_3 \mathrm{d}^2 x$$

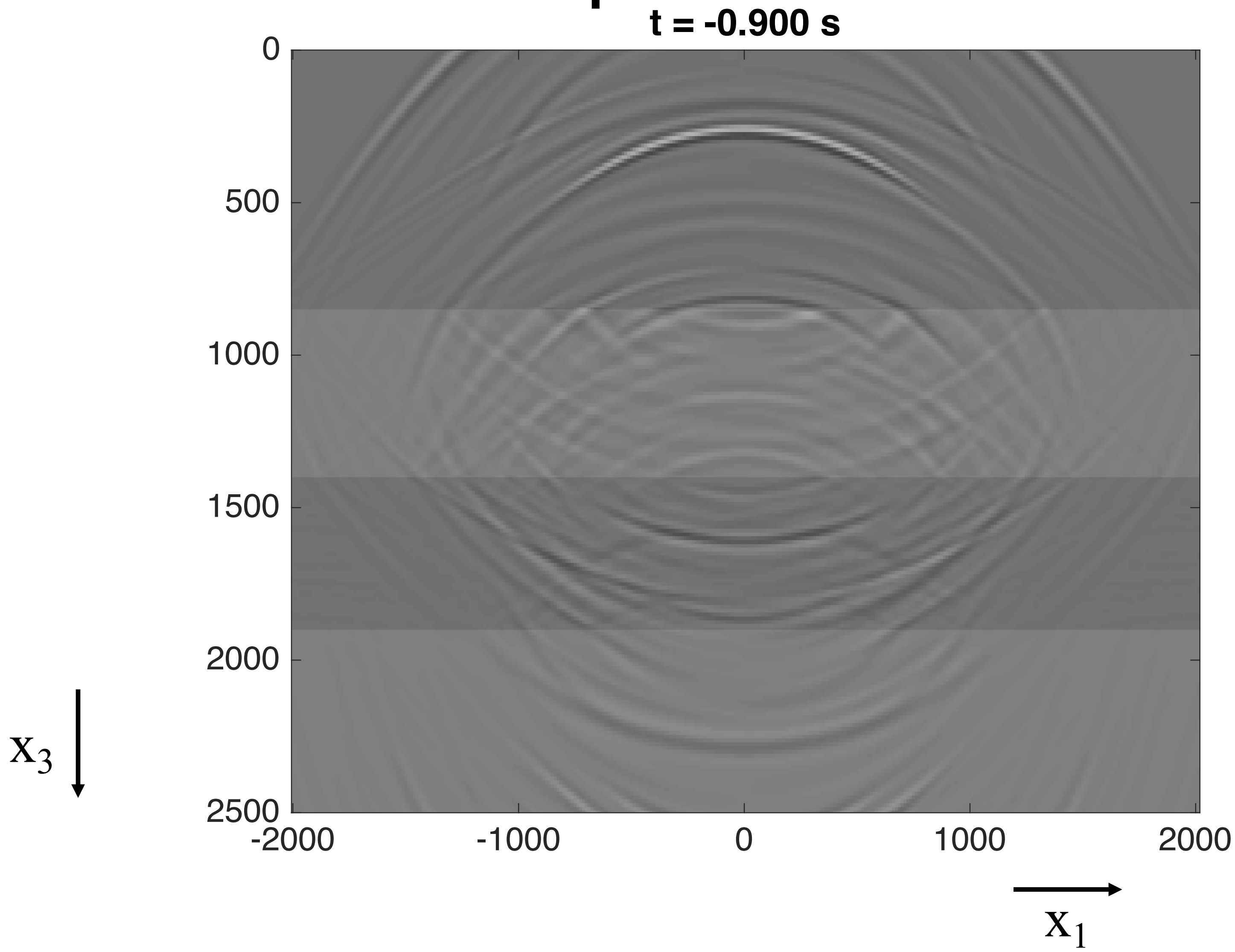


(Wapenaar, Kees, Joost van der Neut, and Evert Slob. "Unified double-and single-sided homogeneous Green's function representations." *Proc. R. Soc. A*. Vol. 472. No. 2190. The Royal Society, 2016.)

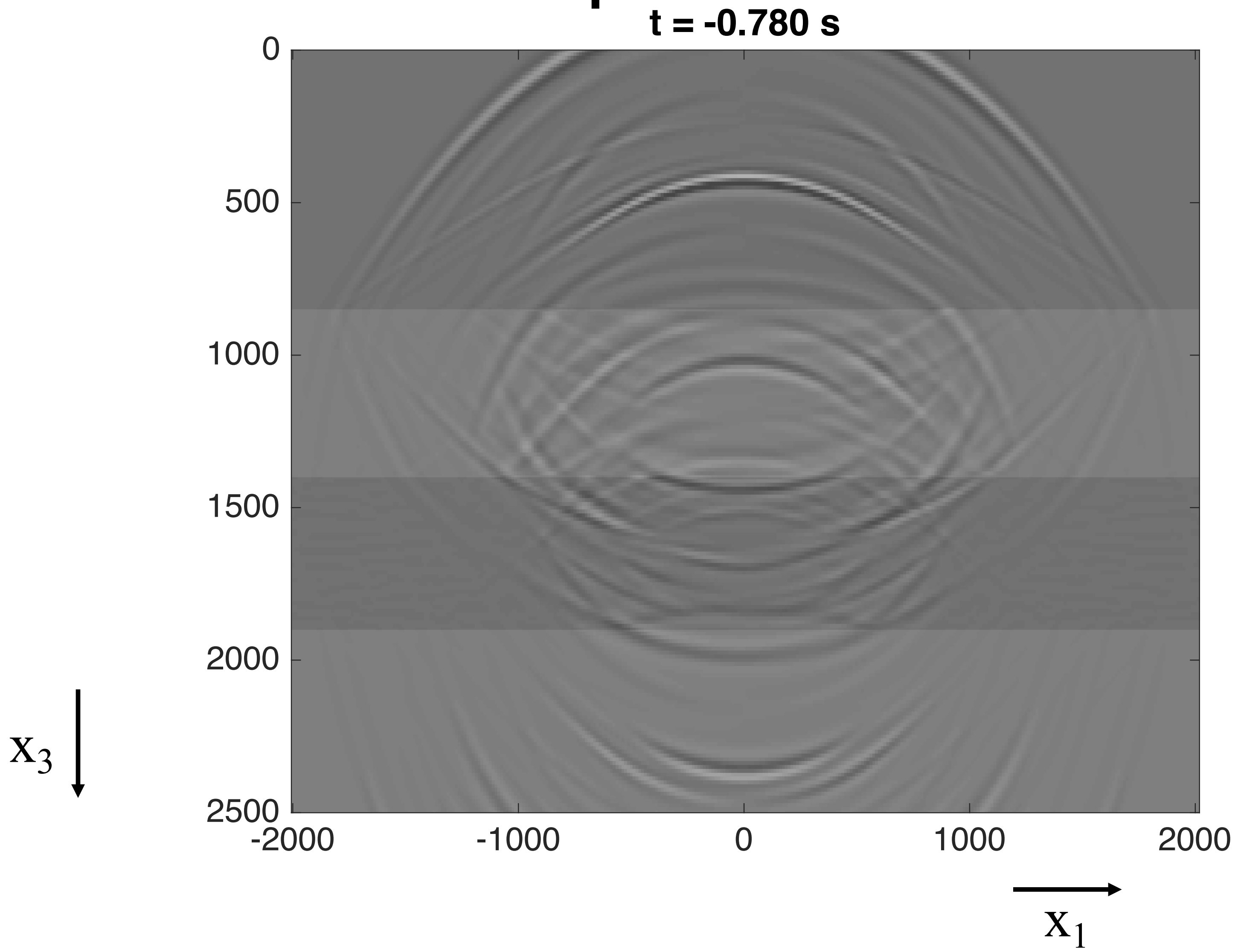
Elastodynamic double-sided homogeneous Green's function representation



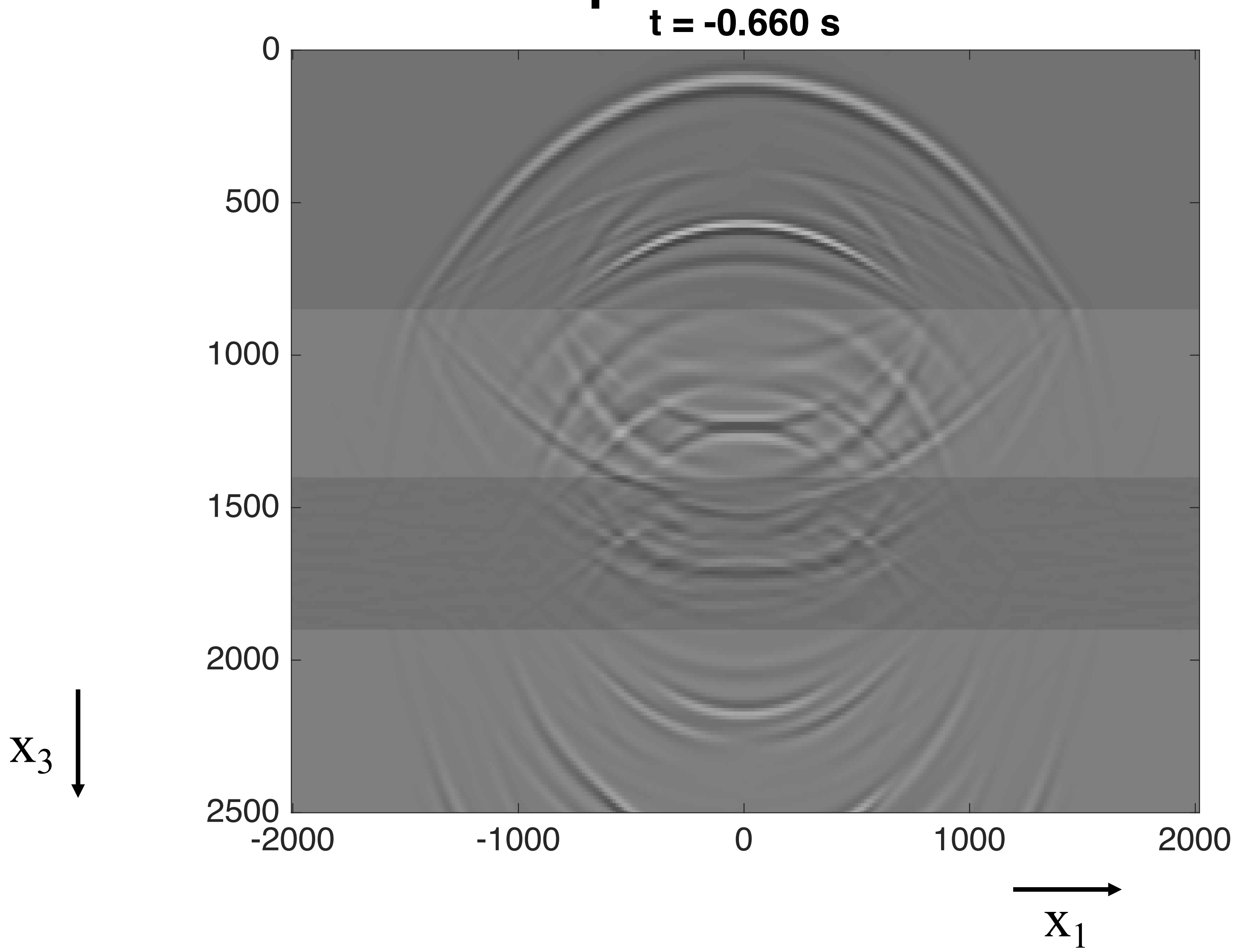
Elastodynamic double-sided homogeneous Green's function representation



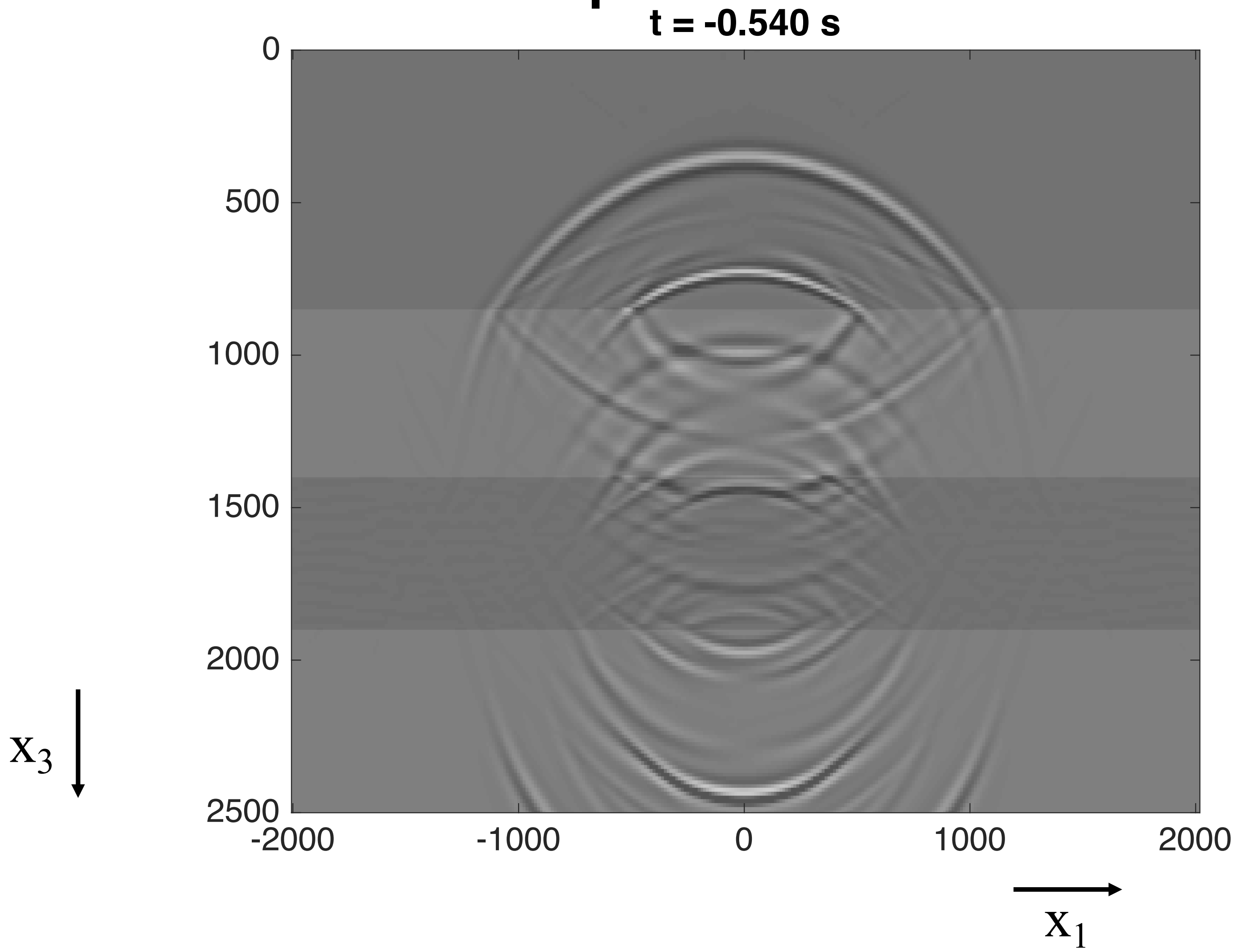
Elastodynamic double-sided homogeneous Green's function representation



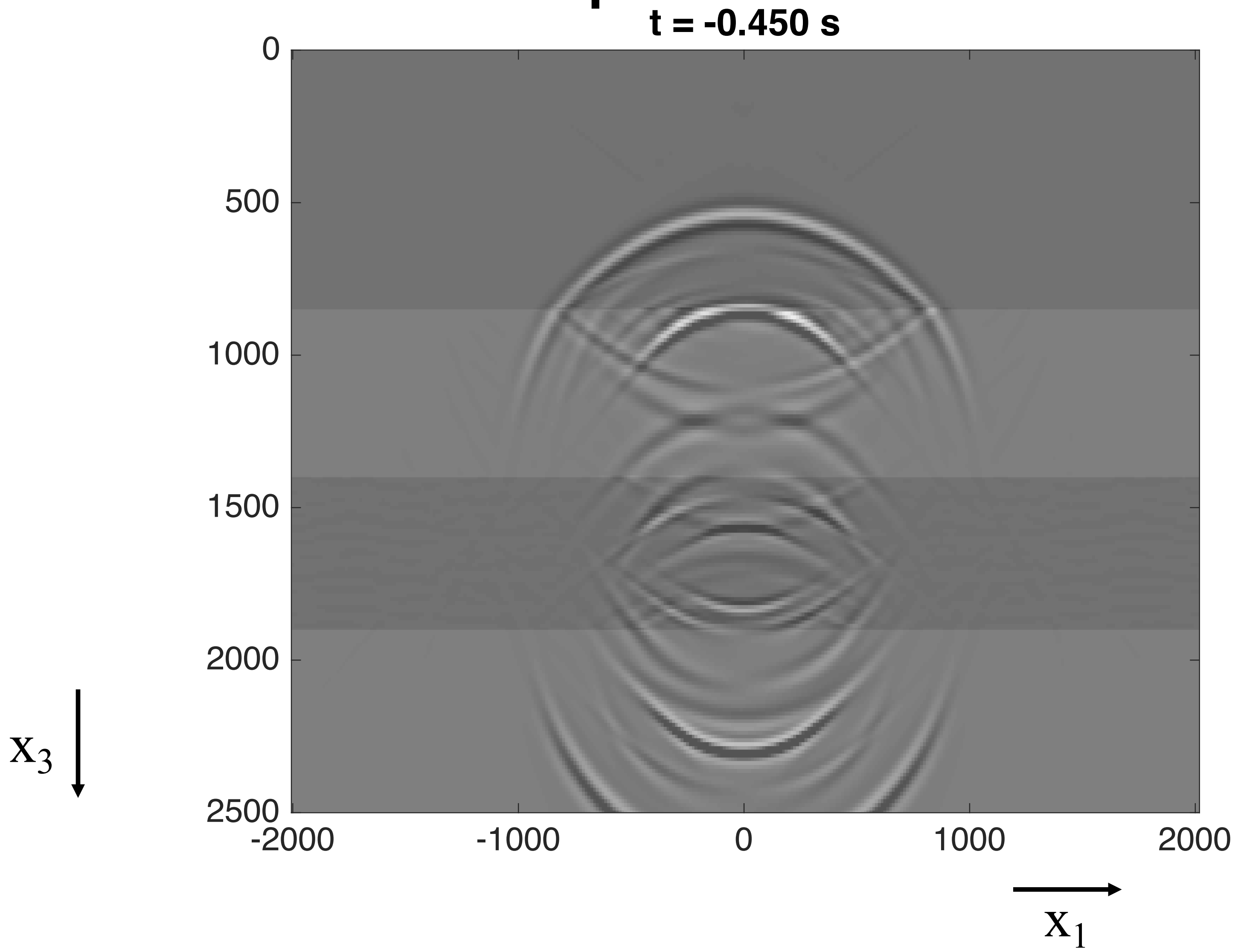
Elastodynamic double-sided homogeneous Green's function representation



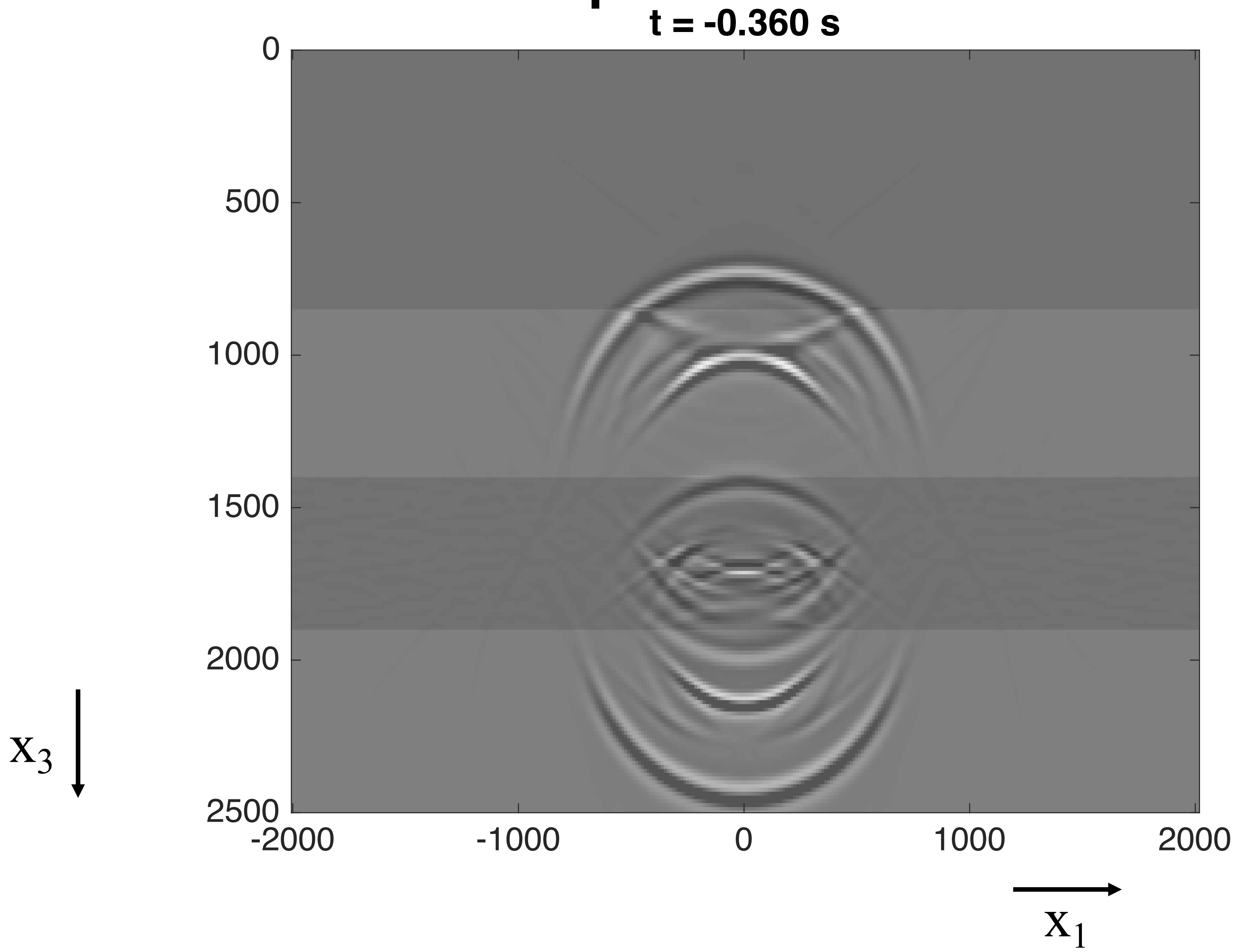
Elastodynamic double-sided homogeneous Green's function representation



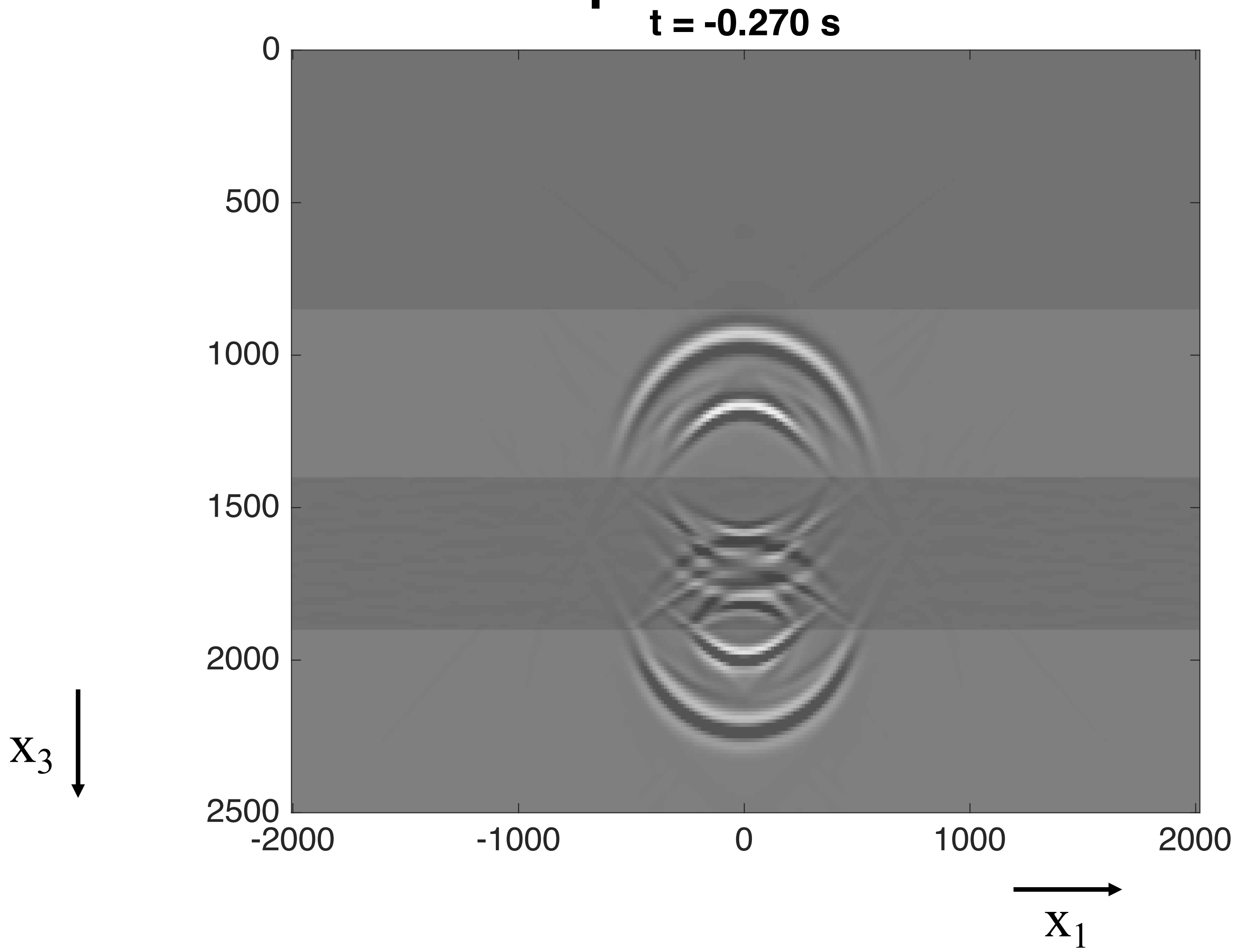
Elastodynamic double-sided homogeneous Green's function representation



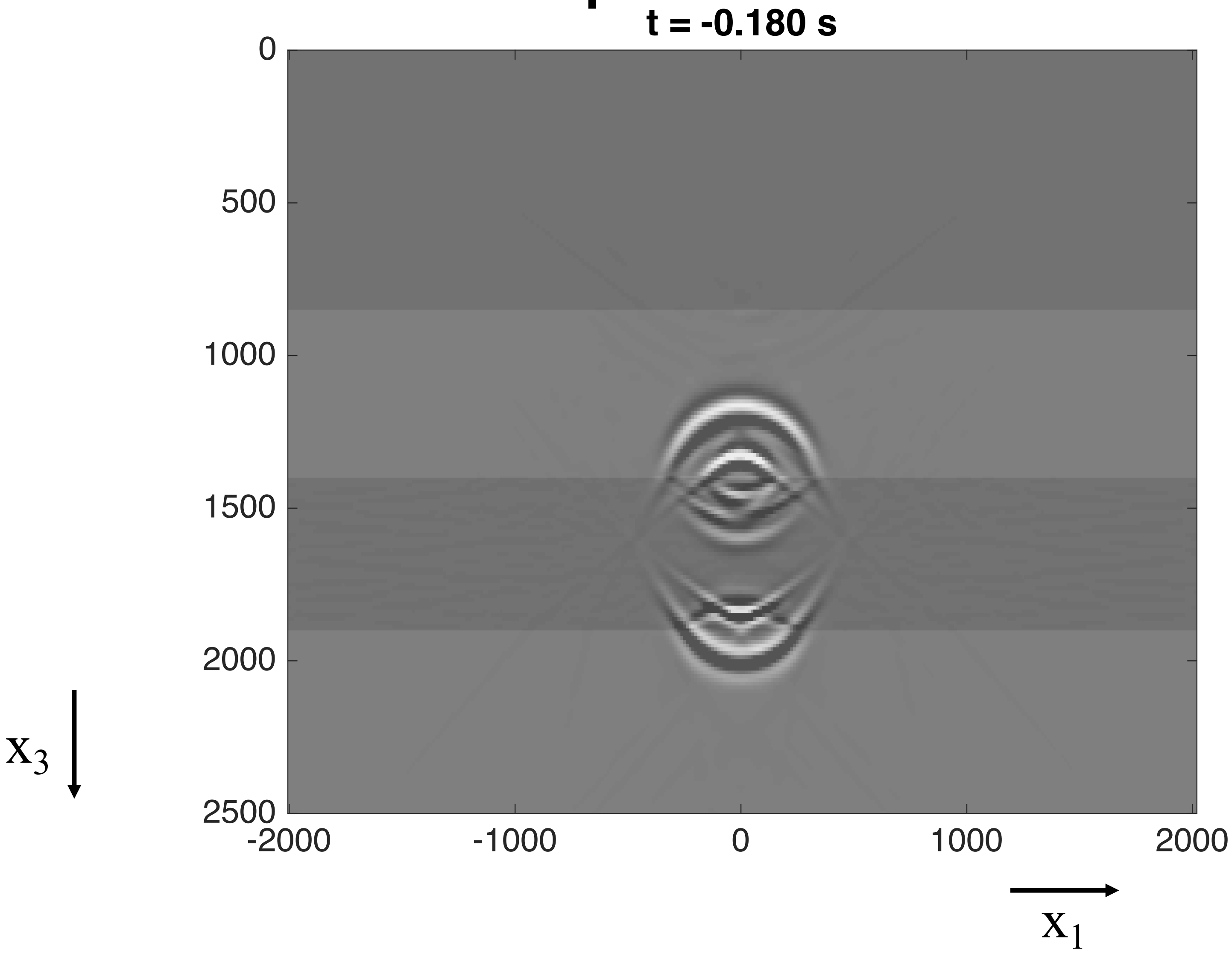
Elastodynamic double-sided homogeneous Green's function representation



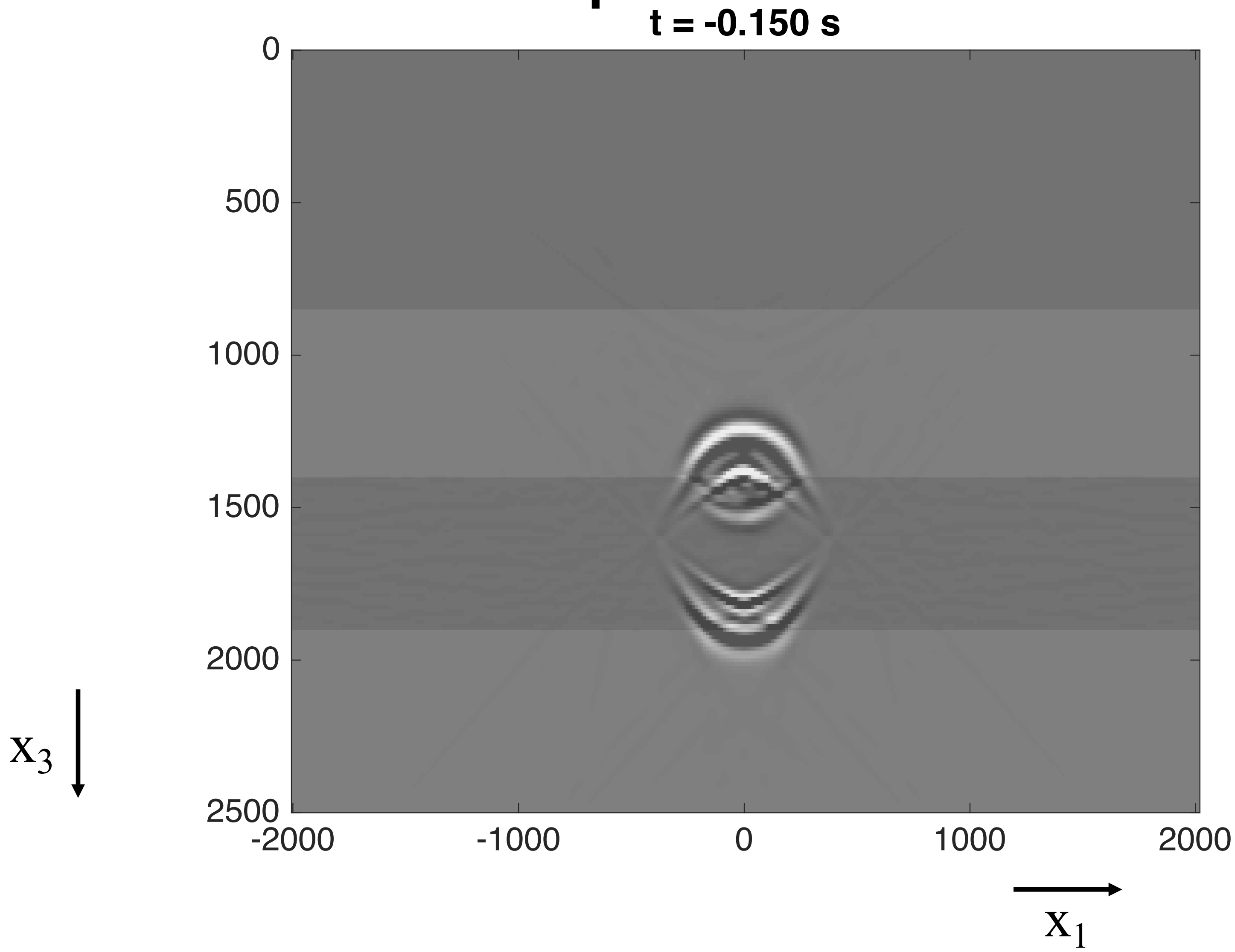
Elastodynamic double-sided homogeneous Green's function representation



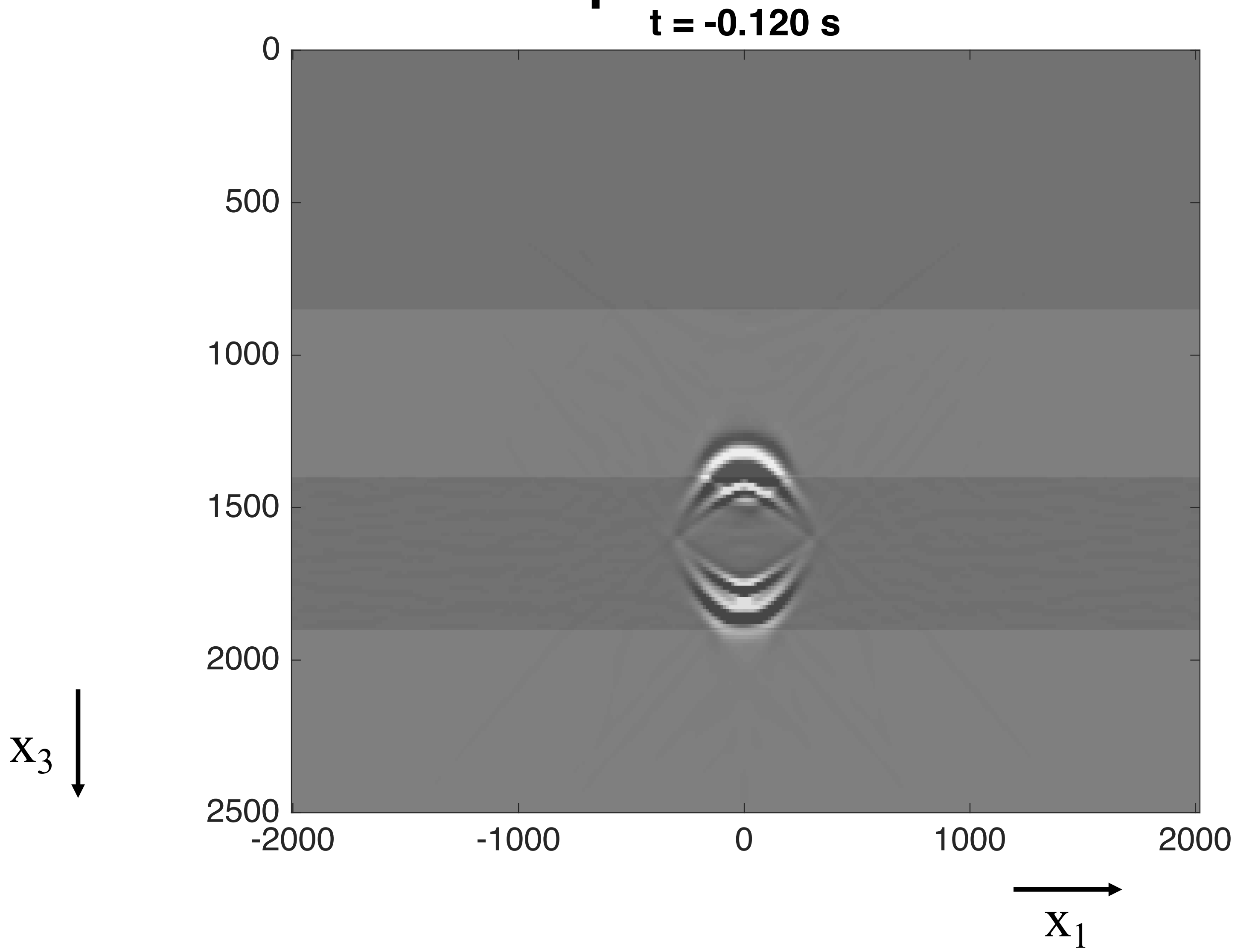
Elastodynamic double-sided homogeneous Green's function representation



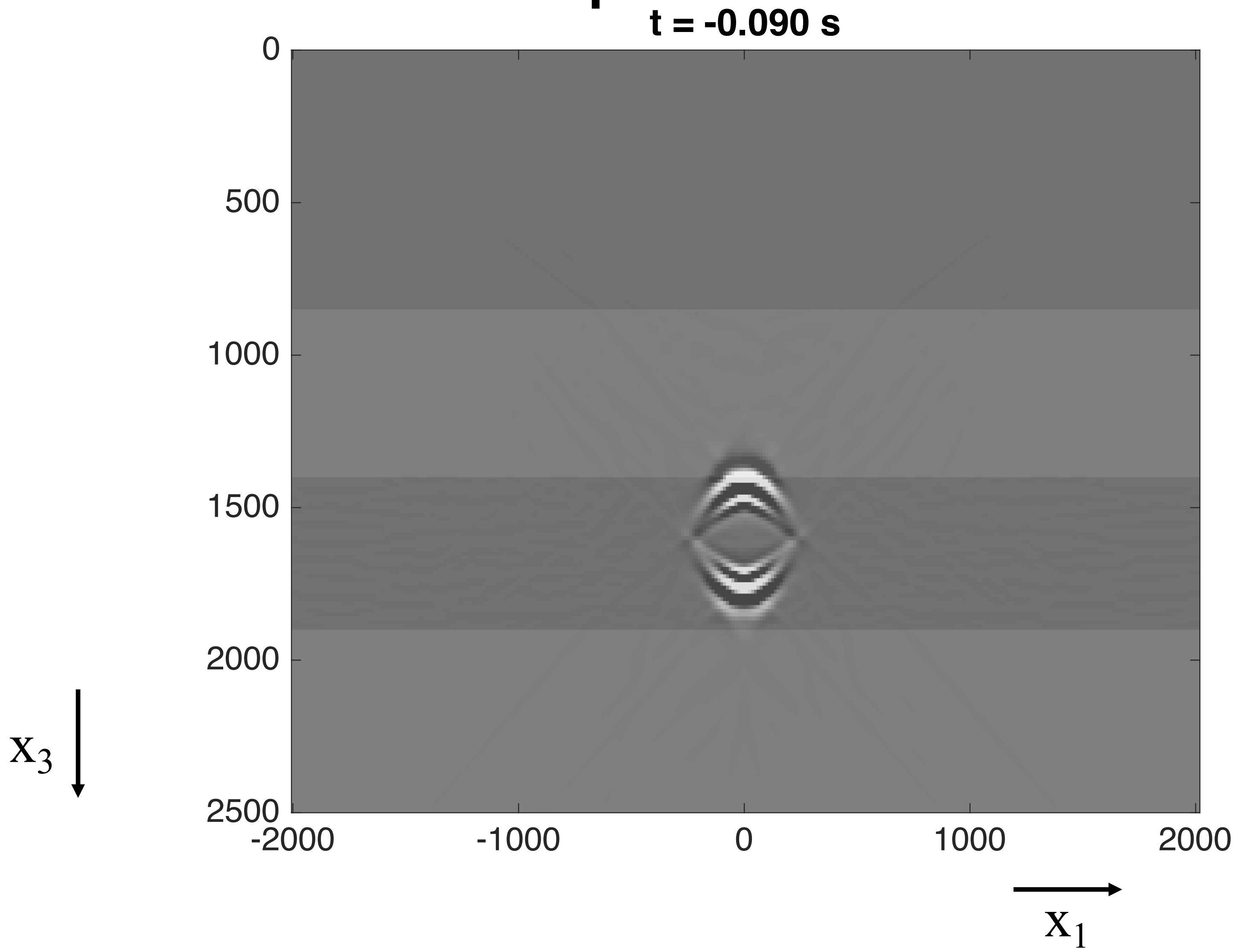
Elastodynamic double-sided homogeneous Green's function representation



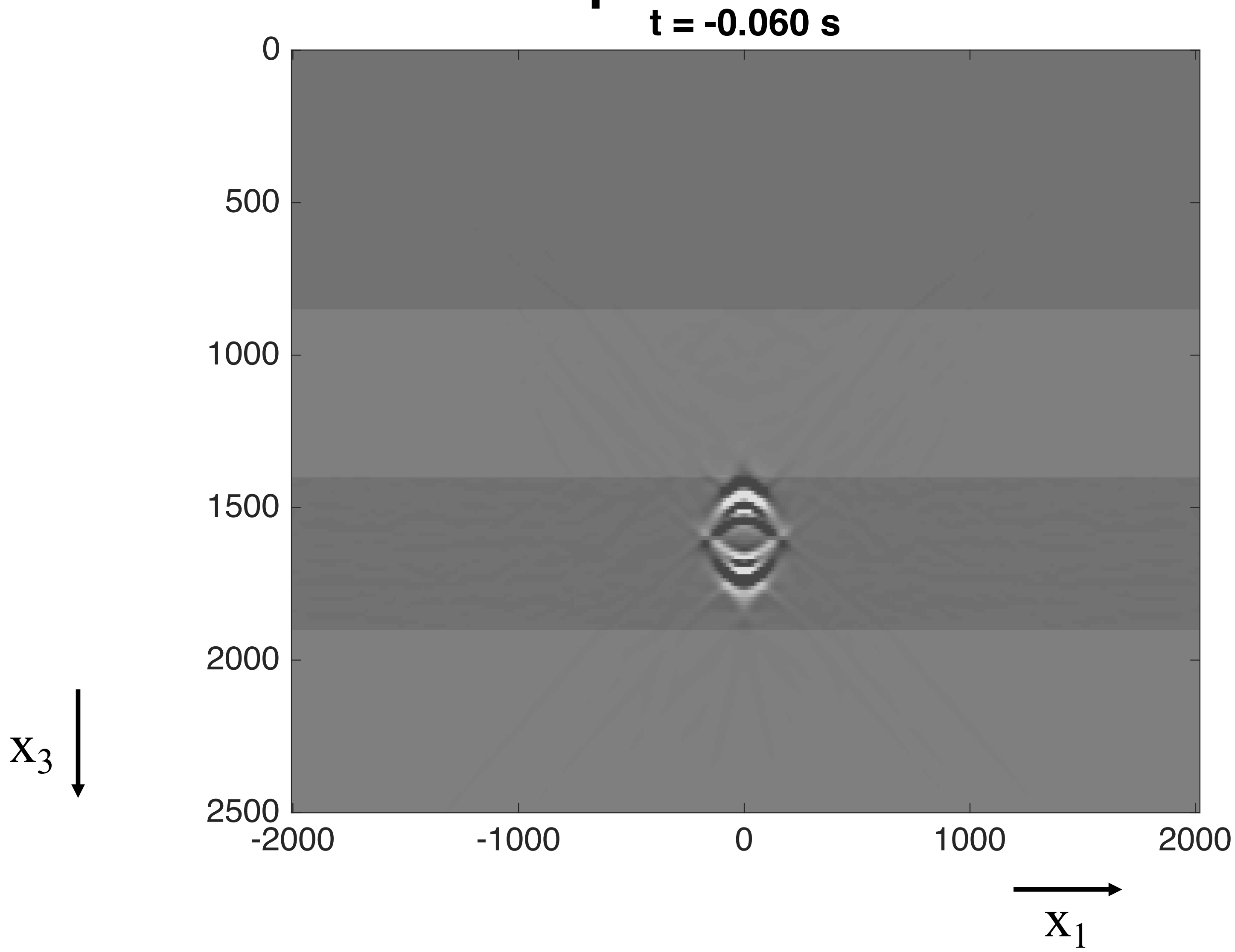
Elastodynamic double-sided homogeneous Green's function representation



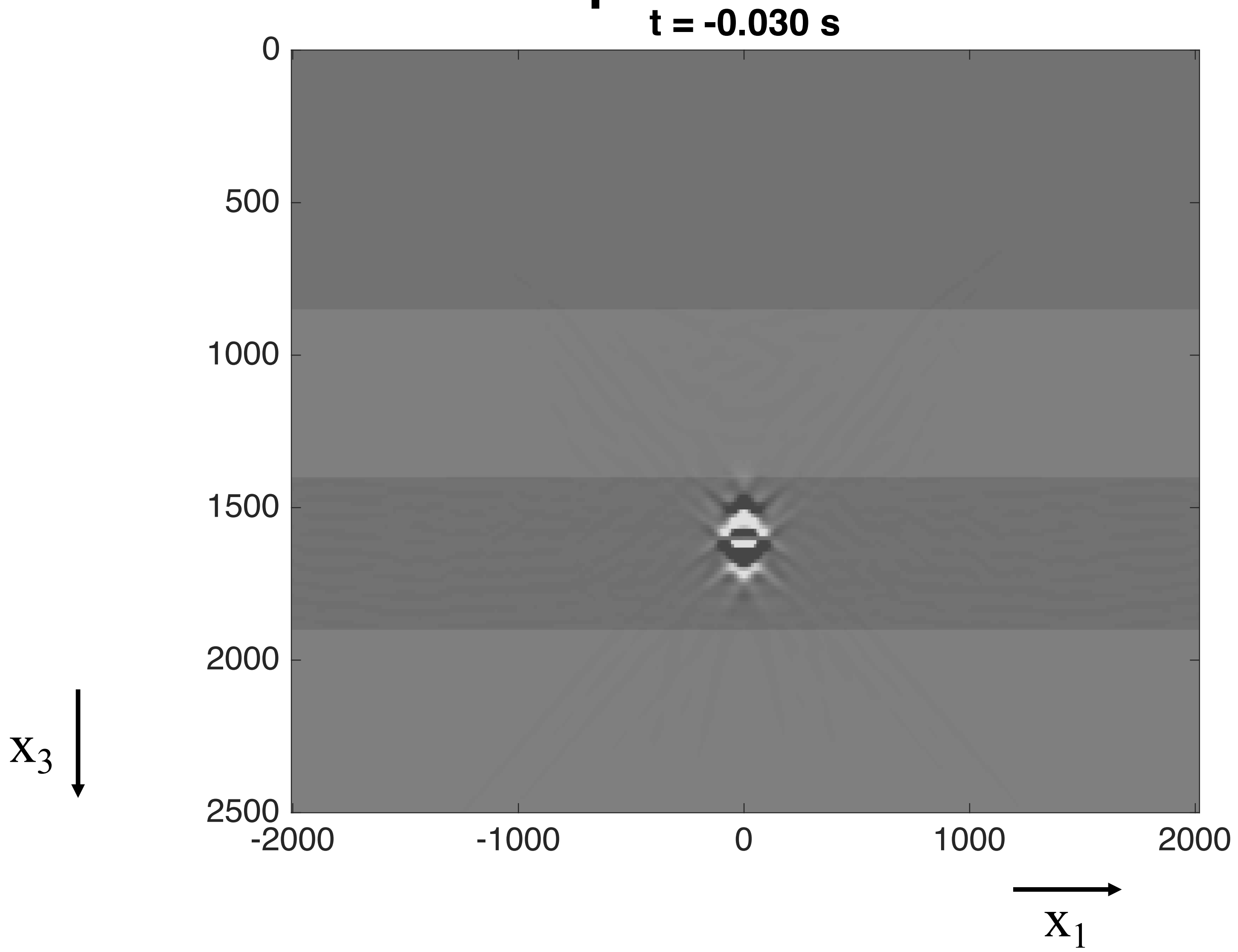
Elastodynamic double-sided homogeneous Green's function representation



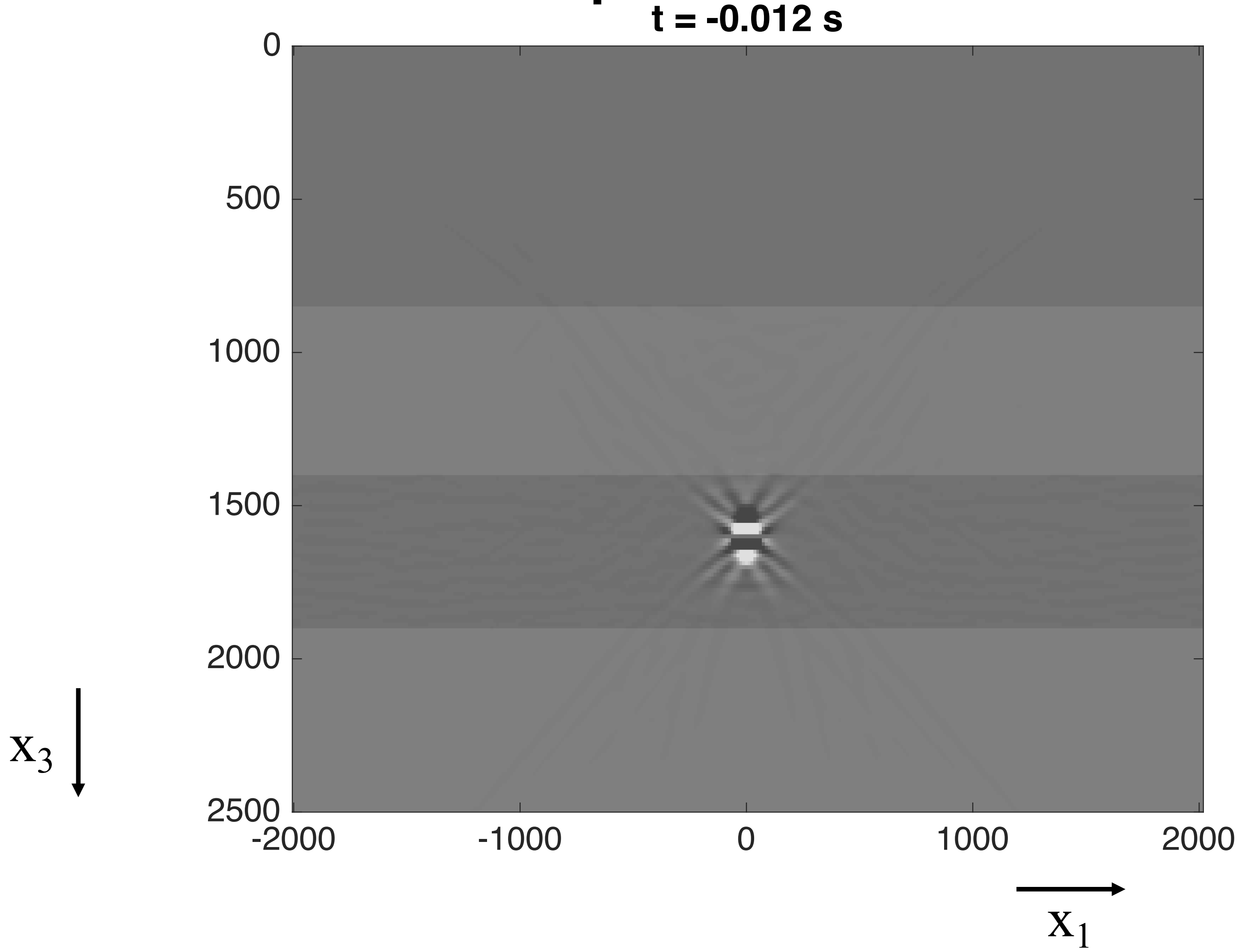
Elastodynamic double-sided homogeneous Green's function representation



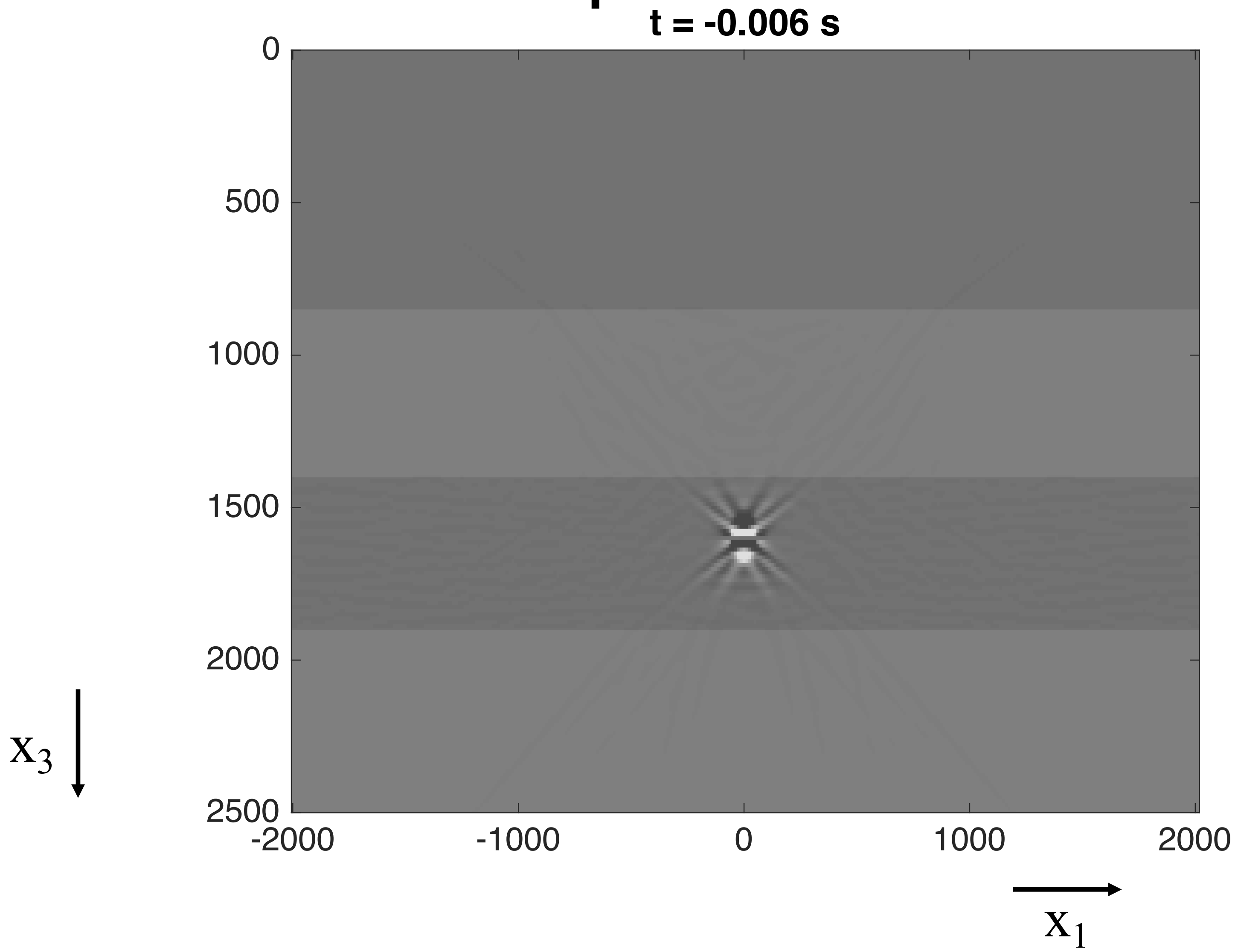
Elastodynamic double-sided homogeneous Green's function representation



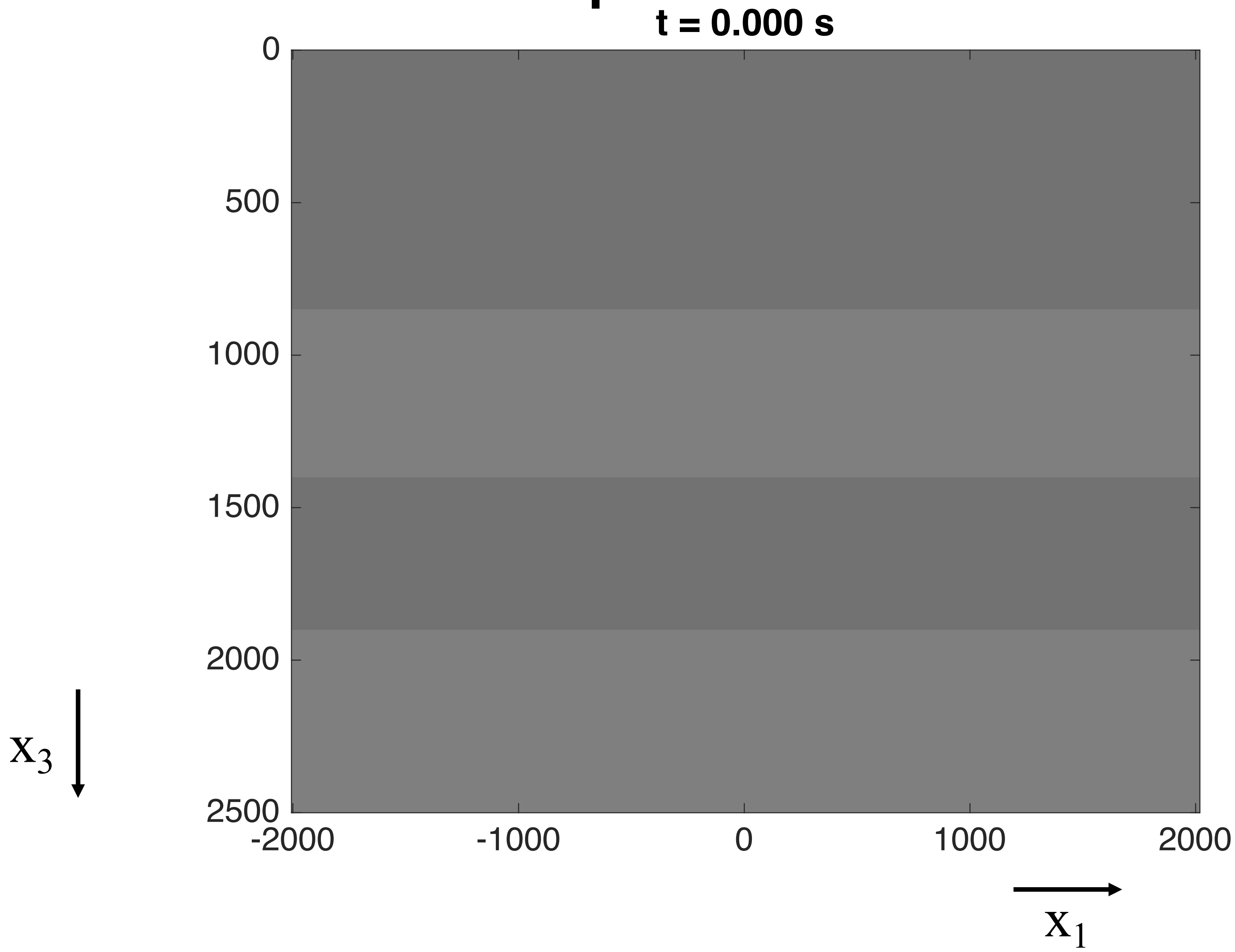
Elastodynamic double-sided homogeneous Green's function representation



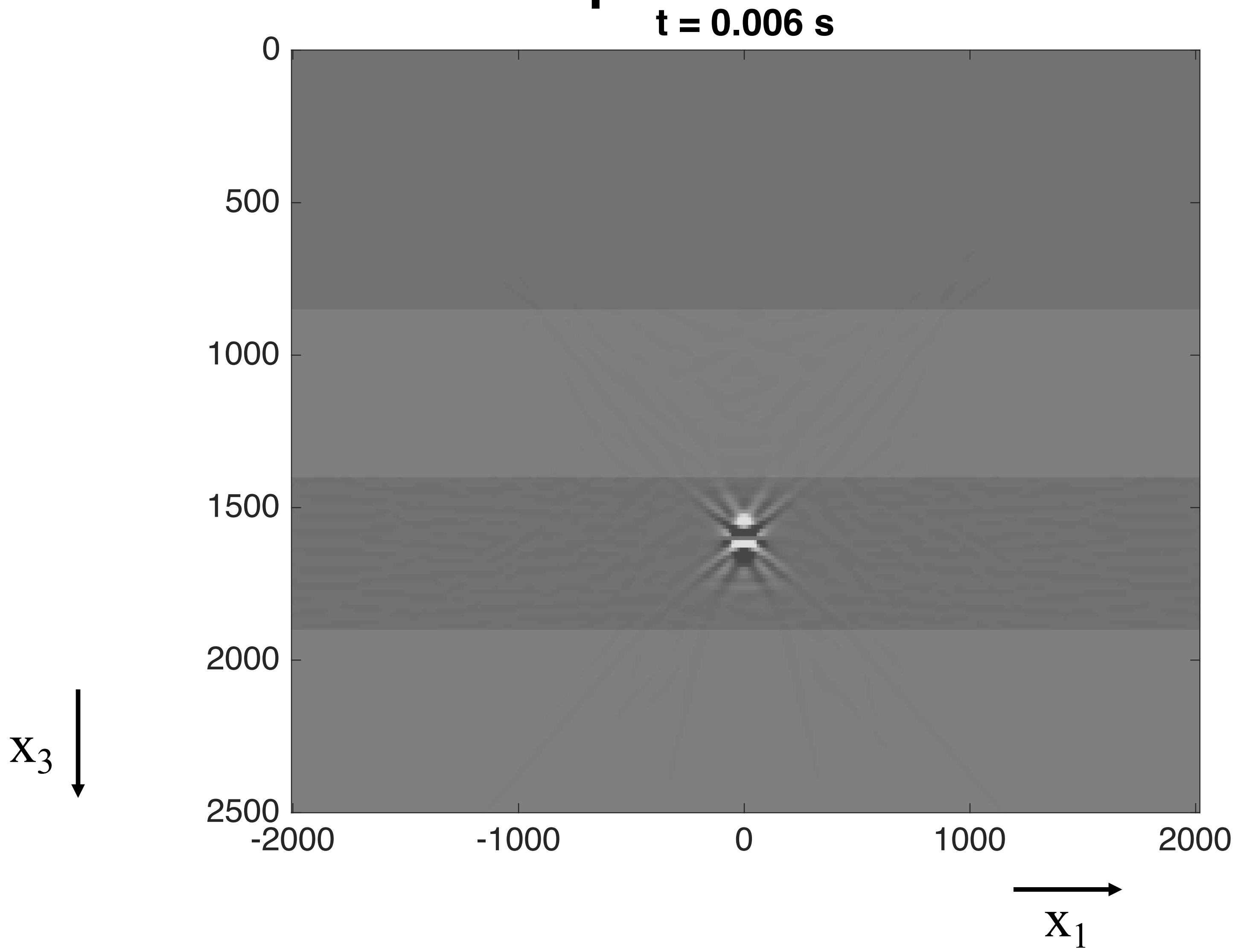
Elastodynamic double-sided homogeneous Green's function representation



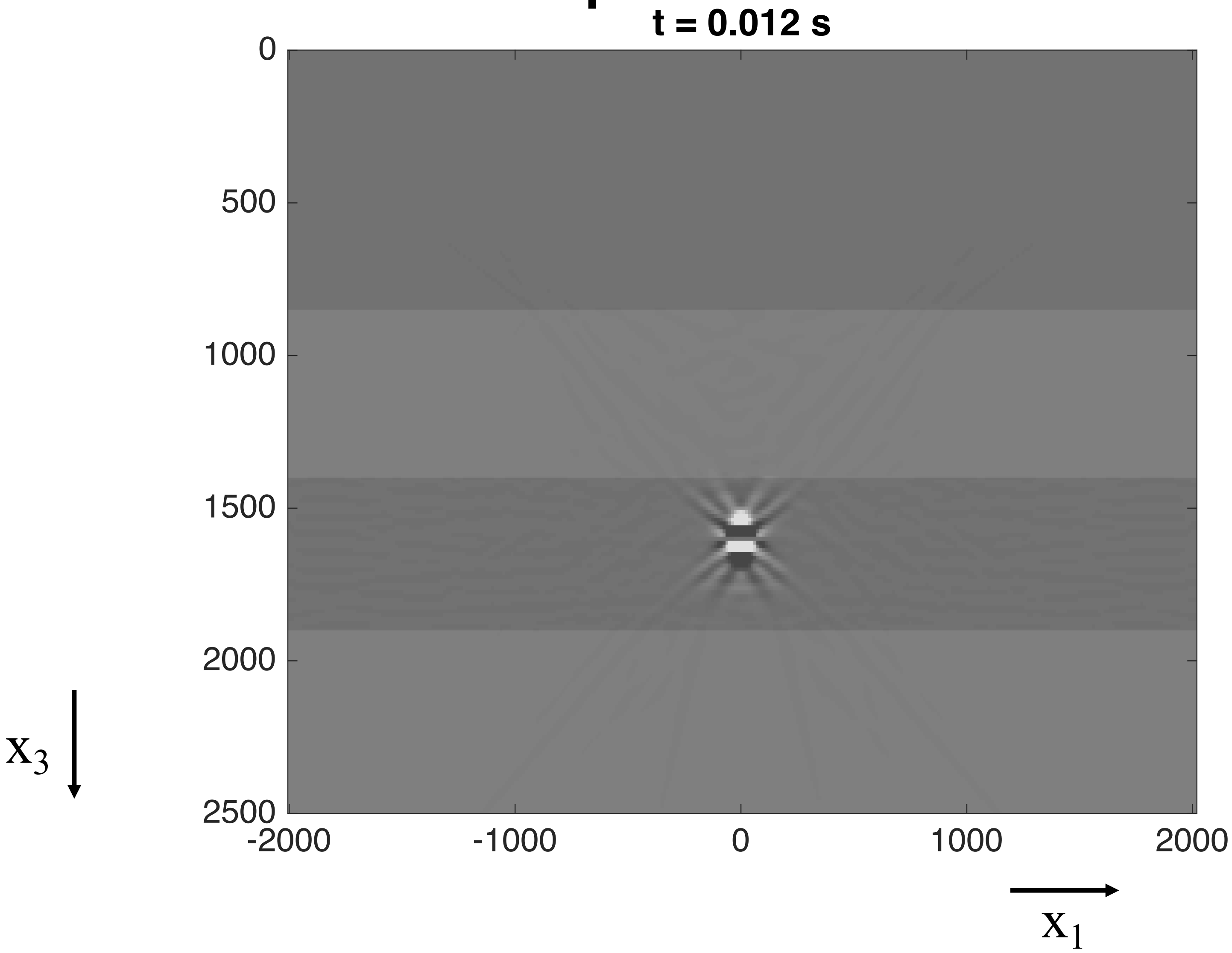
Elastodynamic double-sided homogeneous Green's function representation



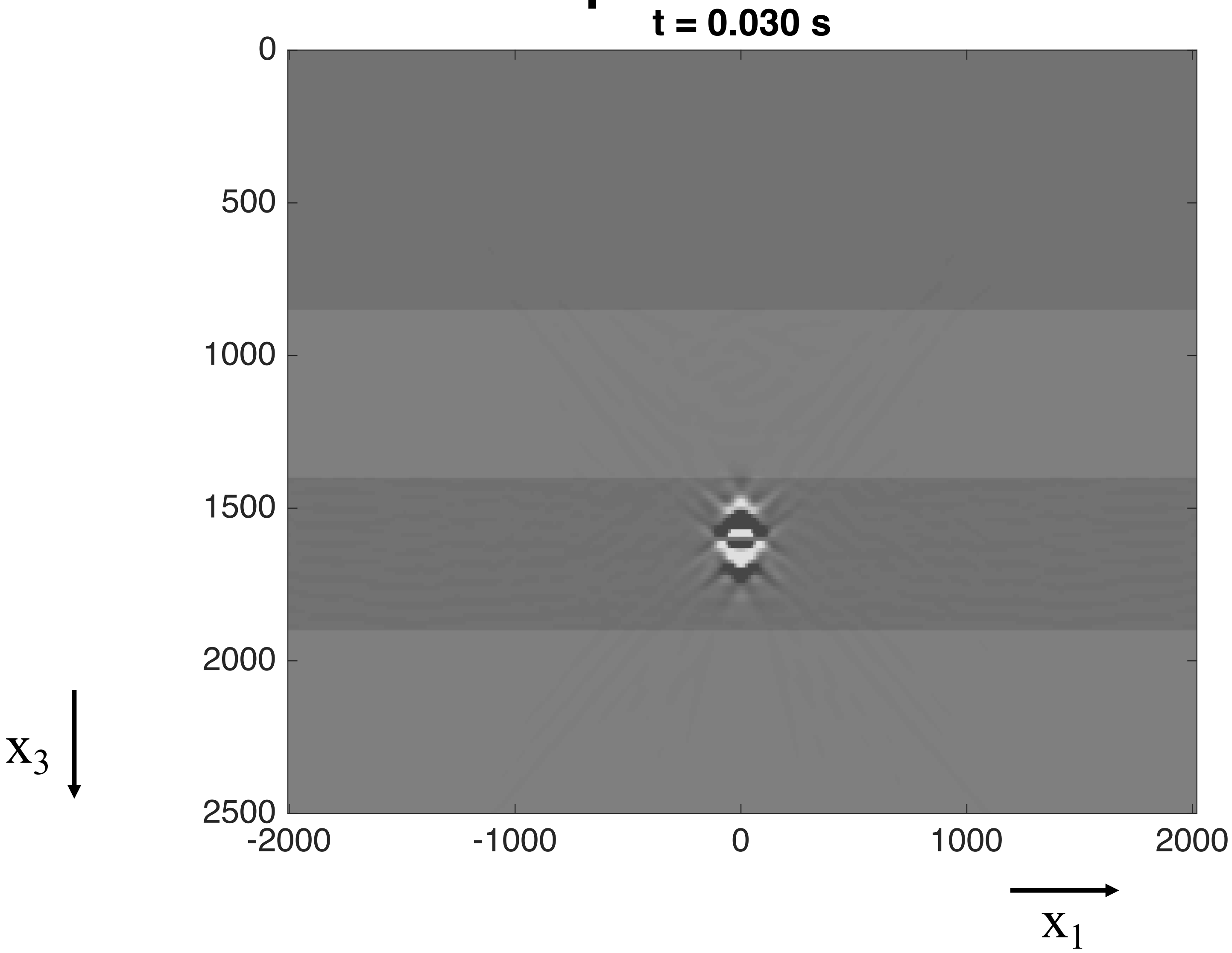
Elastodynamic double-sided homogeneous Green's function representation



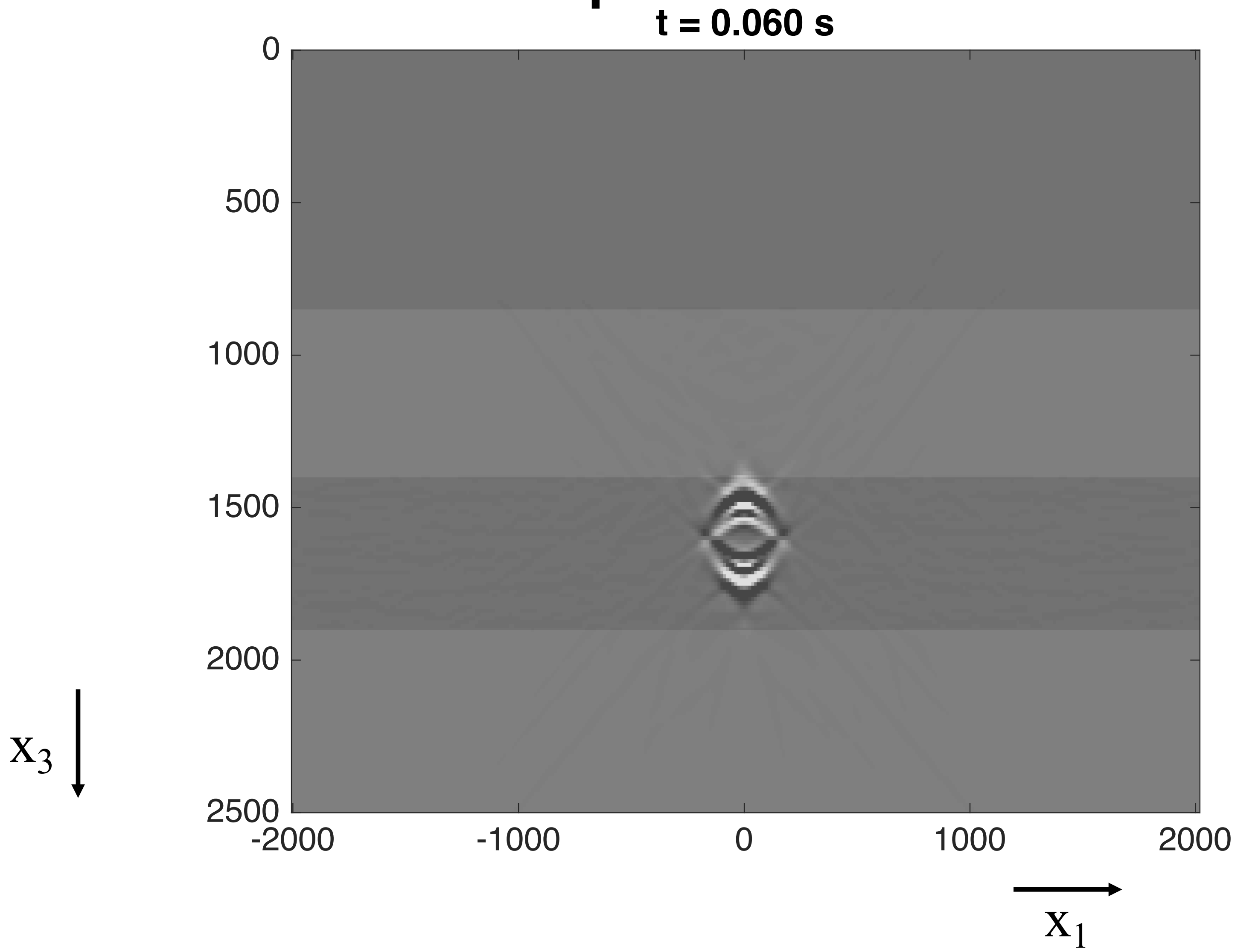
Elastodynamic double-sided homogeneous Green's function representation



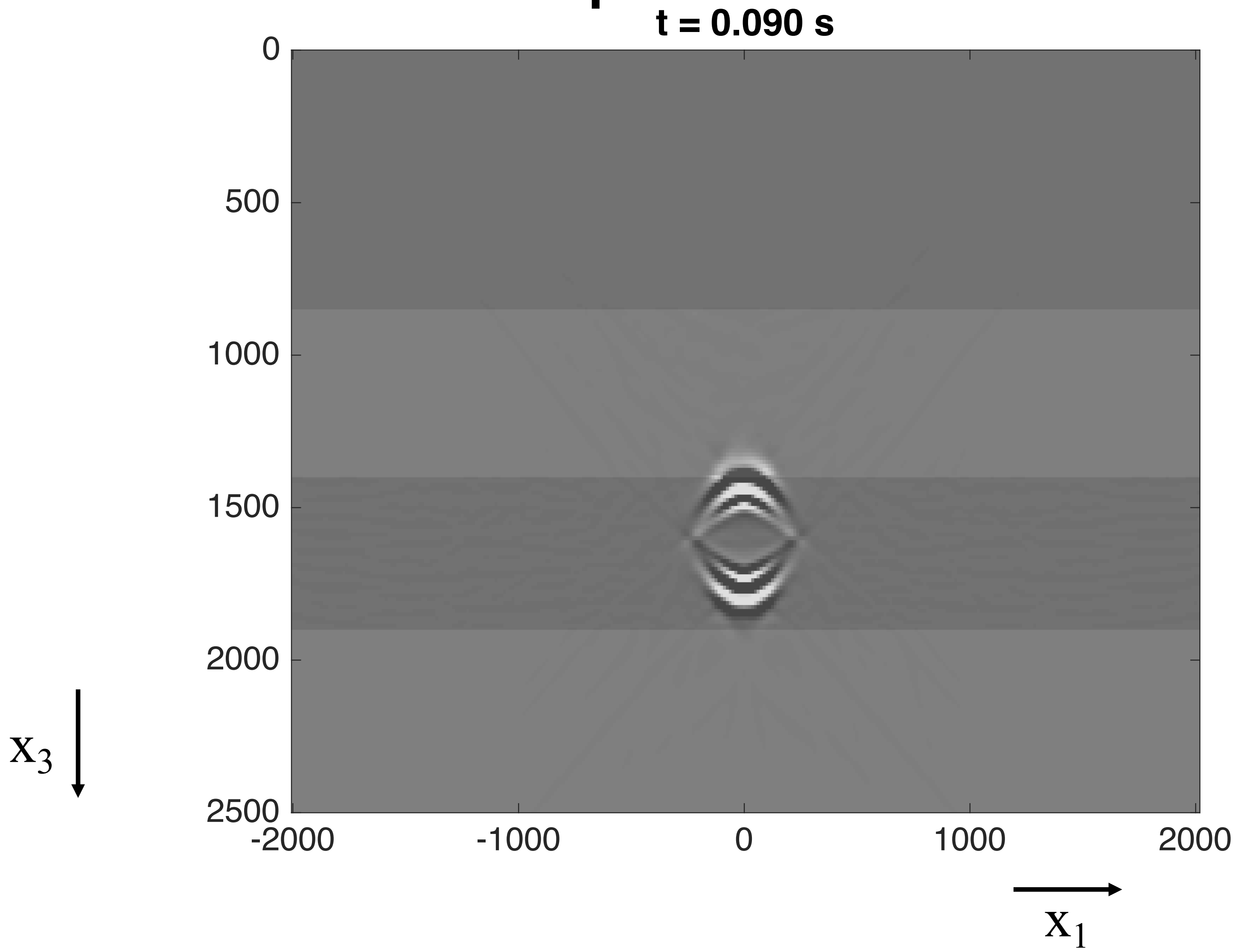
Elastodynamic double-sided homogeneous Green's function representation



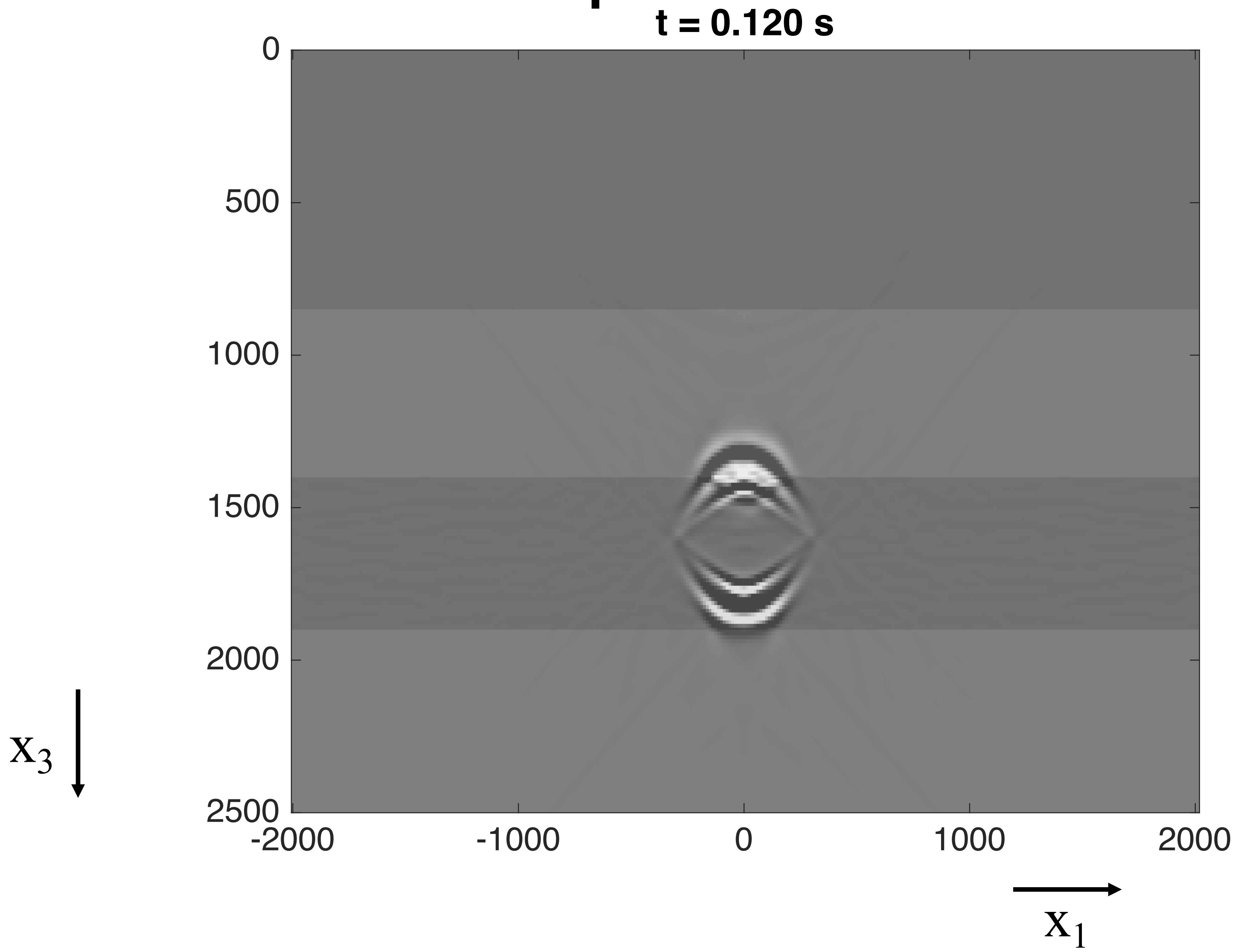
Elastodynamic double-sided homogeneous Green's function representation



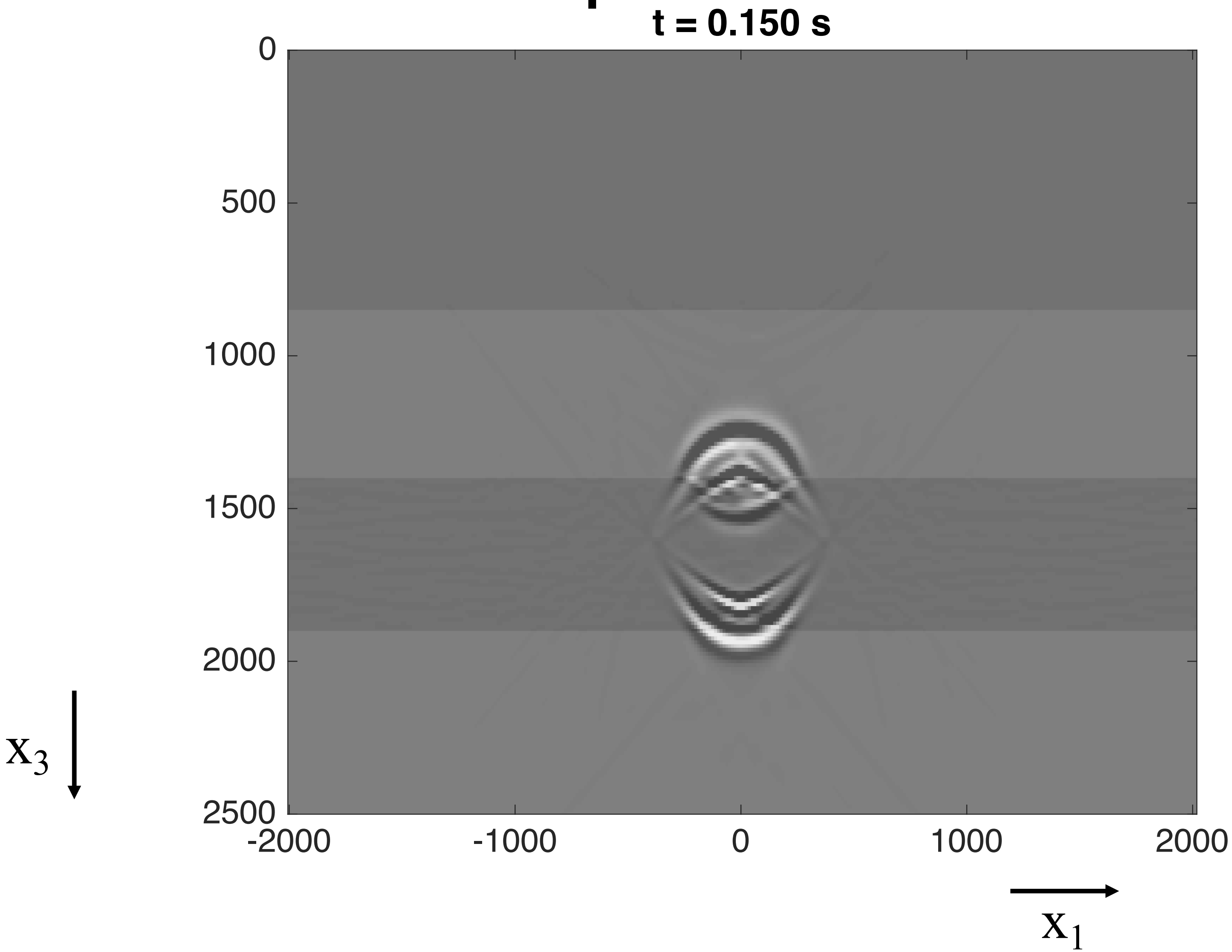
Elastodynamic double-sided homogeneous Green's function representation



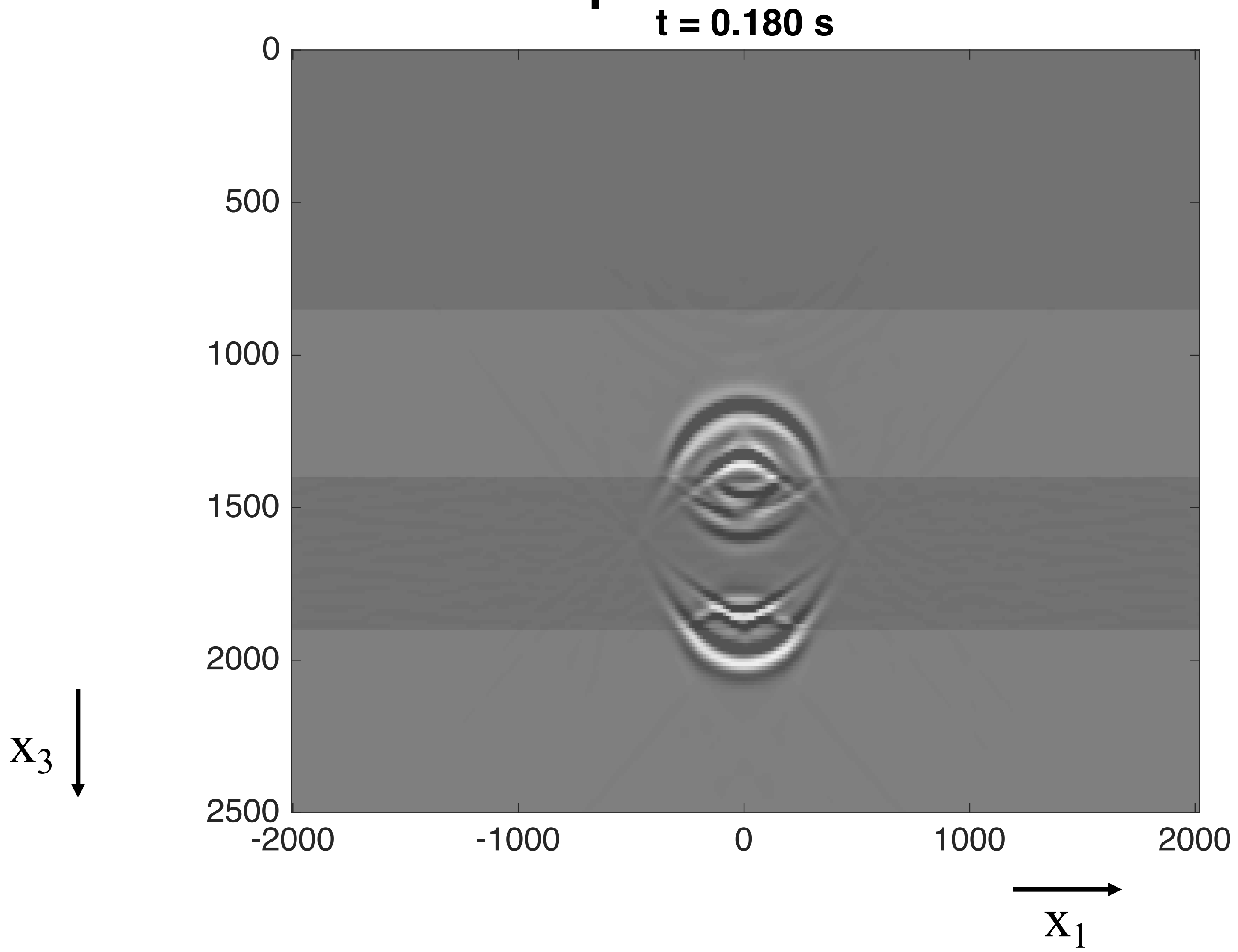
Elastodynamic double-sided homogeneous Green's function representation



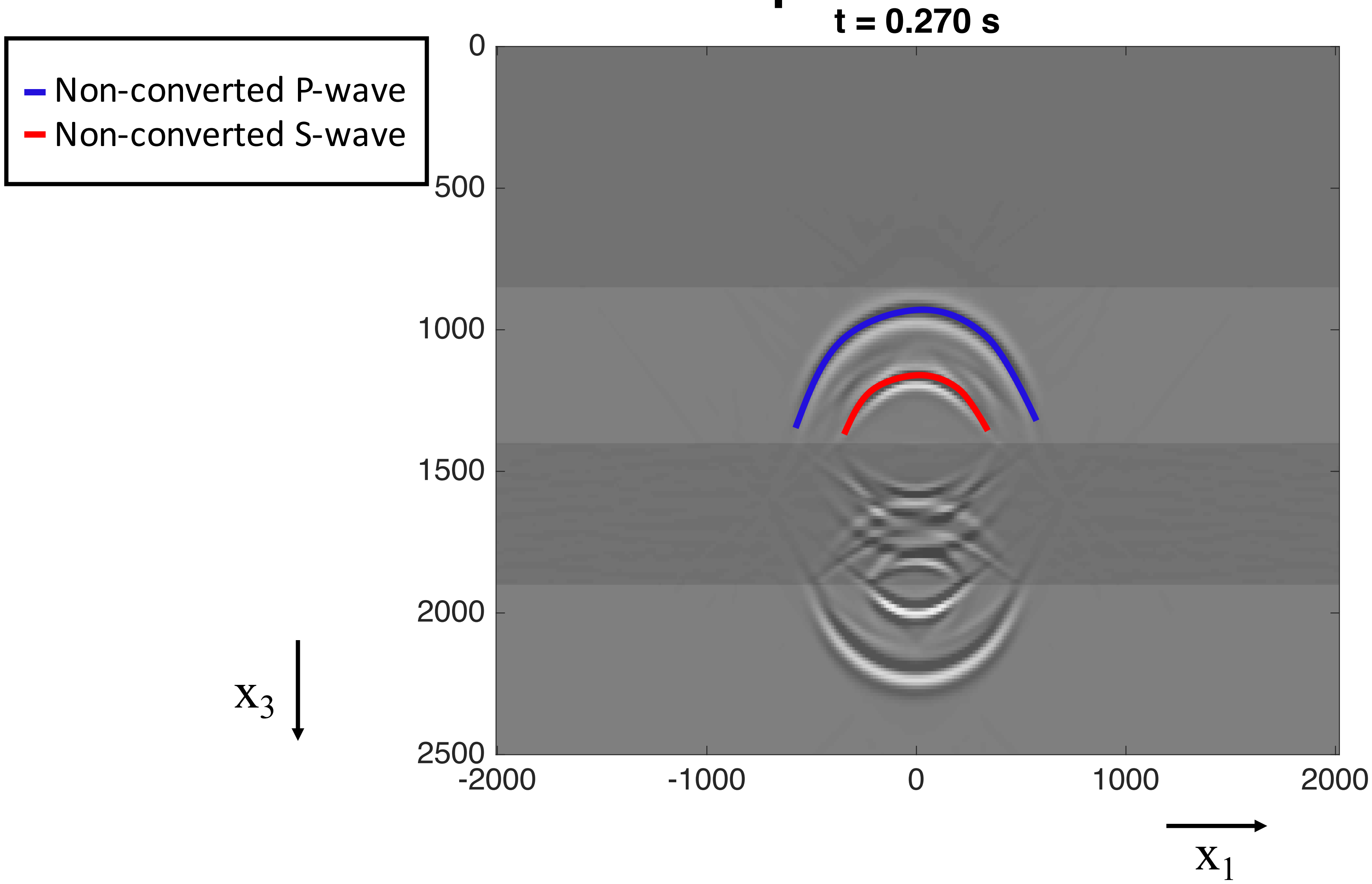
Elastodynamic double-sided homogeneous Green's function representation



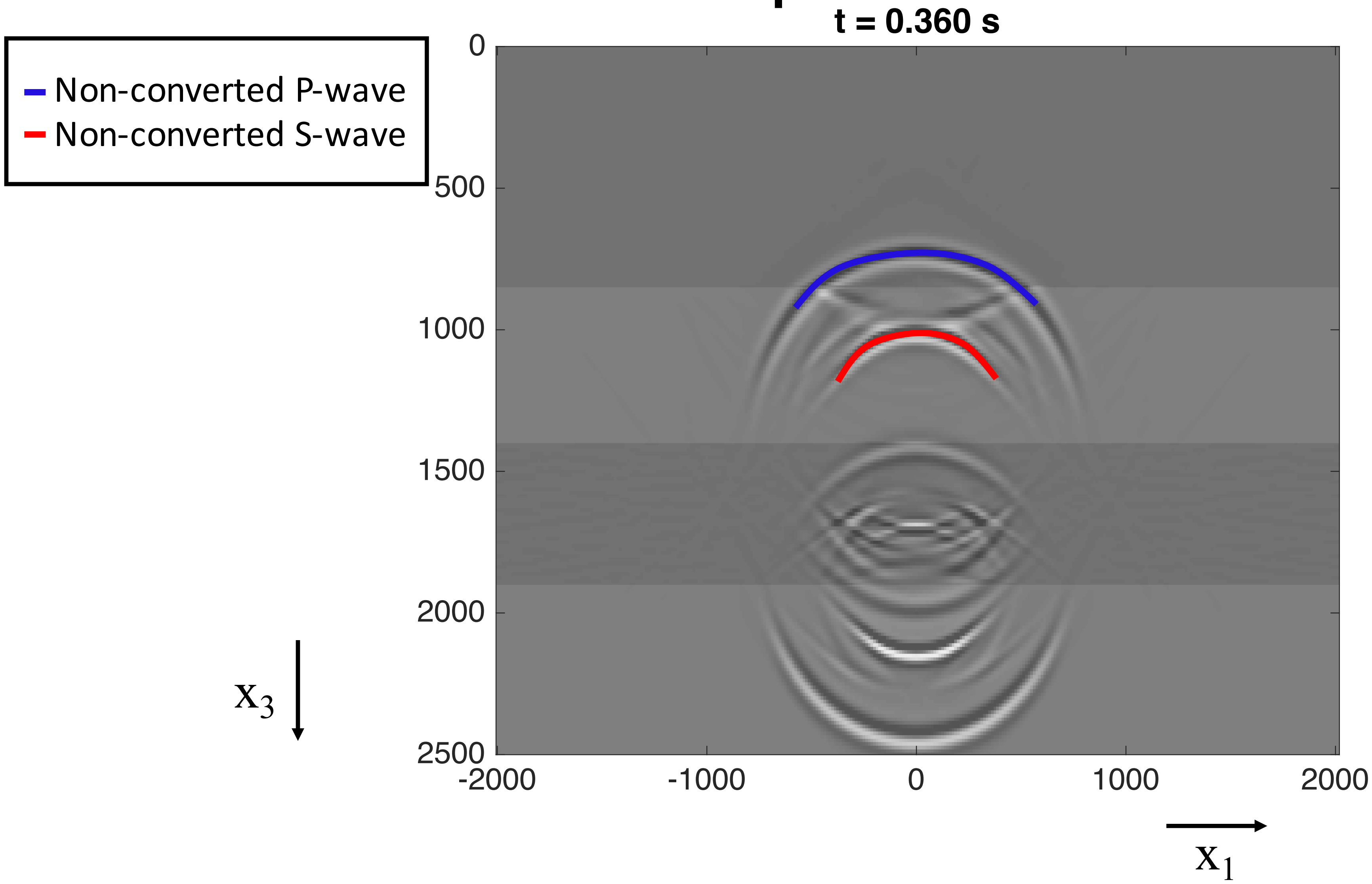
Elastodynamic double-sided homogeneous Green's function representation



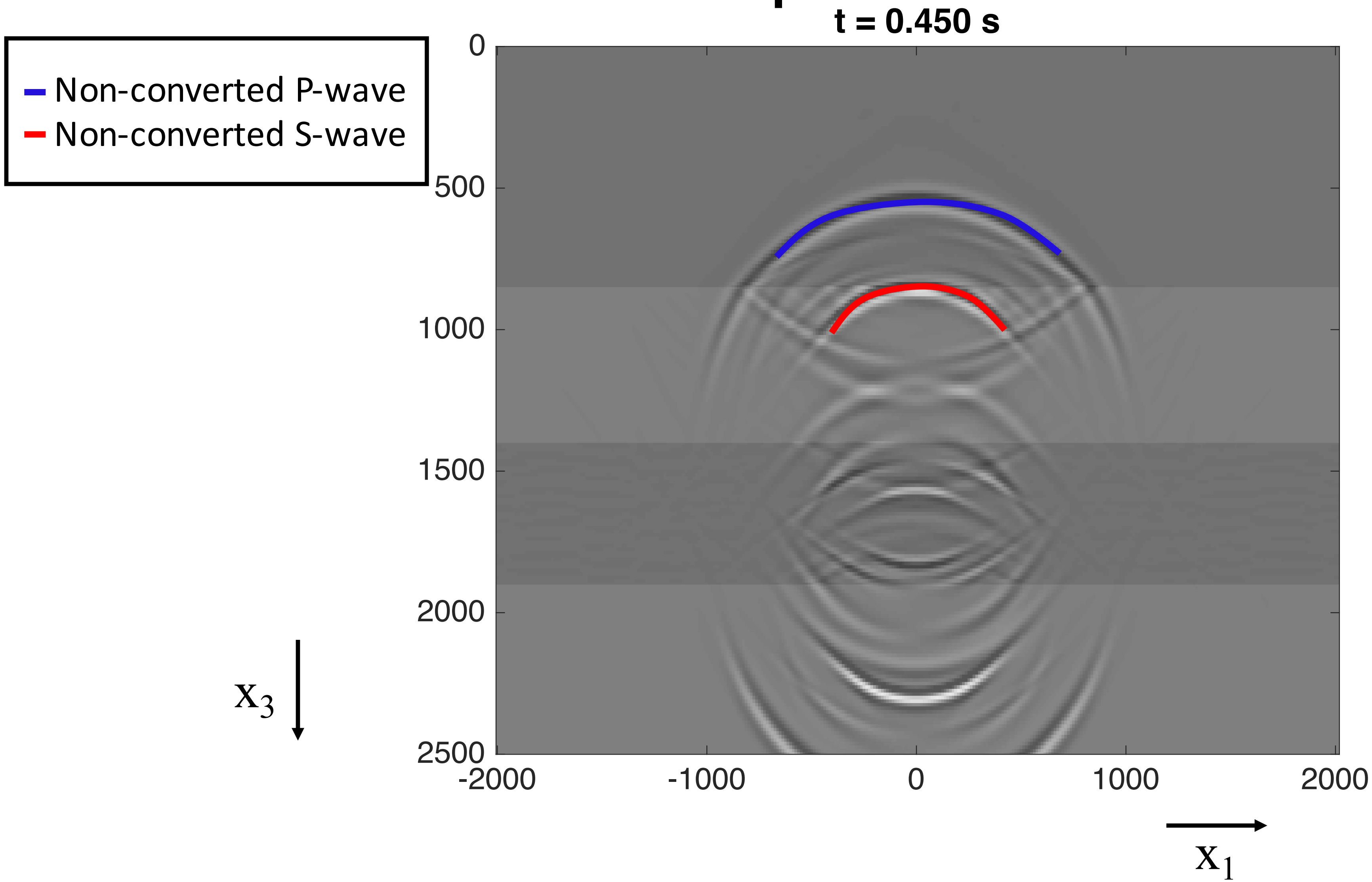
Elastodynamic double-sided homogeneous Green's function representation



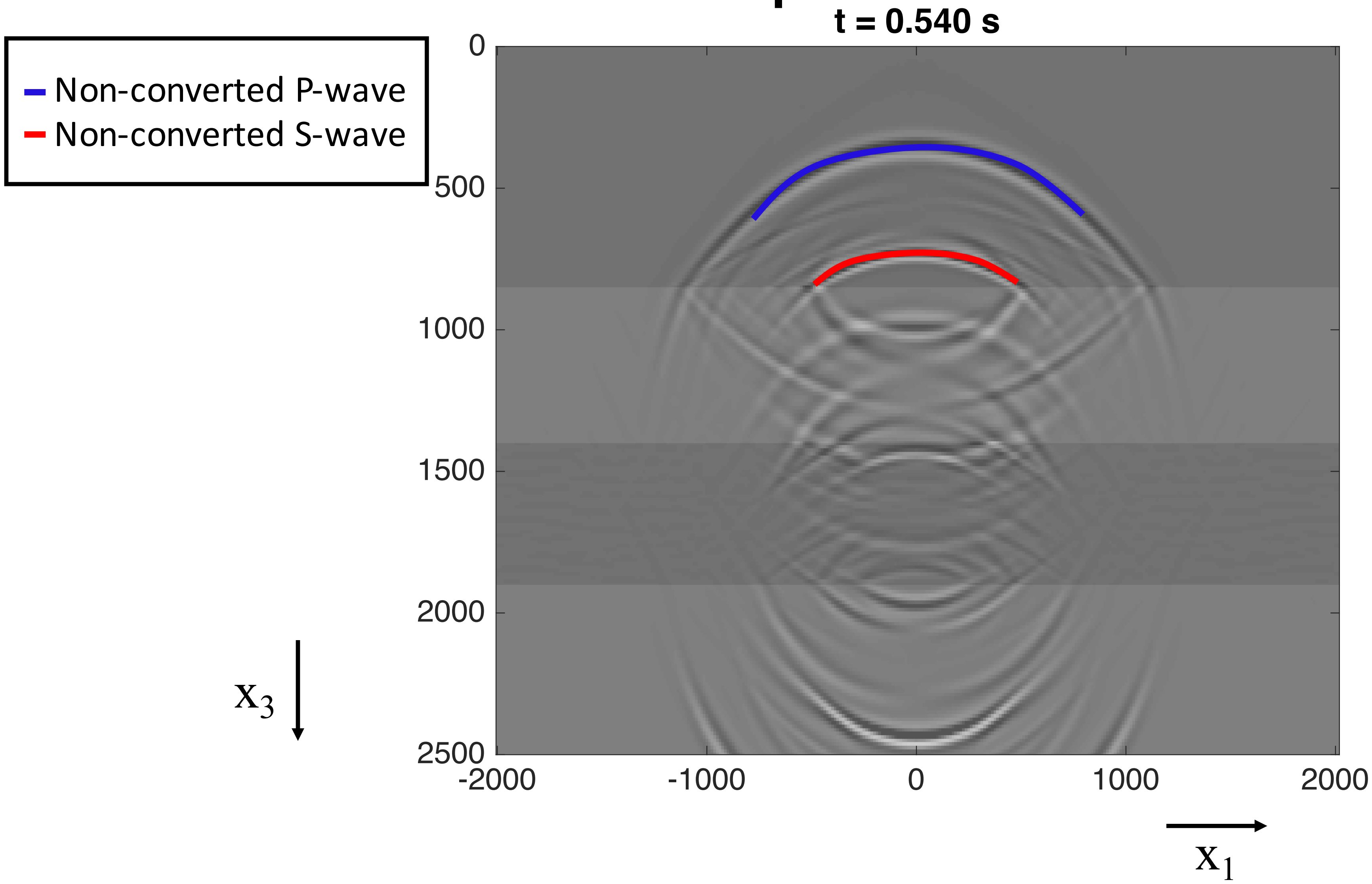
Elastodynamic double-sided homogeneous Green's function representation



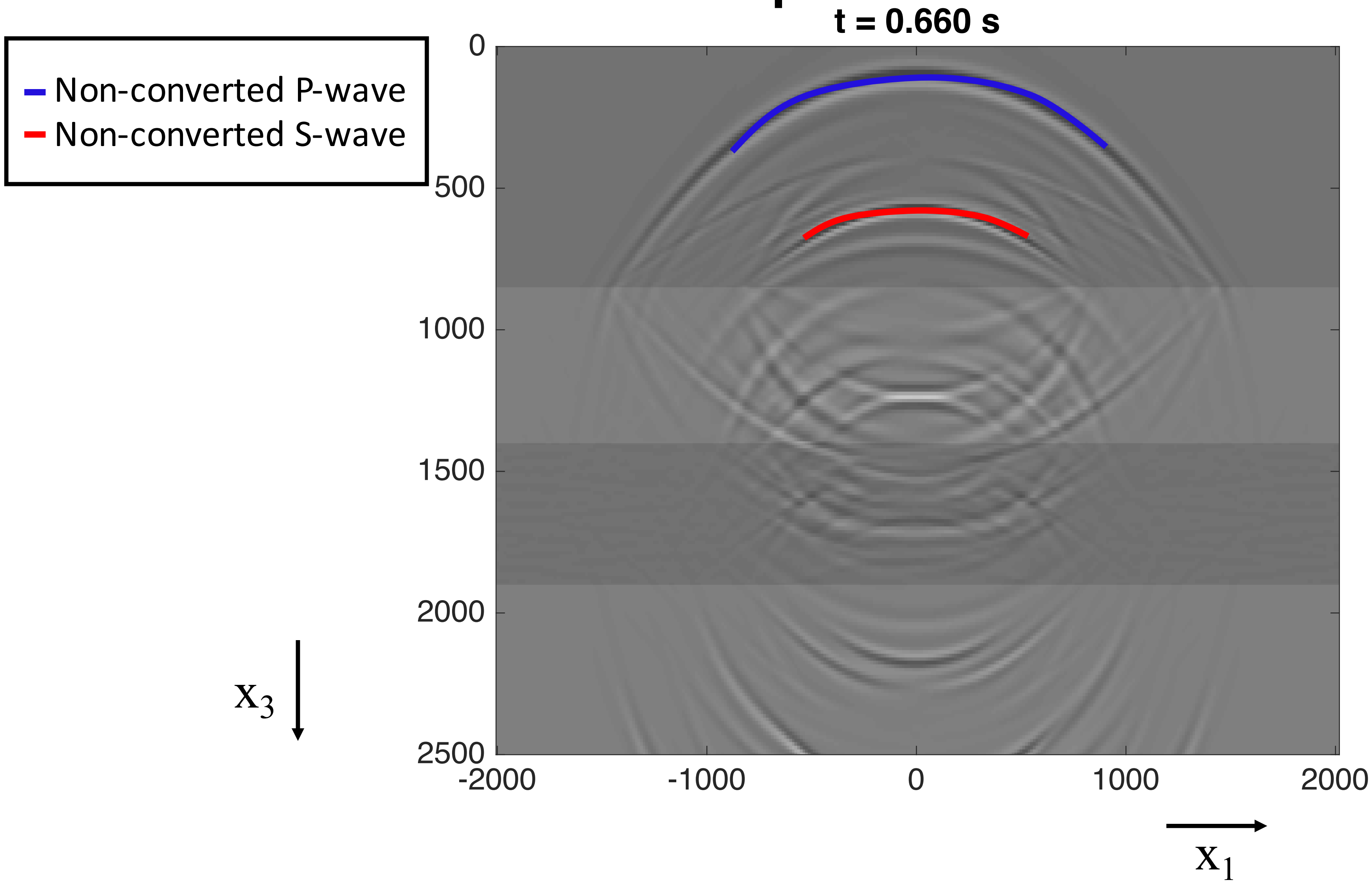
Elastodynamic double-sided homogeneous Green's function representation



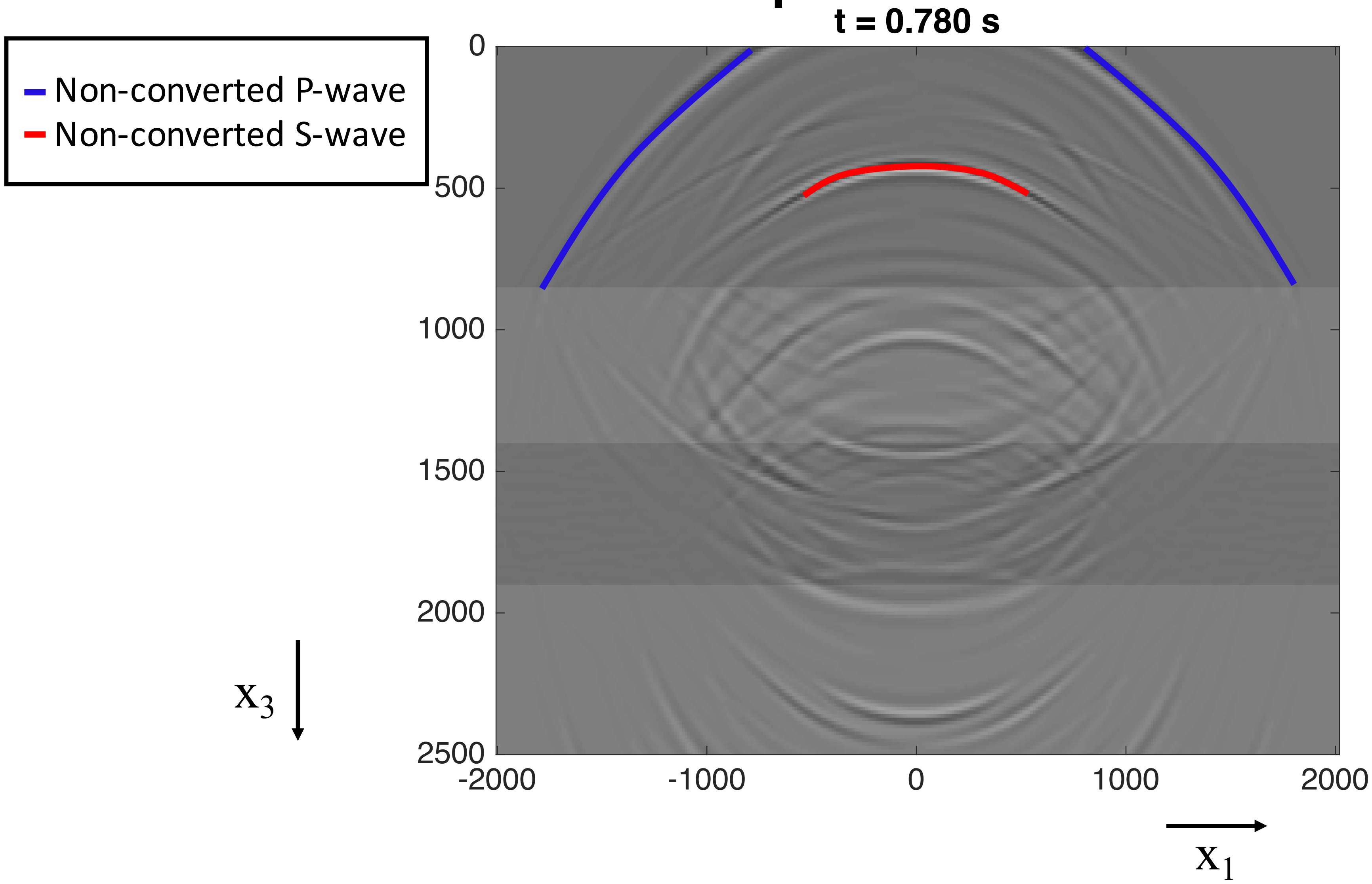
Elastodynamic double-sided homogeneous Green's function representation



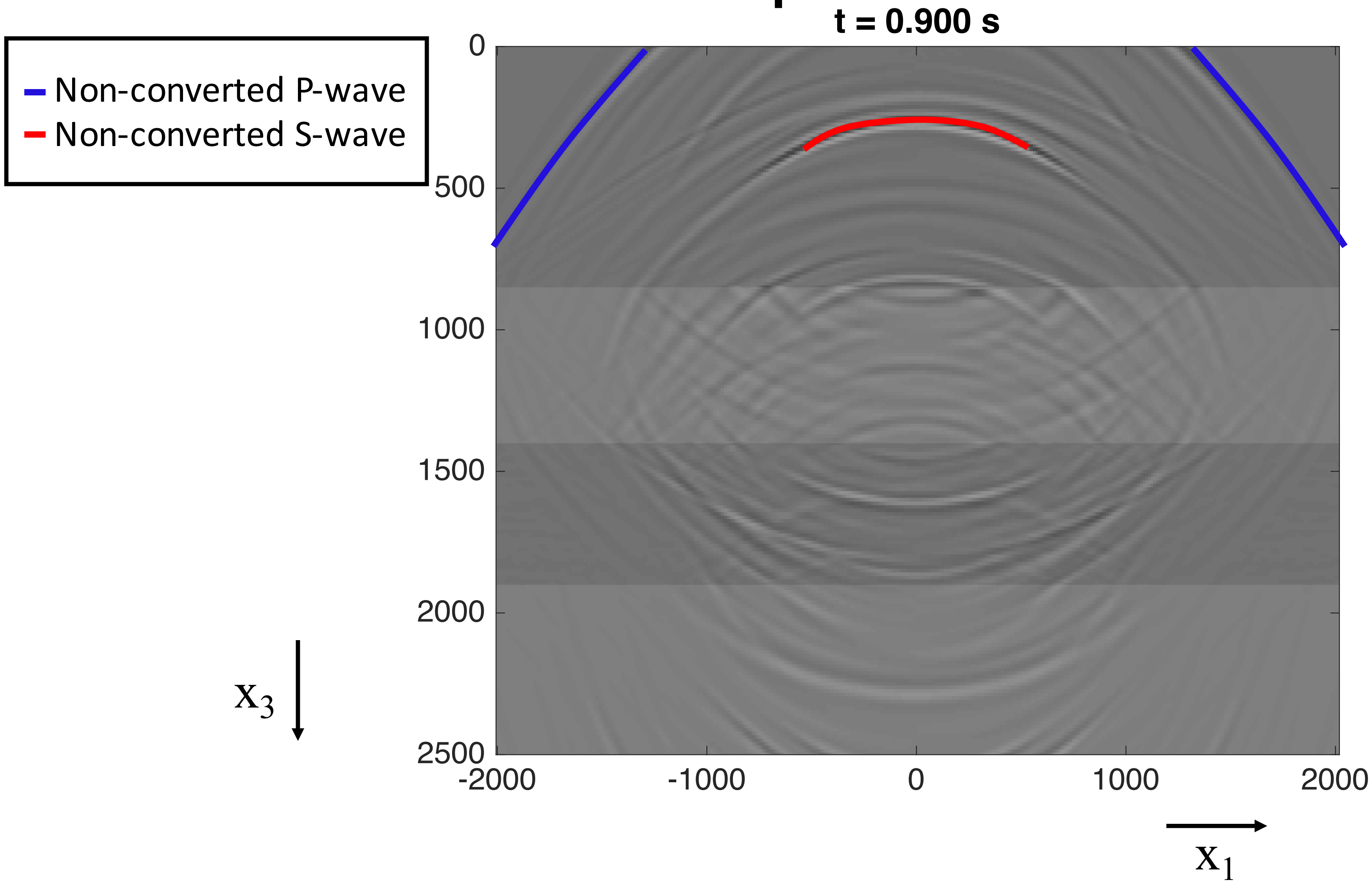
Elastodynamic double-sided homogeneous Green's function representation



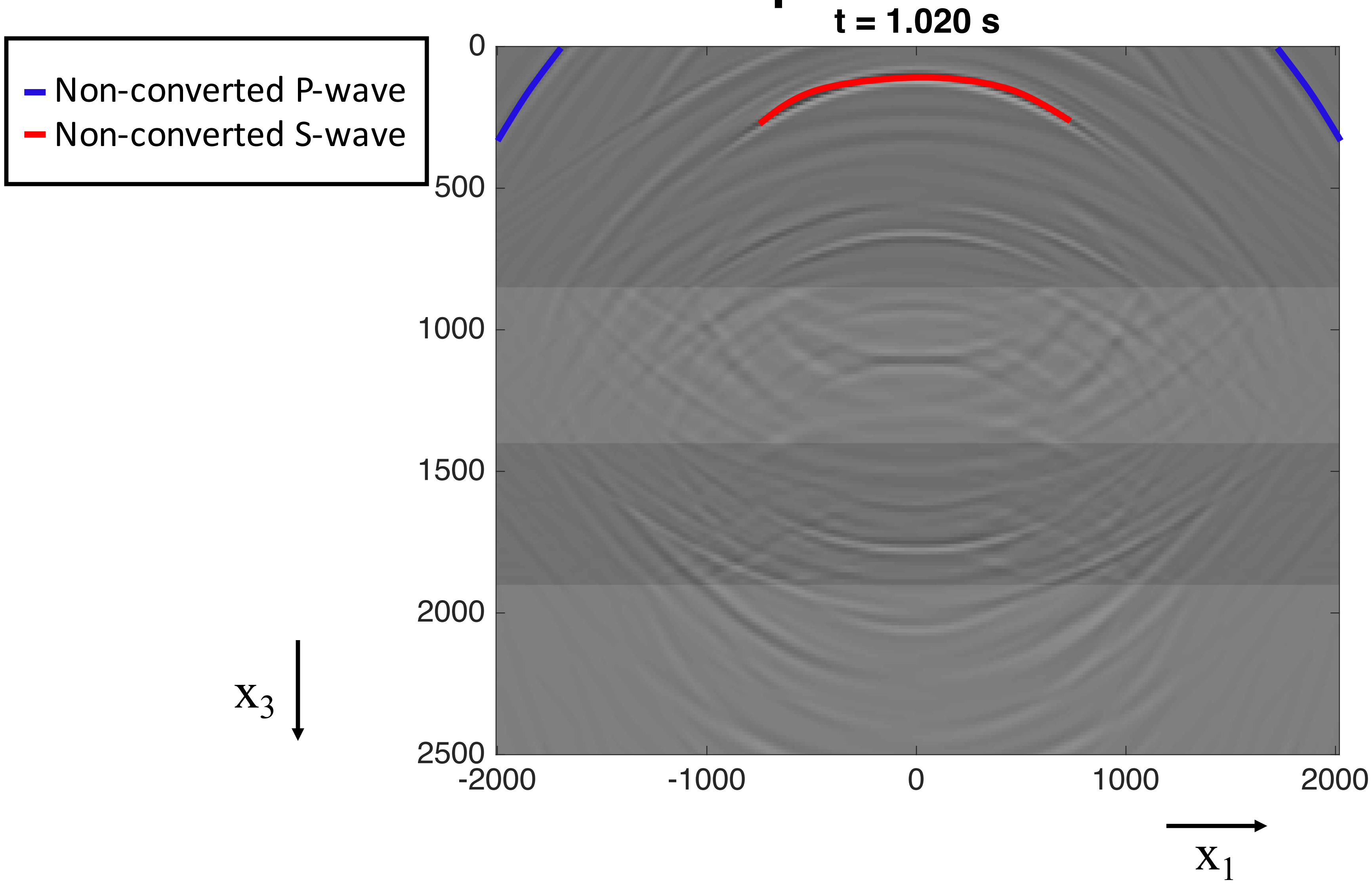
Elastodynamic double-sided homogeneous Green's function representation



Elastodynamic double-sided homogeneous Green's function representation



Elastodynamic double-sided homogeneous Green's function representation



Contents

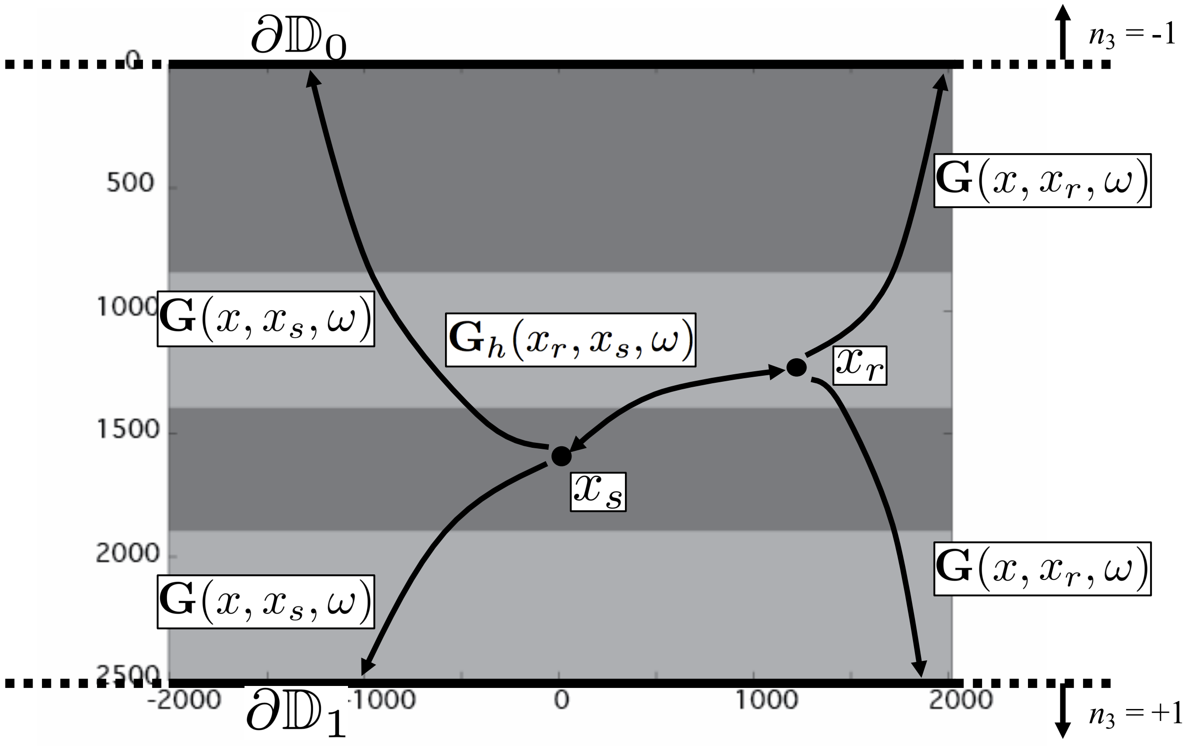
1. Introduction
2. Homogeneous Green's function
3. Elastodynamic double-sided homogeneous Green's function representation
4. **Elastodynamic single-sided homogeneous Green's function representation**
5. Conclusions

TU Delft

WAVES

Elastodynamic double-sided homogeneous Green's function representation

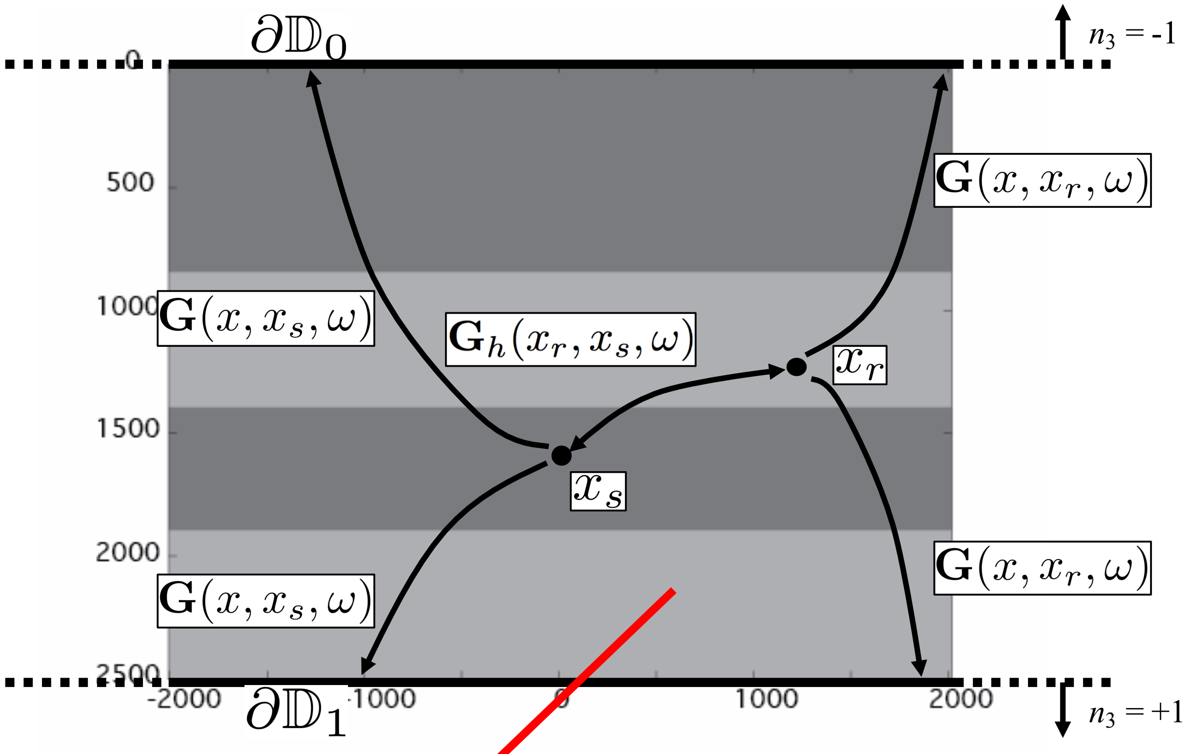
$$\mathbf{G}_h(x_r, x_s, \omega) = \int_{\partial\mathbb{D}_0 \cup \partial\mathbb{D}_1} \mathbf{J}\mathbf{G}^{\dagger}(x, x_r, \omega)\mathbf{J}\mathbf{G}(x, x_s, \omega) n_3 \mathrm{d}^2 x$$



(Wapenaar, Kees, Joost van der Neut, and Evert Slob. "Unified double-and single-sided homogeneous Green's function representations." *Proc. R. Soc. A*. Vol. 472. No. 2190. The Royal Society, 2016.)

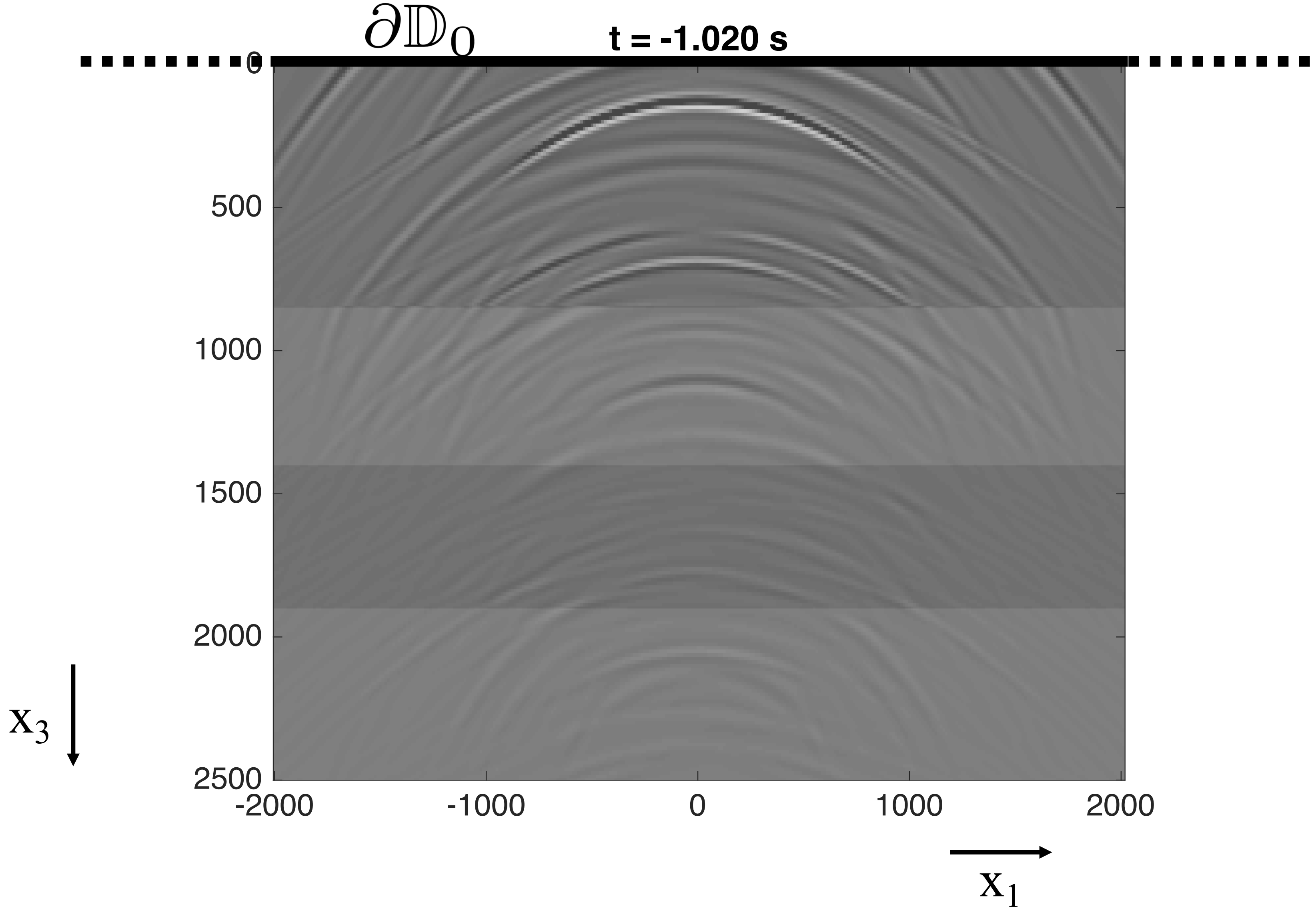
Remove bottom boundary

$$\mathbf{G}_h(x_r, x_s, \omega) \cancel{=} \int_{\partial\mathbb{D}_0 \cup \cancel{\partial\mathbb{D}_1}} \mathbf{J}\mathbf{G}^{\dagger}(x, x_r, \omega)\mathbf{J}\mathbf{G}(x, x_s, \omega) n_3 \mathrm{d}^2 x$$

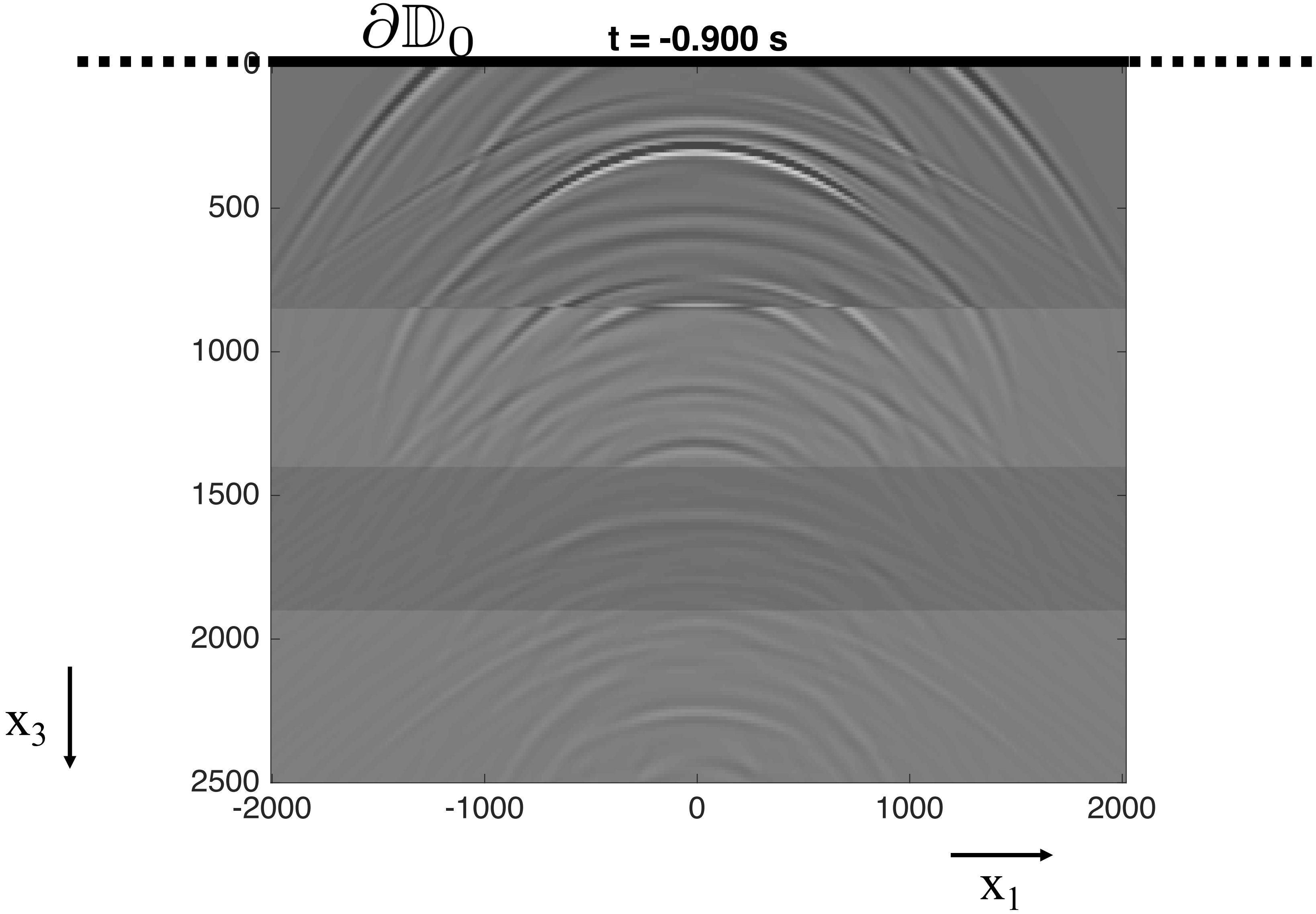


(Wapenaar, Kees, Joost van der Neut, and Evert Slob. "Unified double-and single-sided homogeneous Green's function representations." *Proc. R. Soc. A*. Vol. 472. No. 2190. The Royal Society, 2016.)

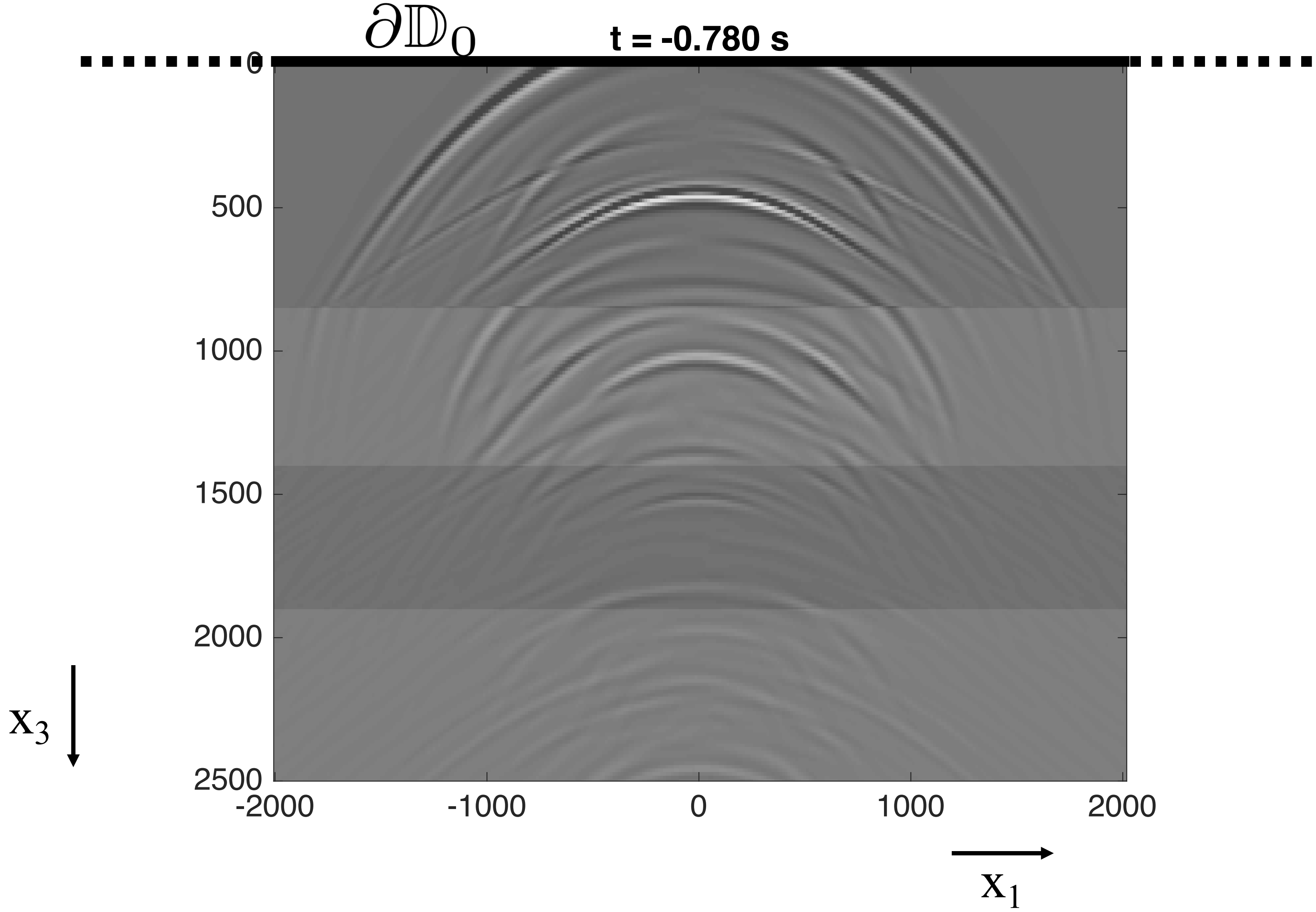
Remove bottom boundary



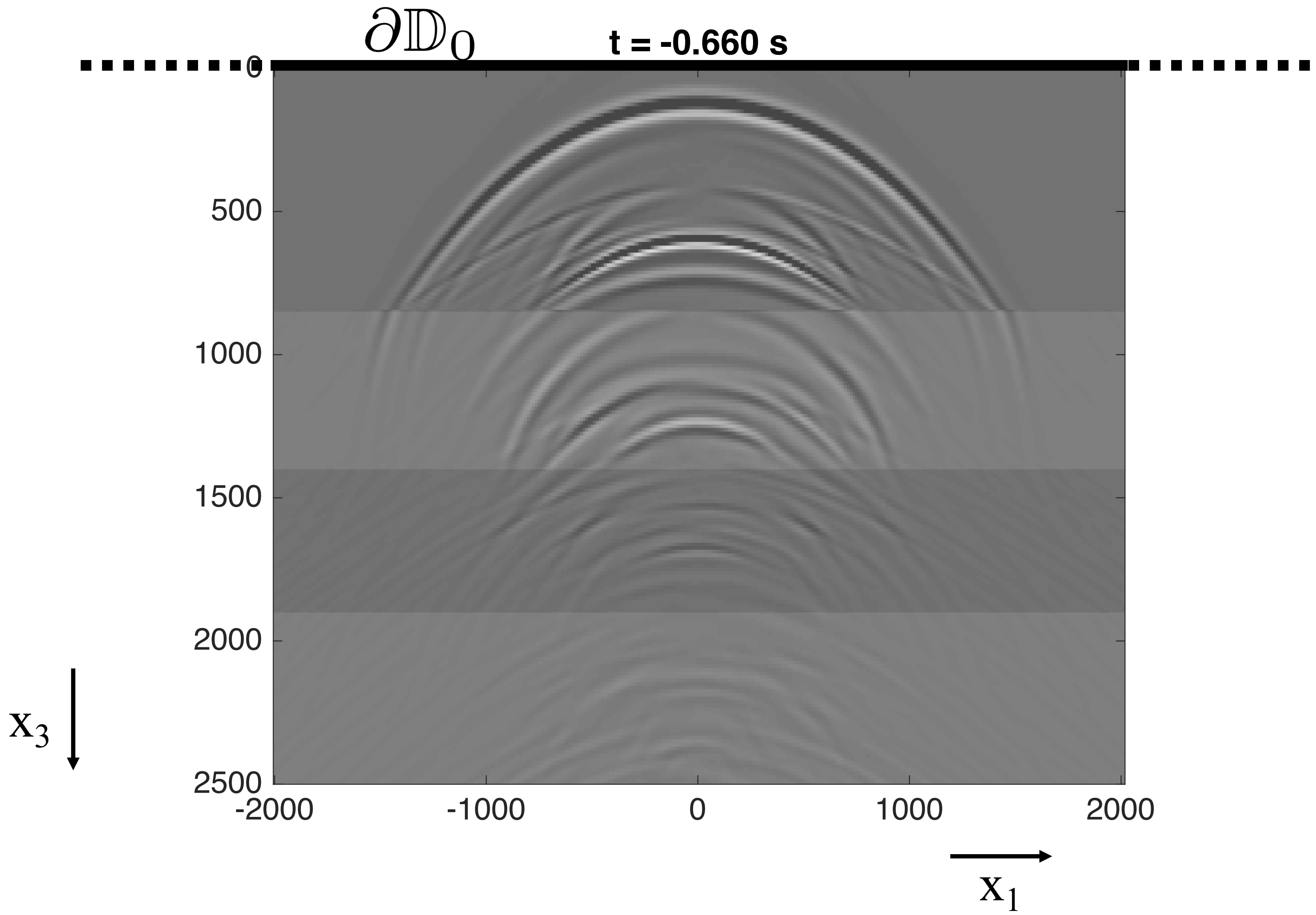
Remove bottom boundary



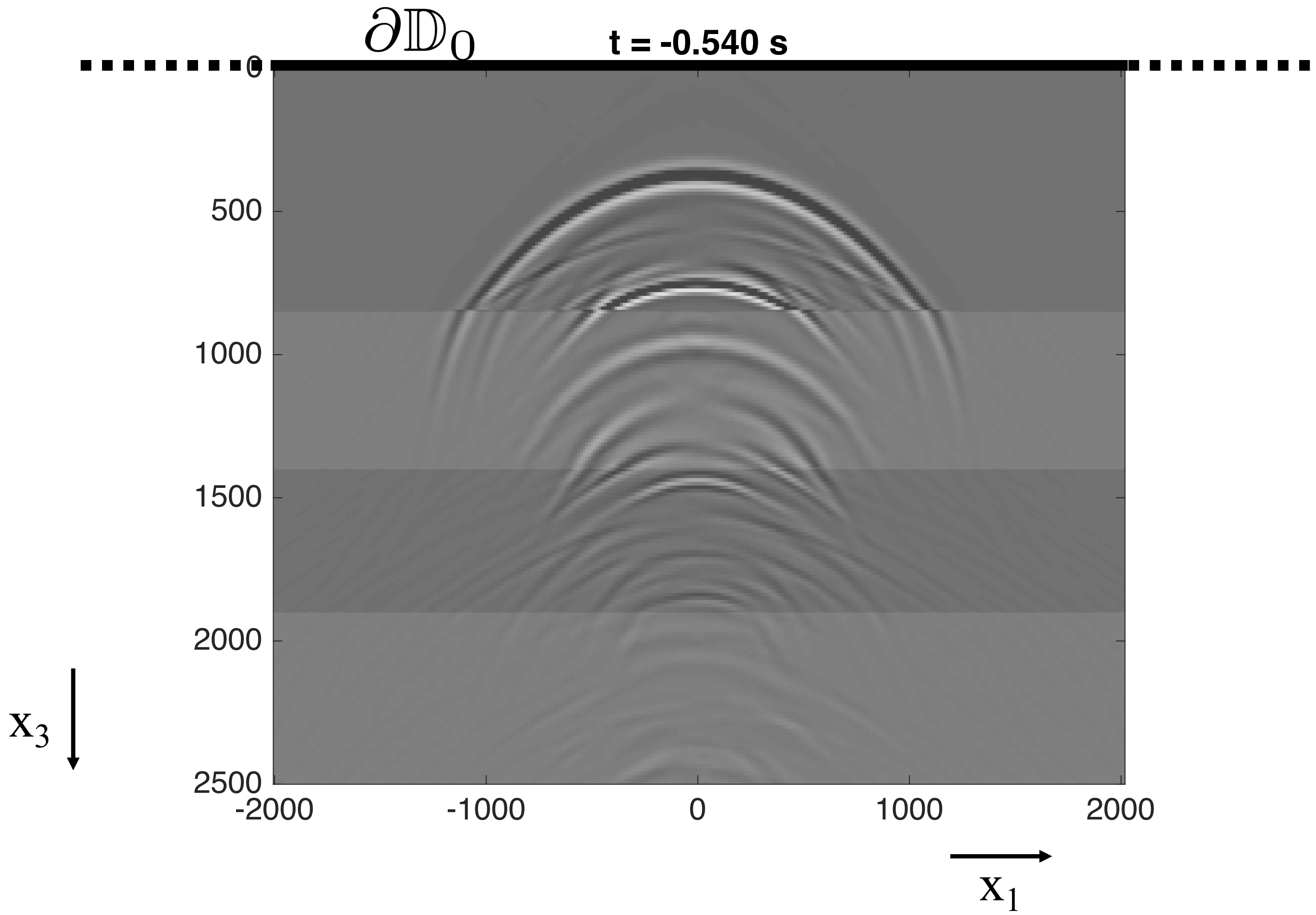
Remove bottom boundary



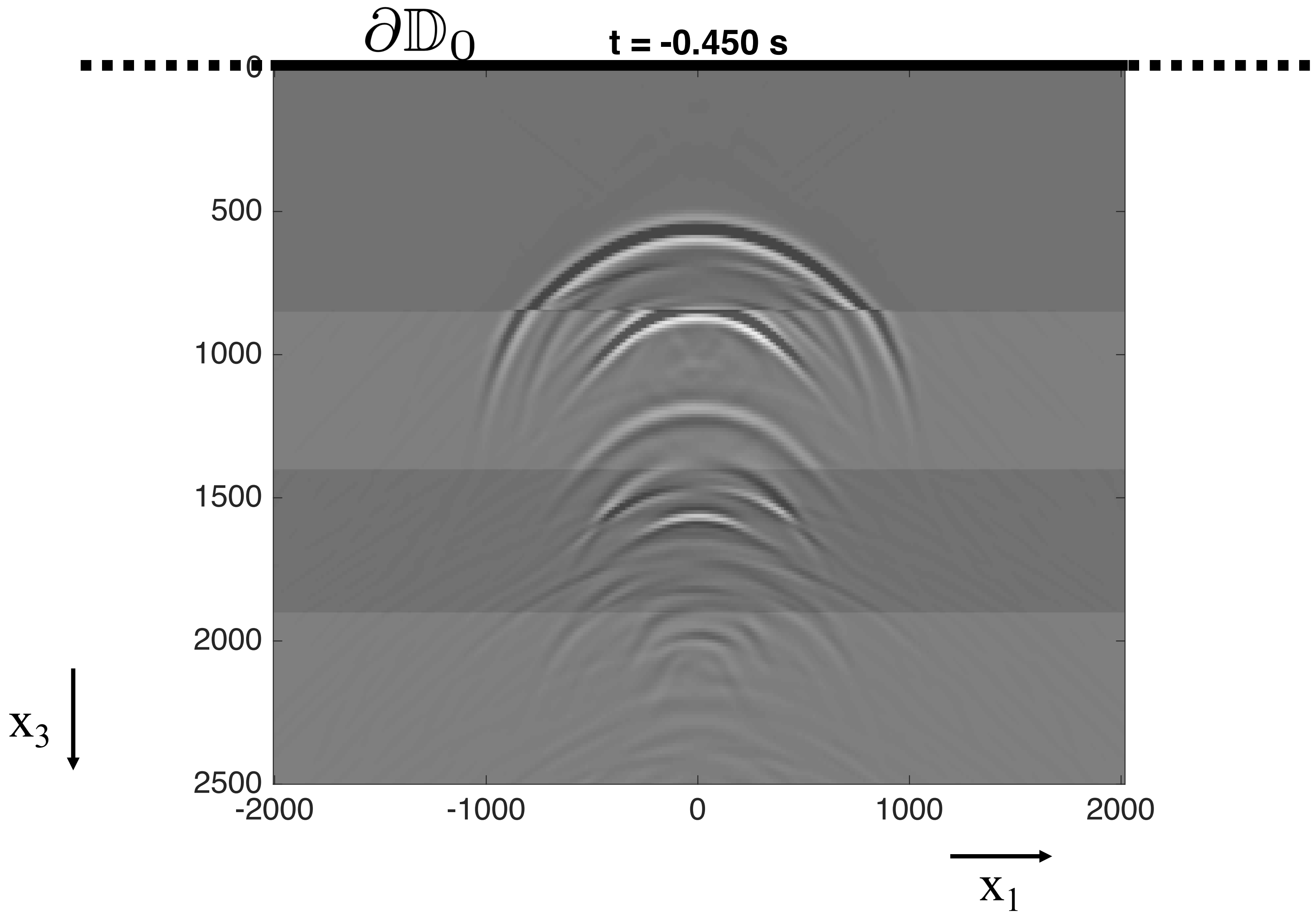
Remove bottom boundary



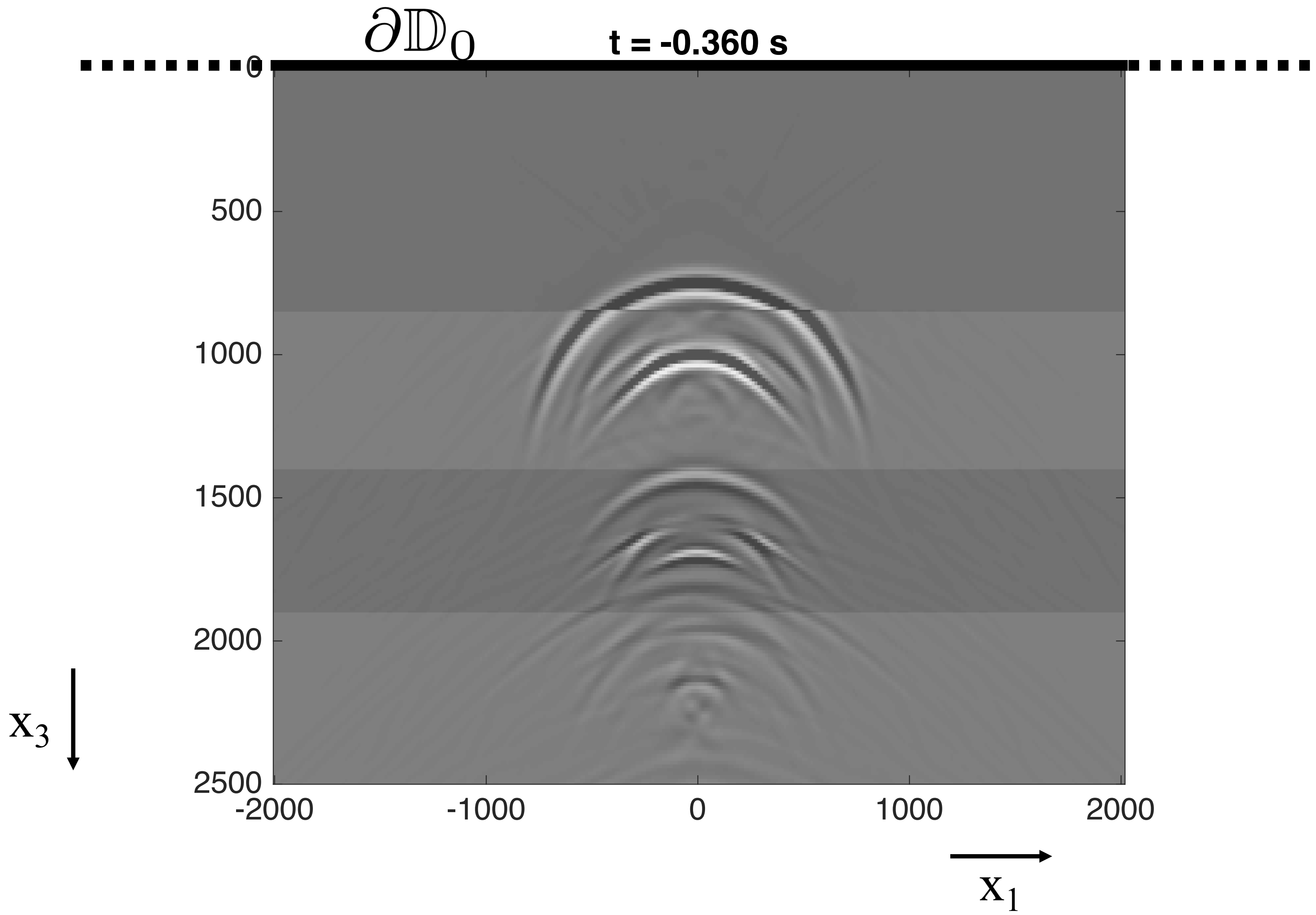
Remove bottom boundary



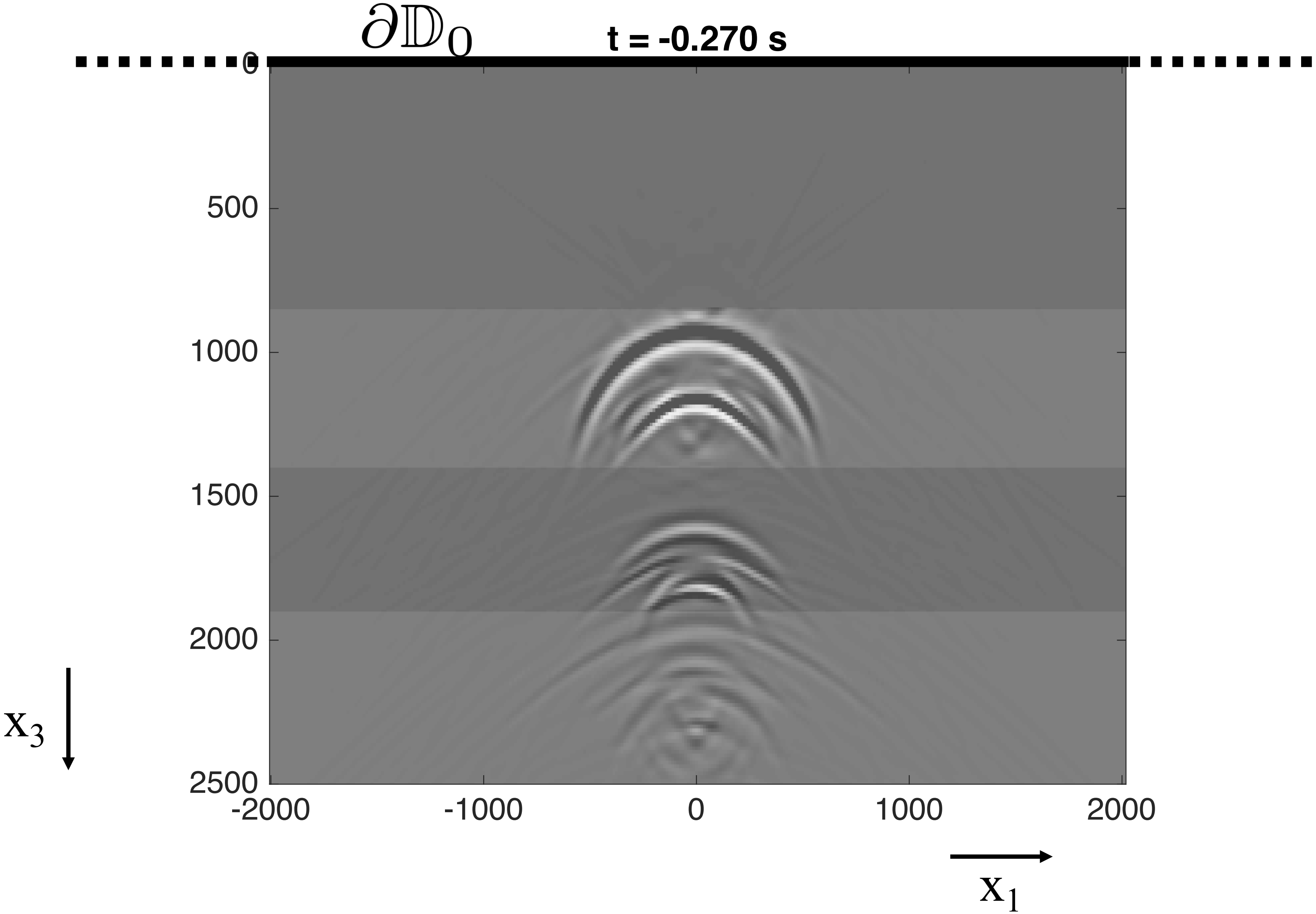
Remove bottom boundary



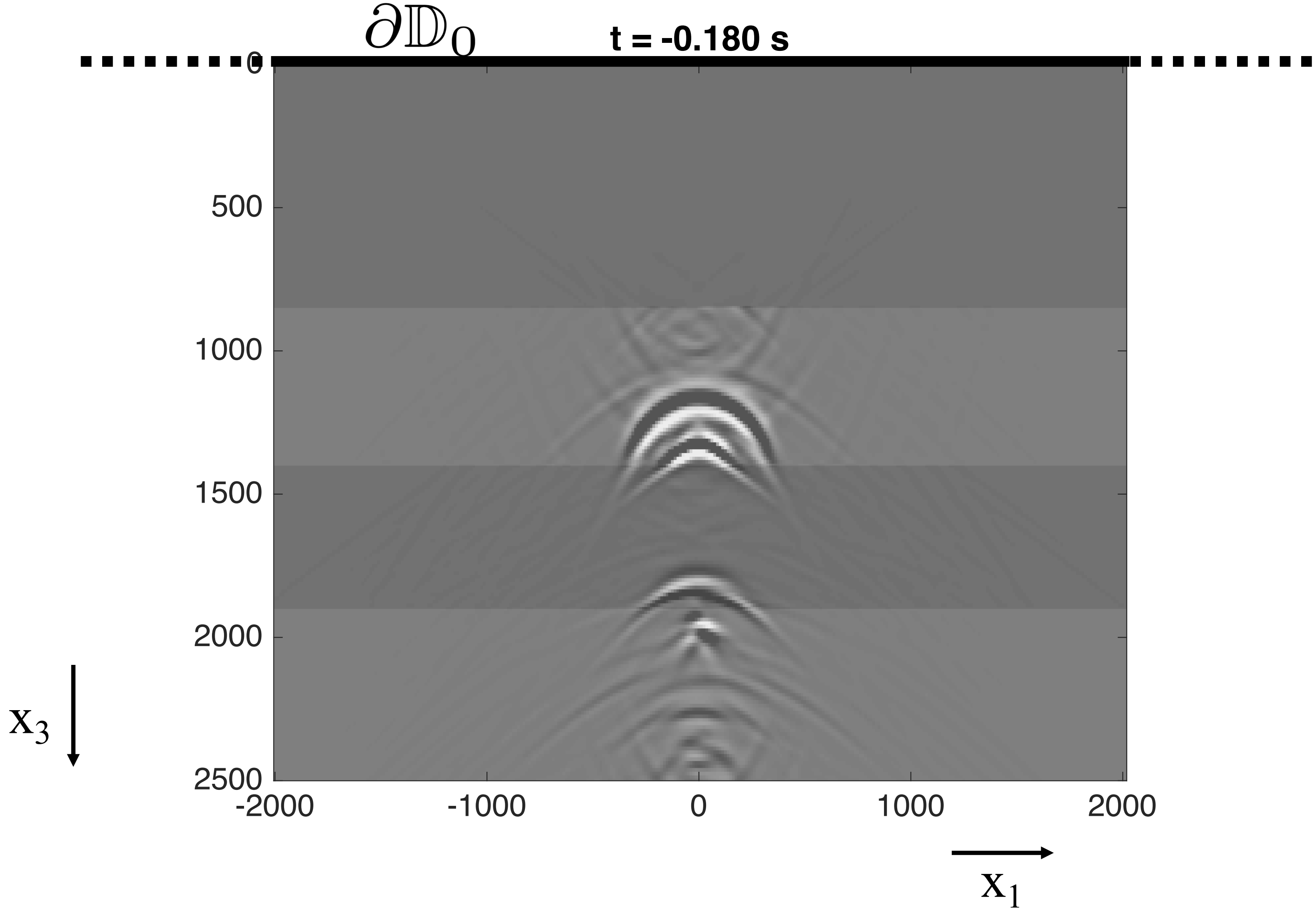
Remove bottom boundary



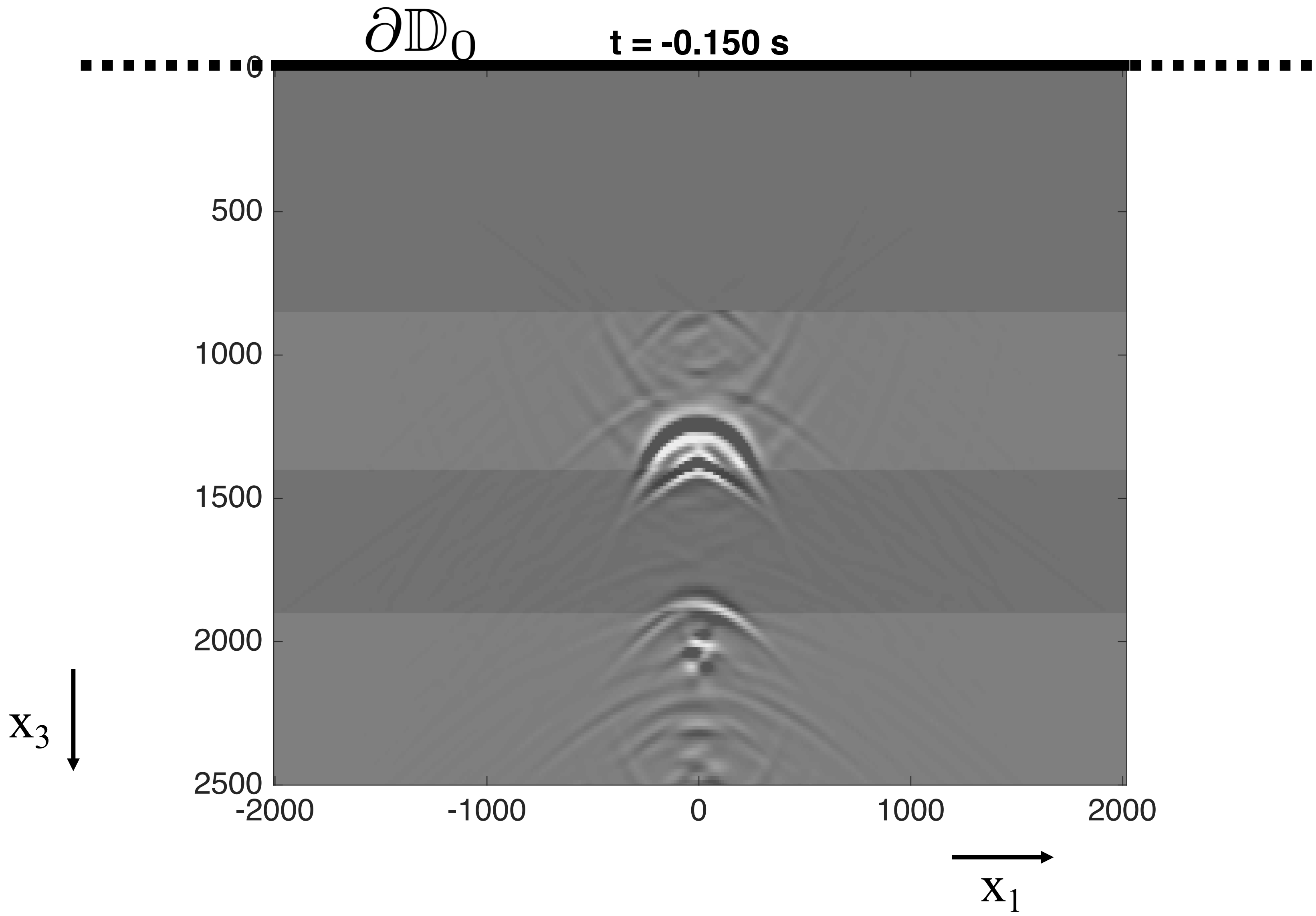
Remove bottom boundary



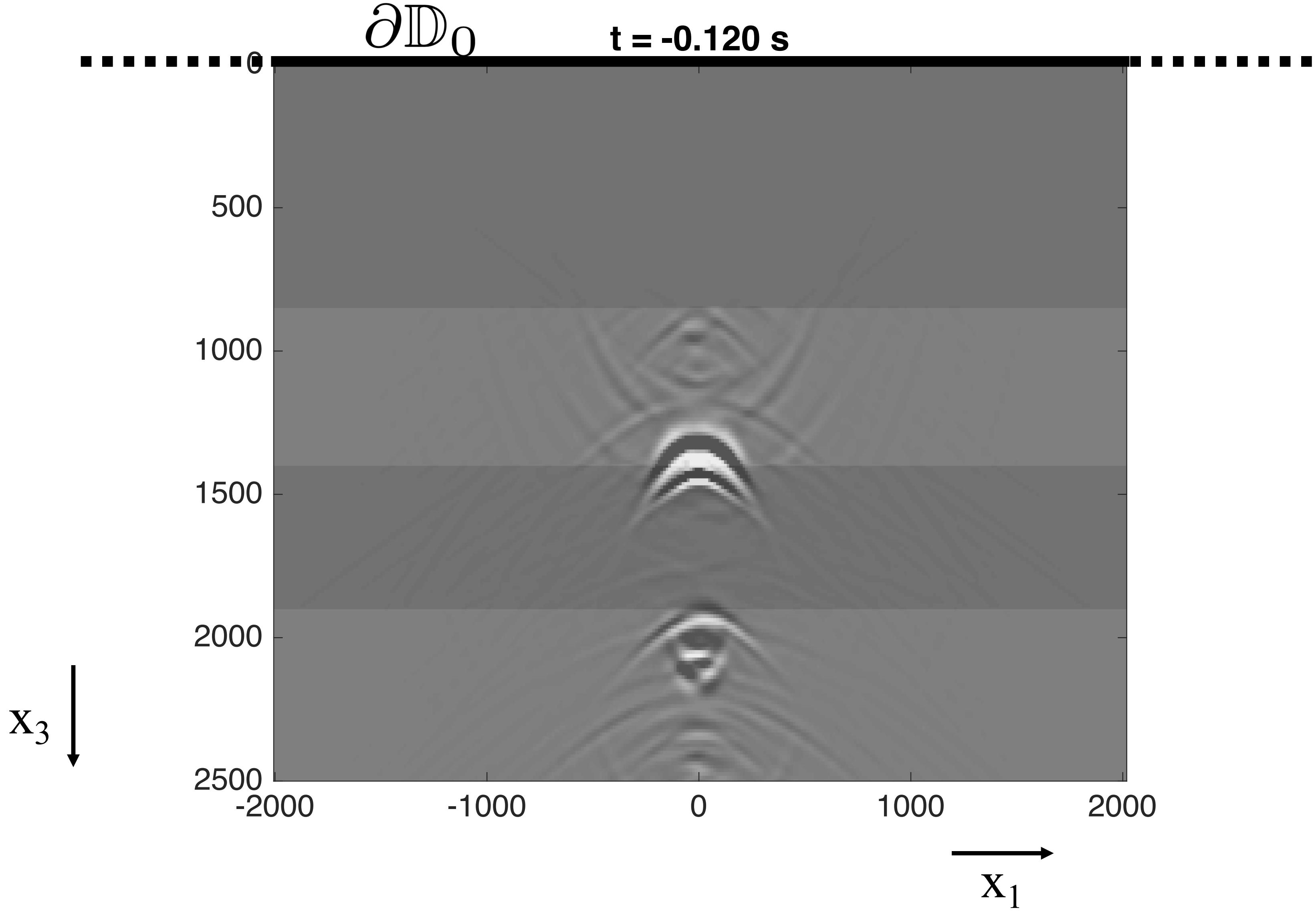
Remove bottom boundary



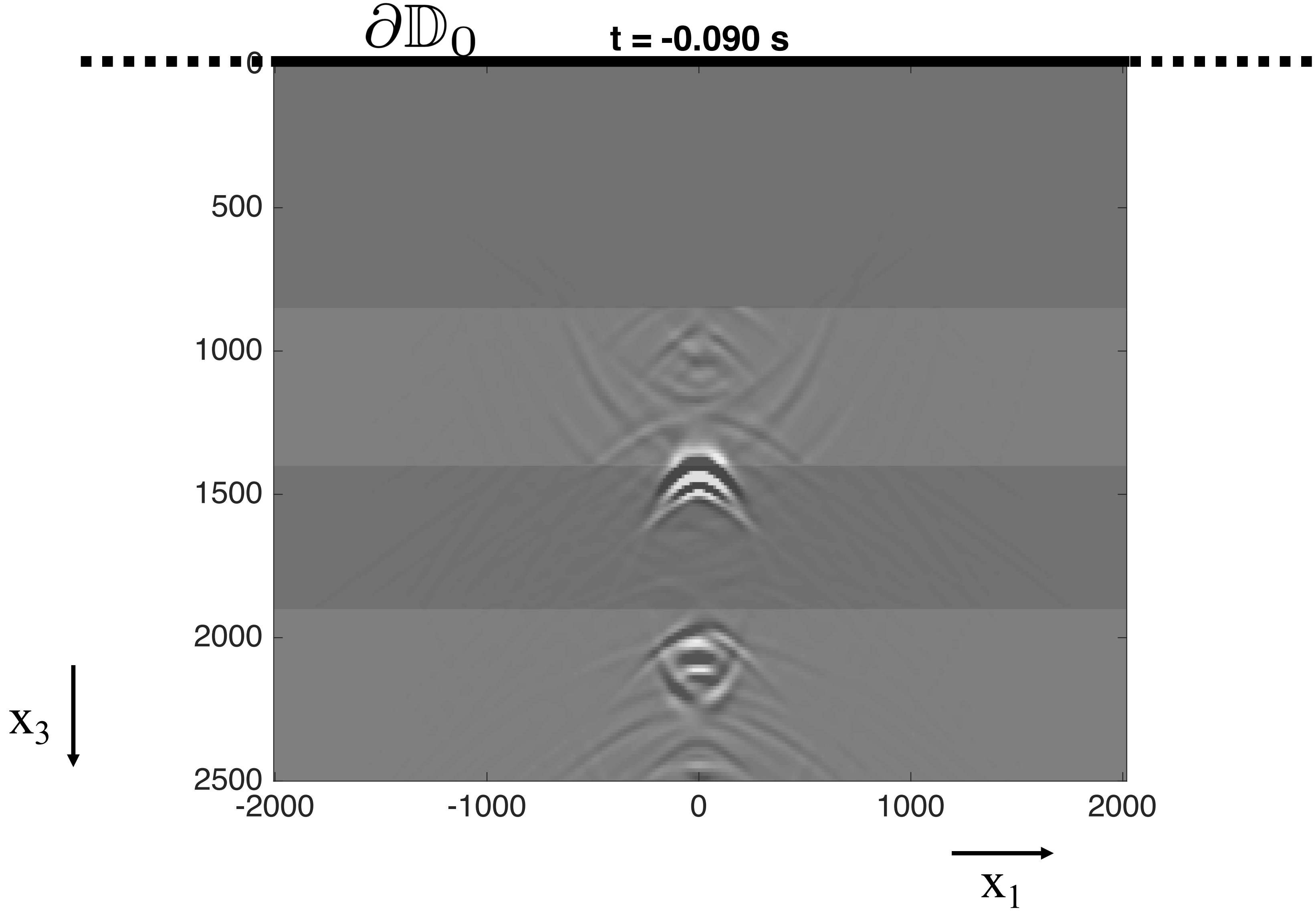
Remove bottom boundary



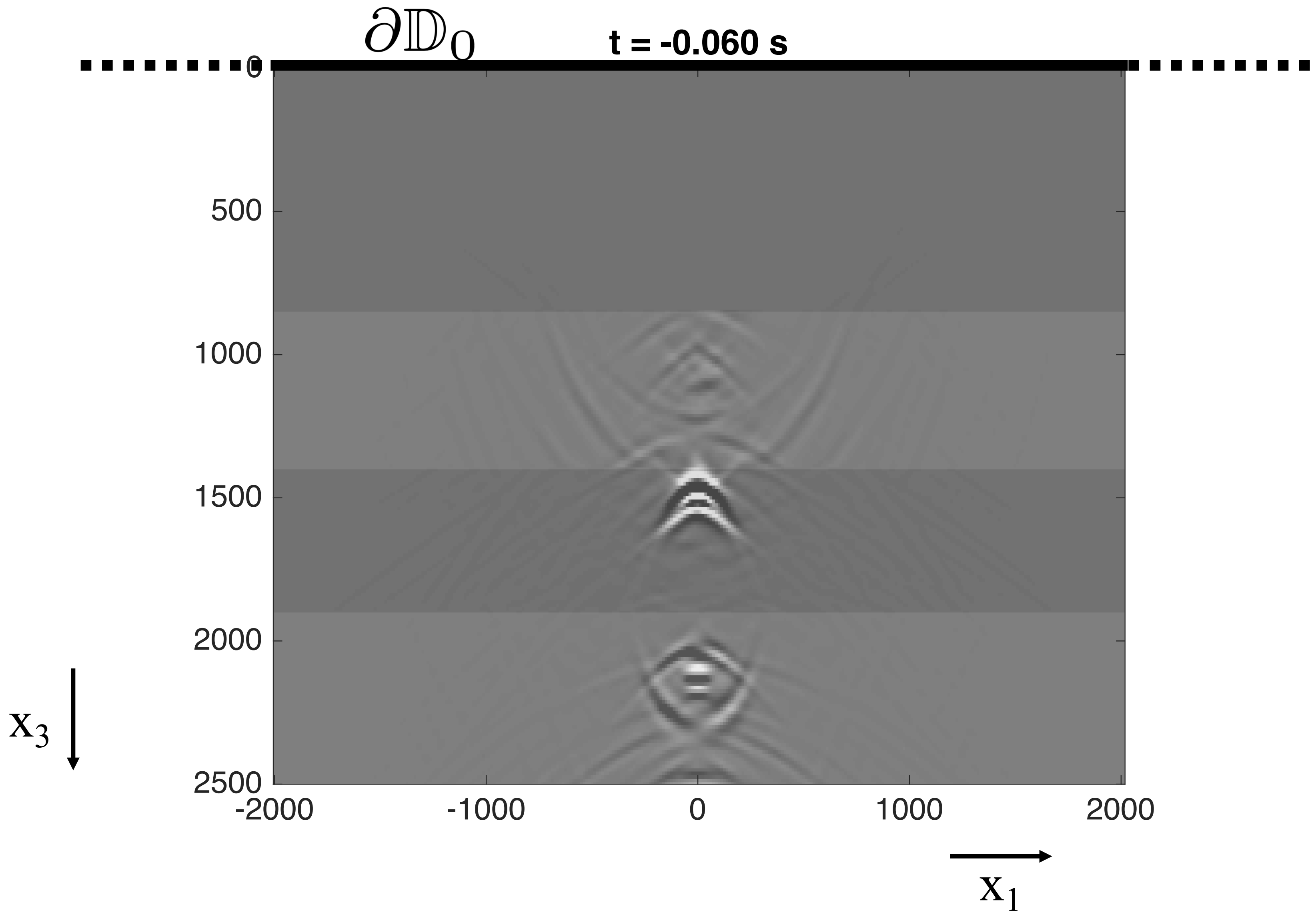
Remove bottom boundary



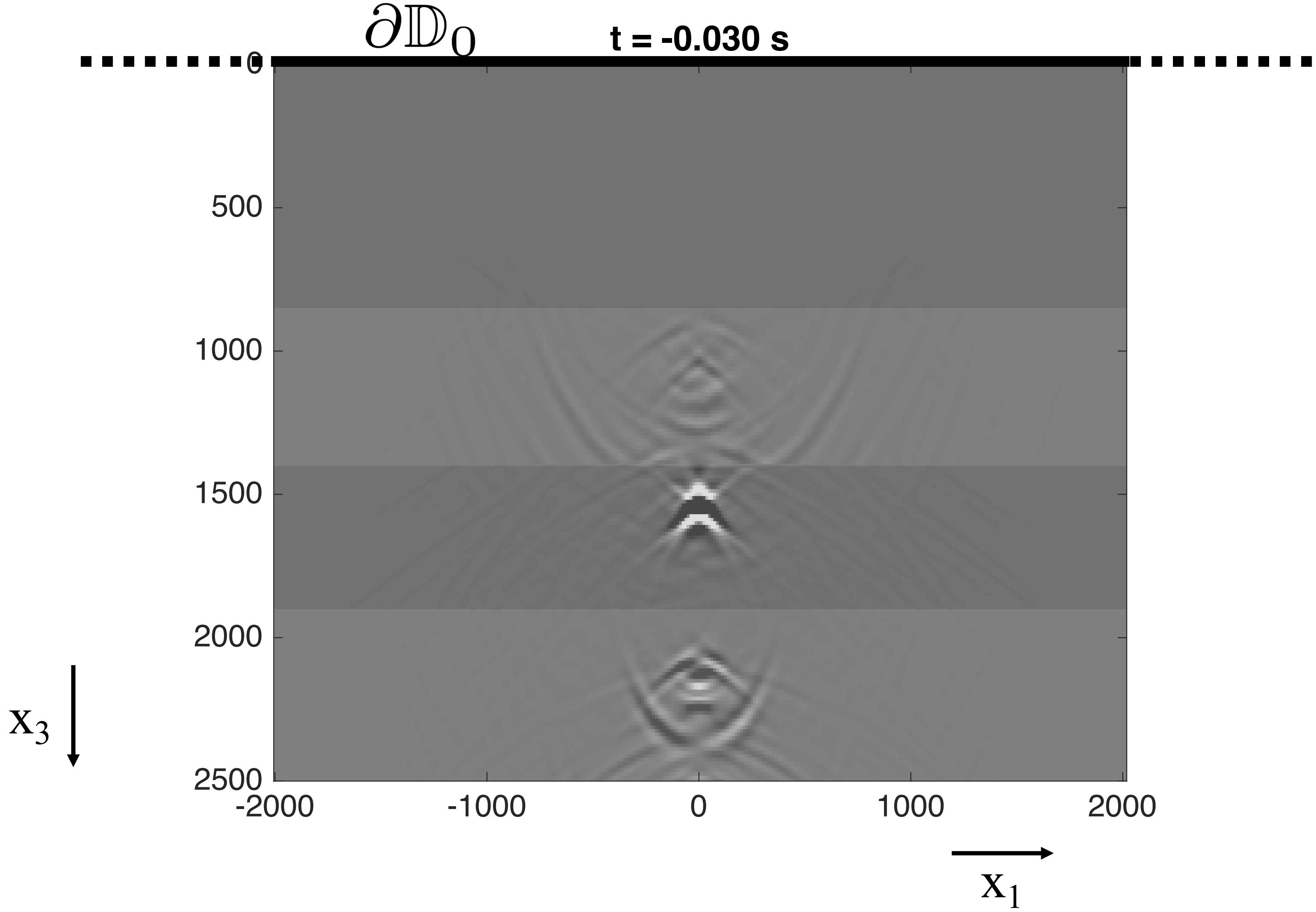
Remove bottom boundary



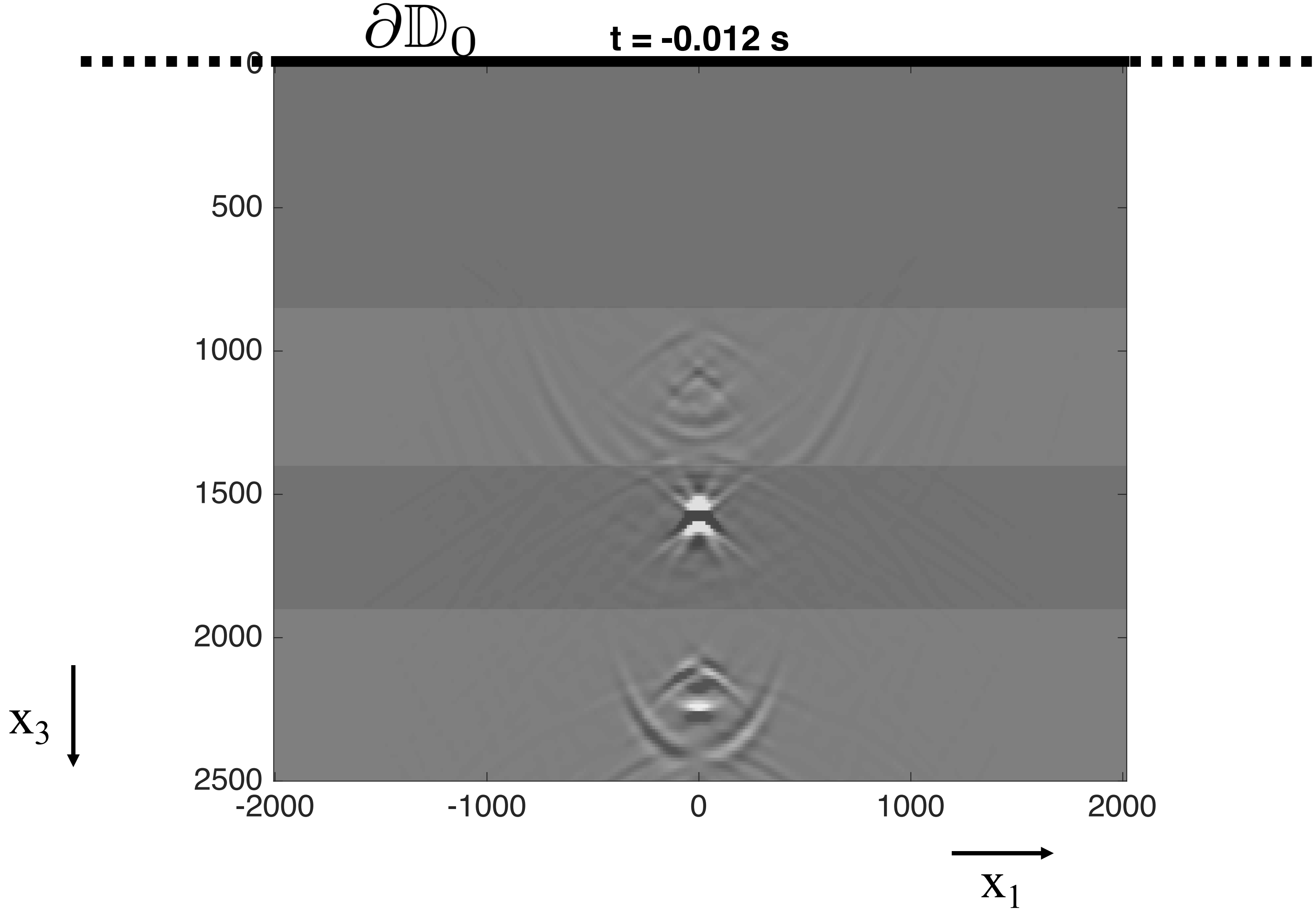
Remove bottom boundary



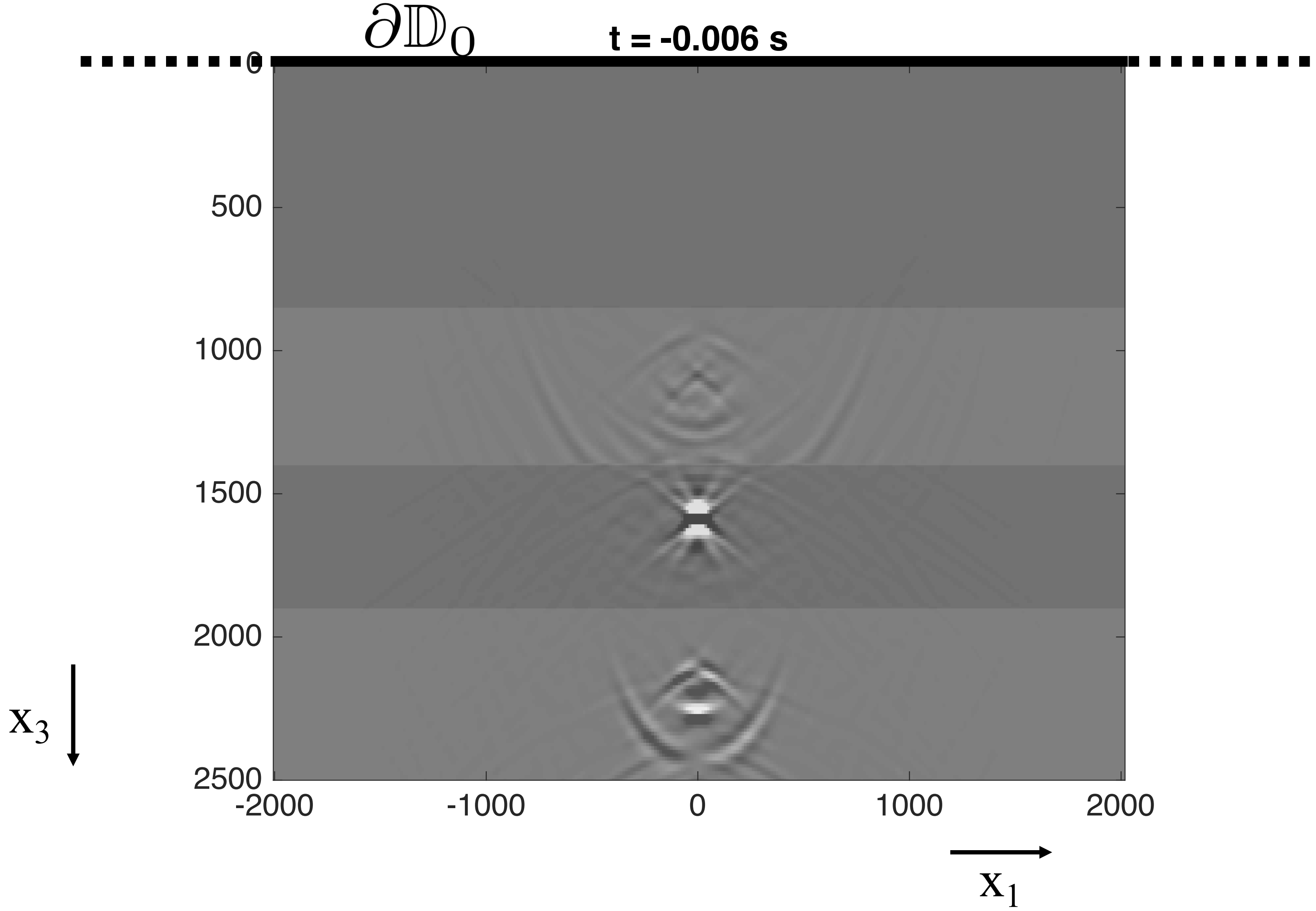
Remove bottom boundary



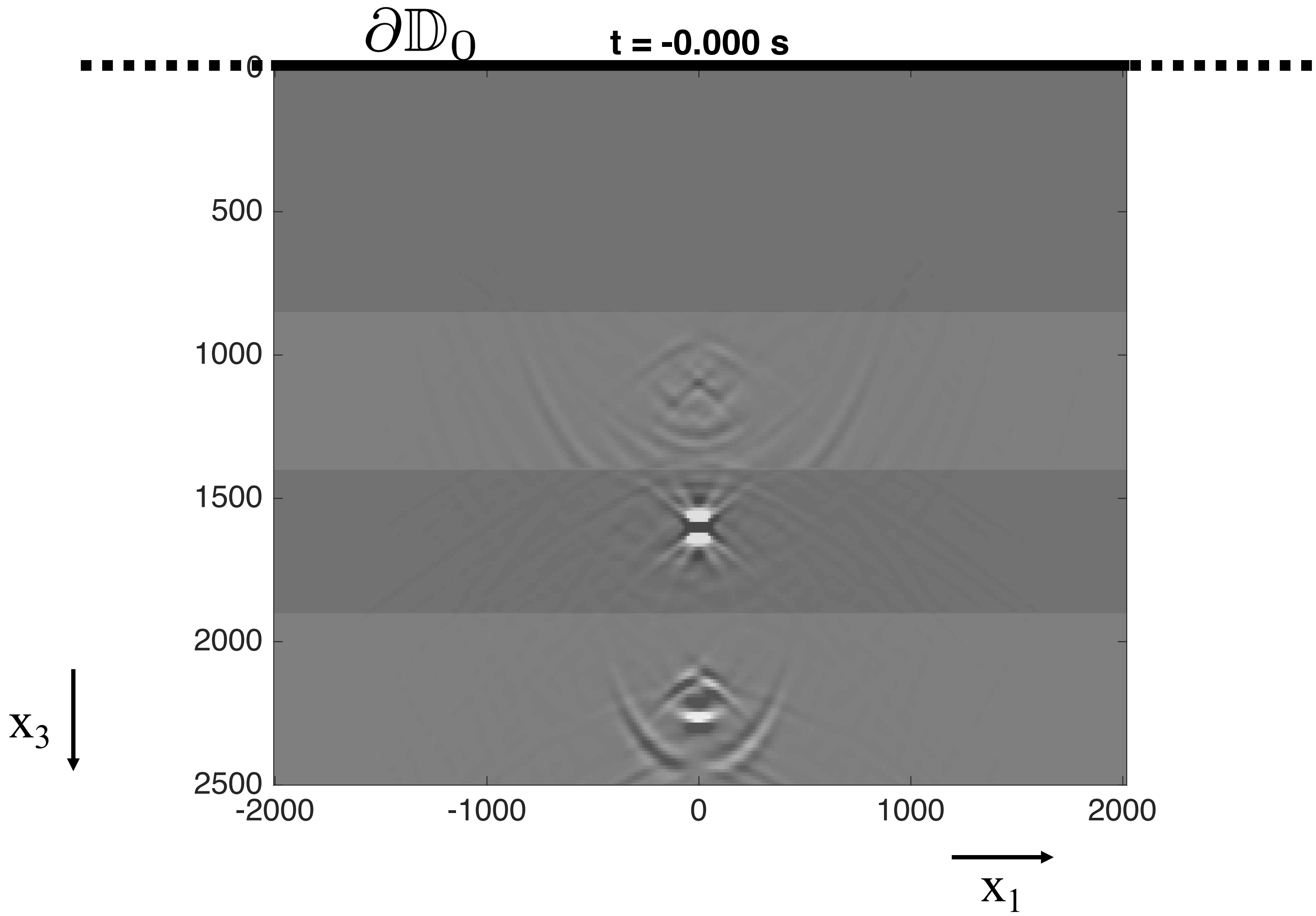
Remove bottom boundary



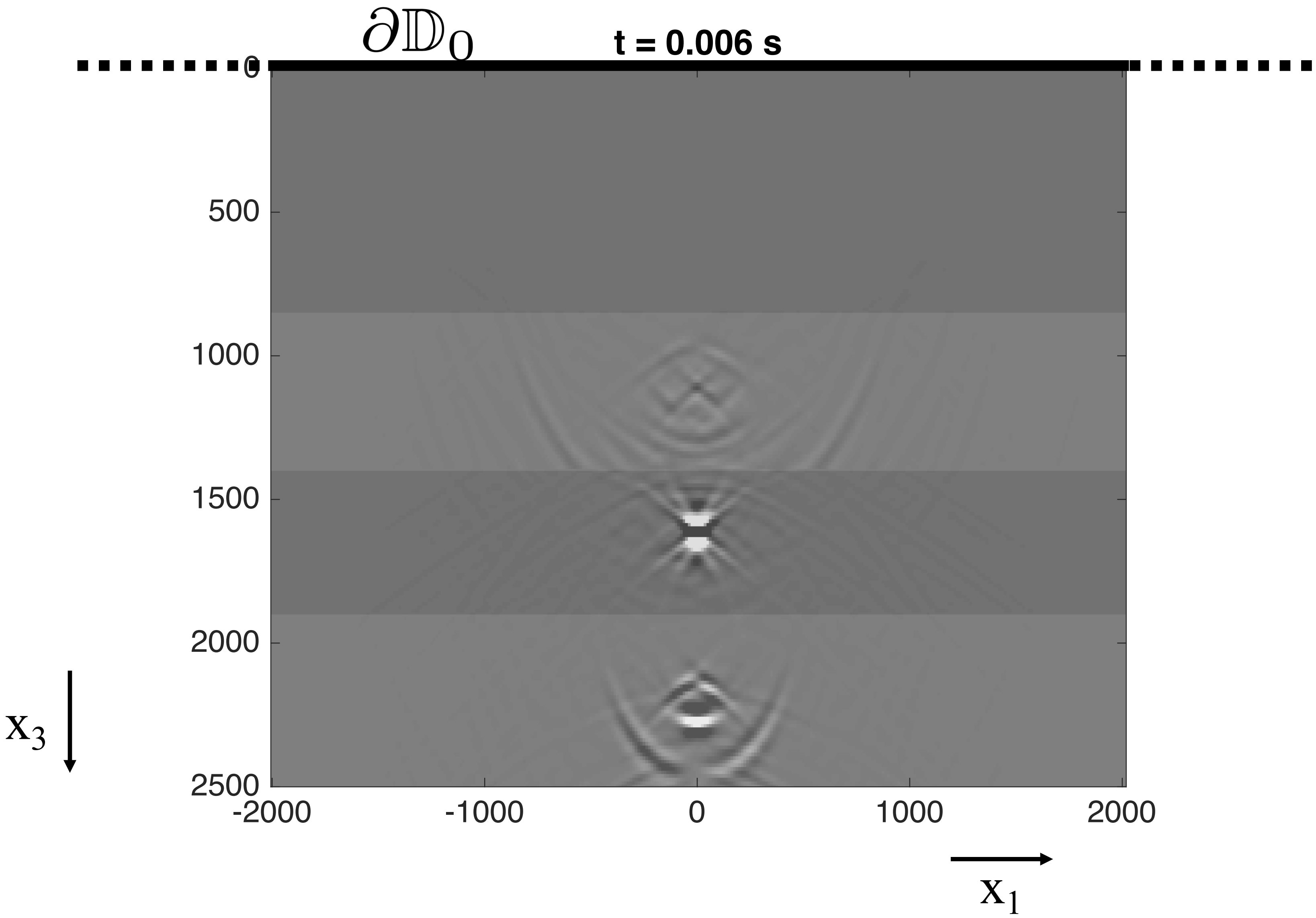
Remove bottom boundary



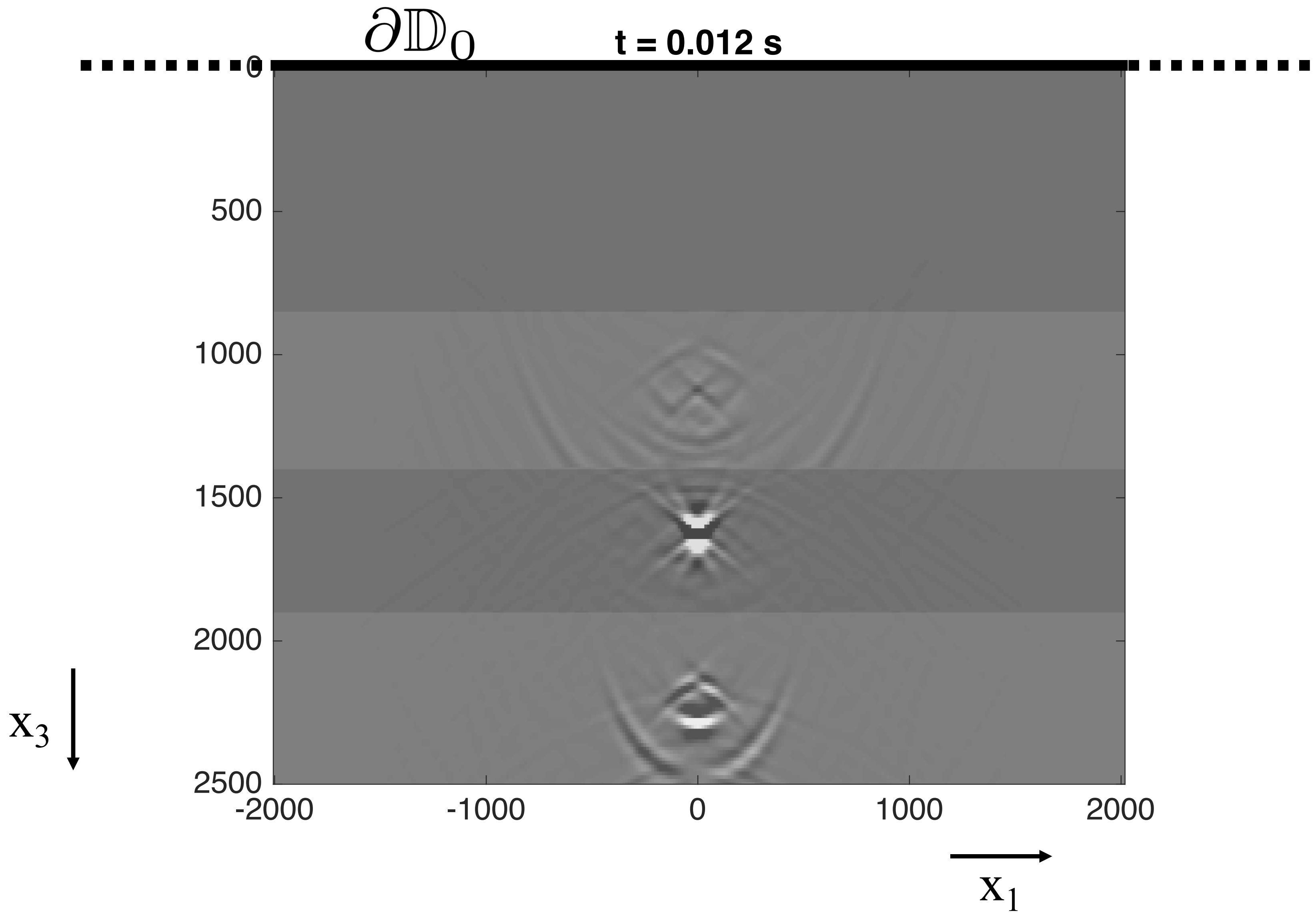
Remove bottom boundary



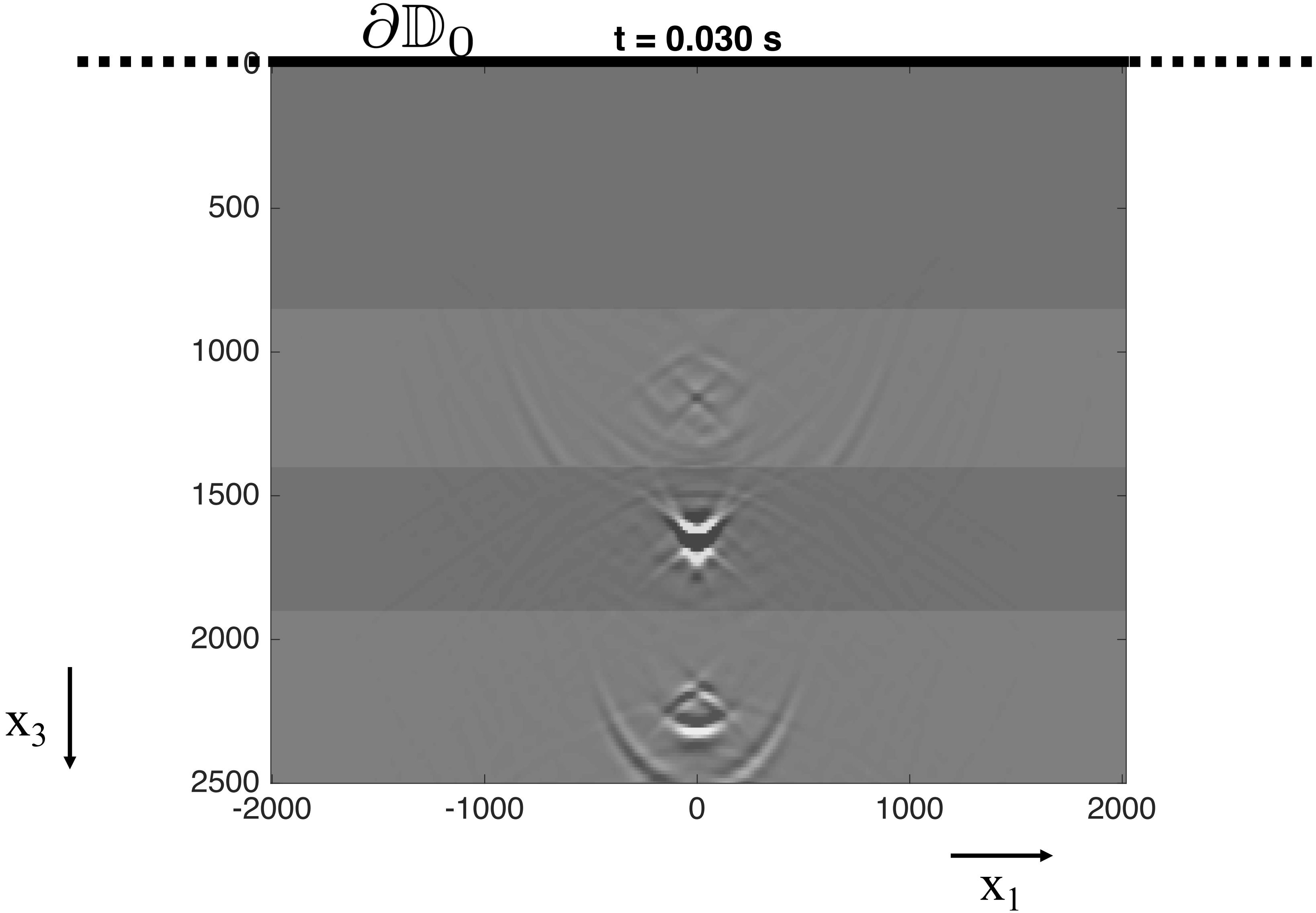
Remove bottom boundary



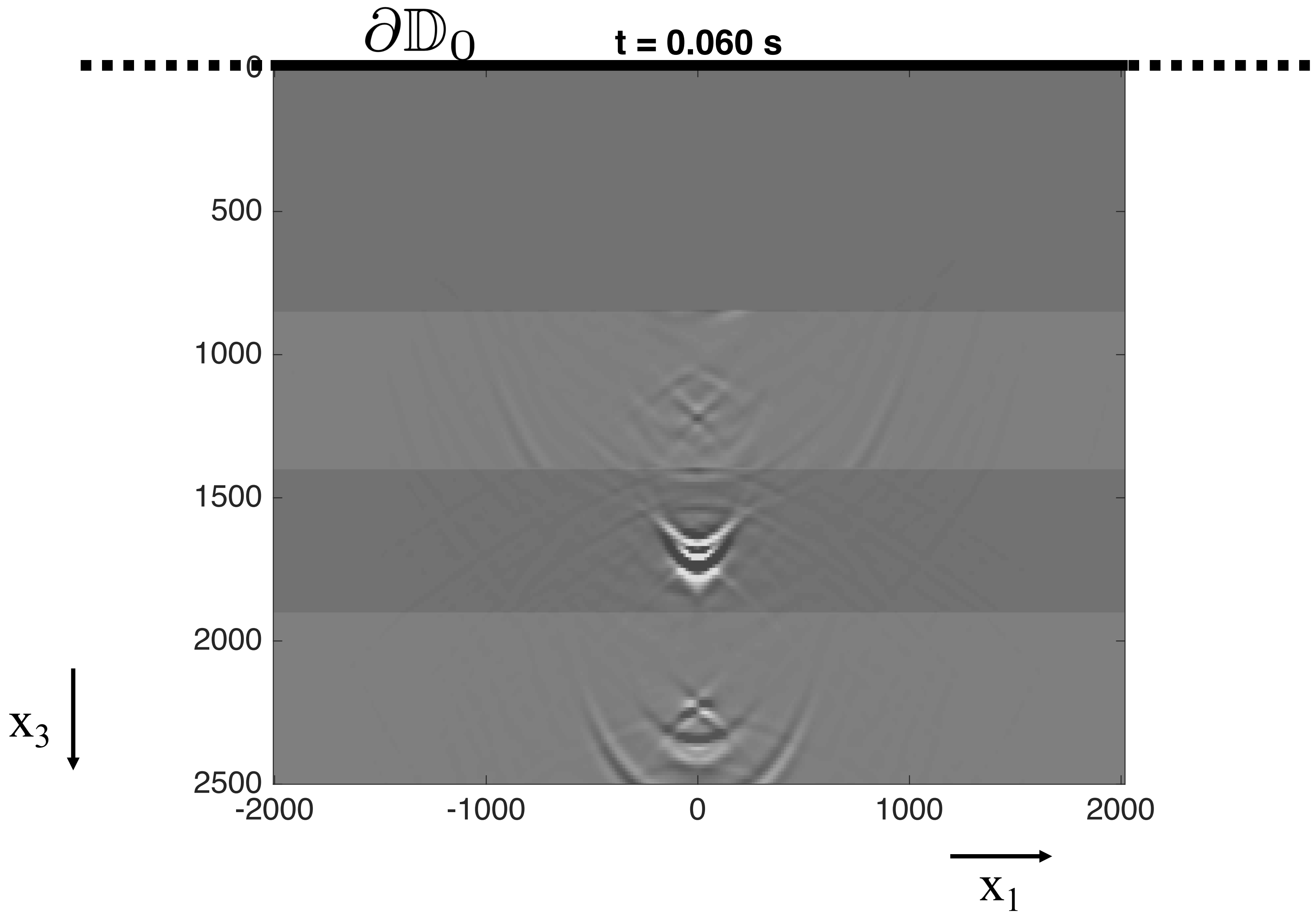
Remove bottom boundary



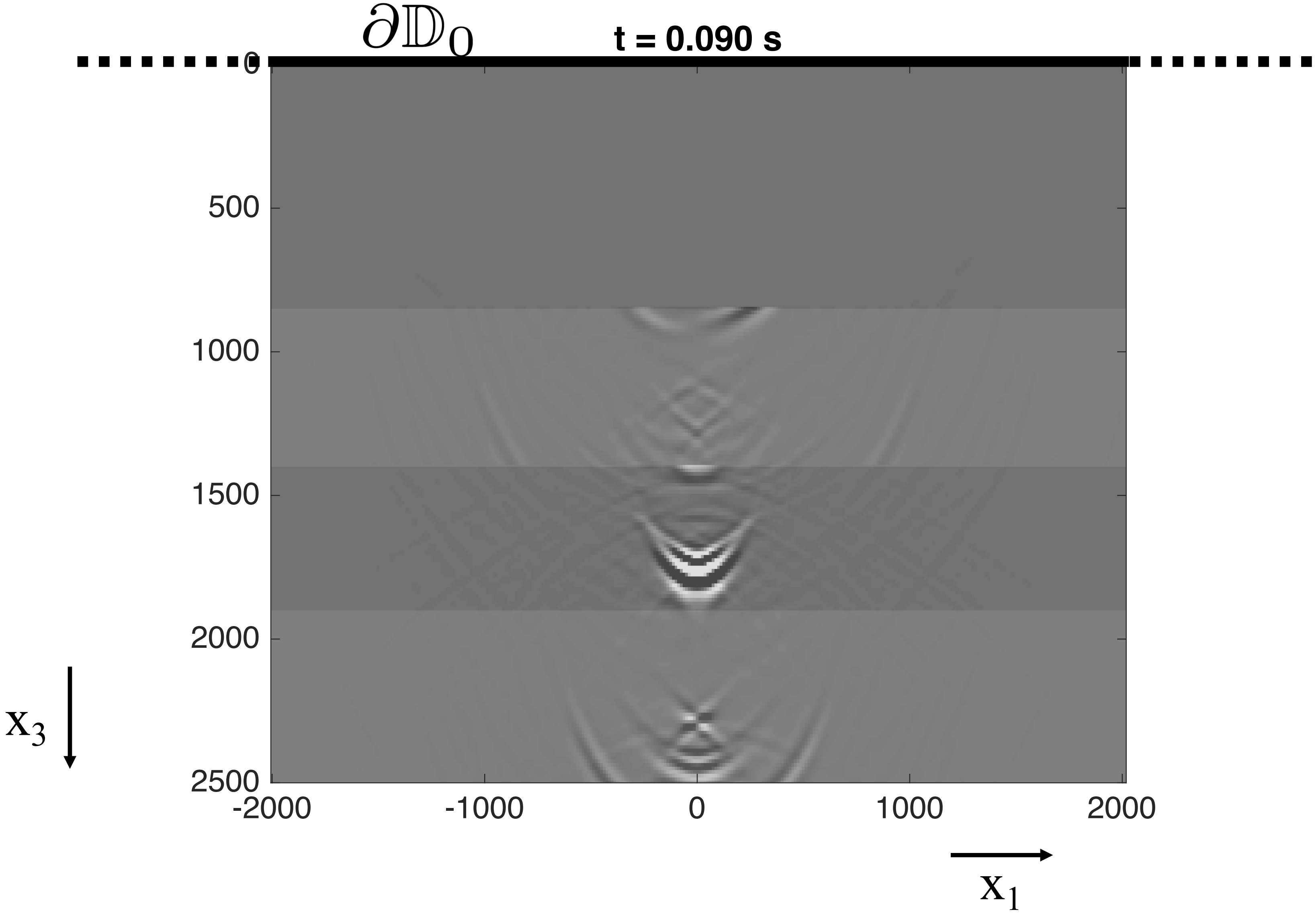
Remove bottom boundary



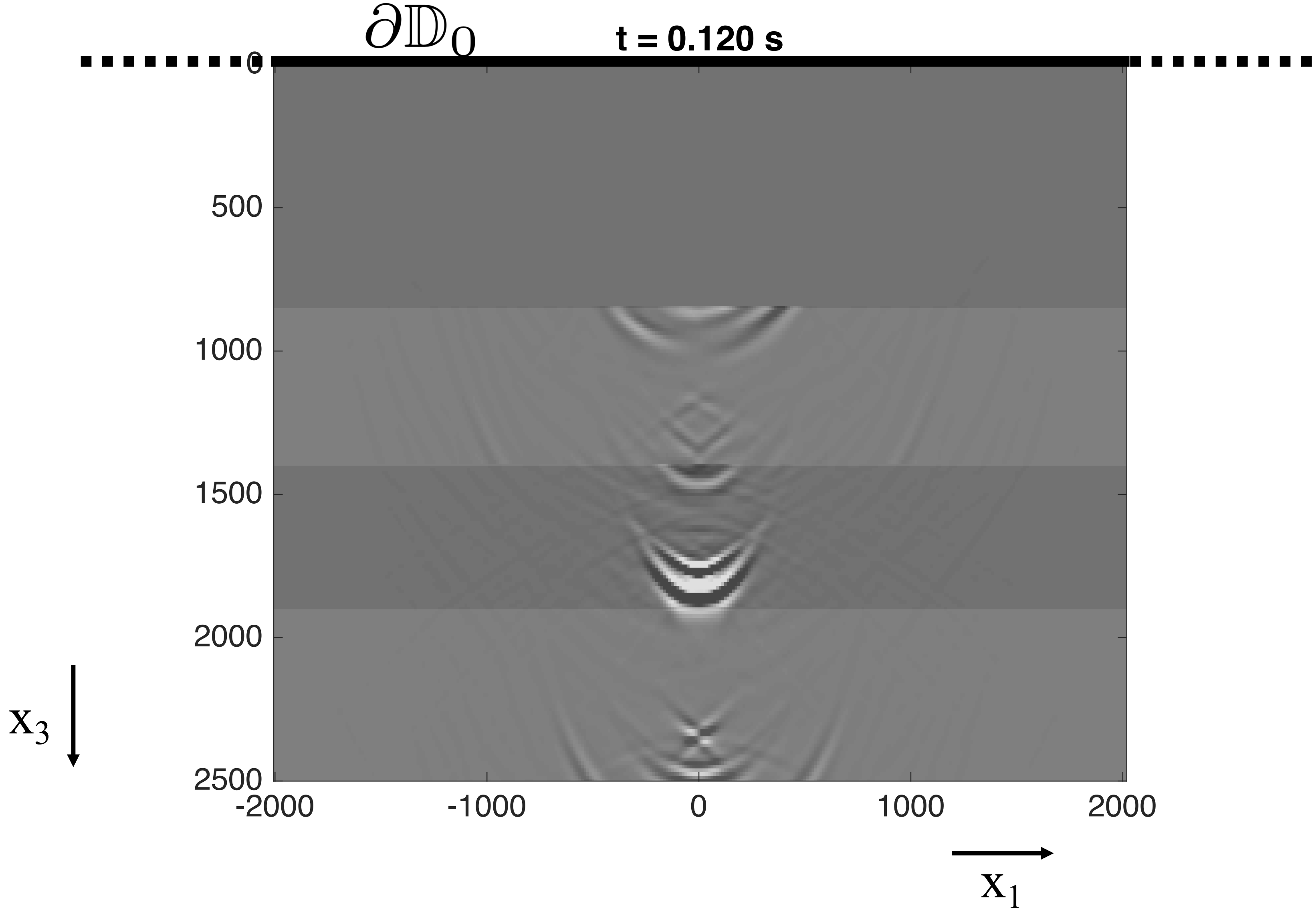
Remove bottom boundary



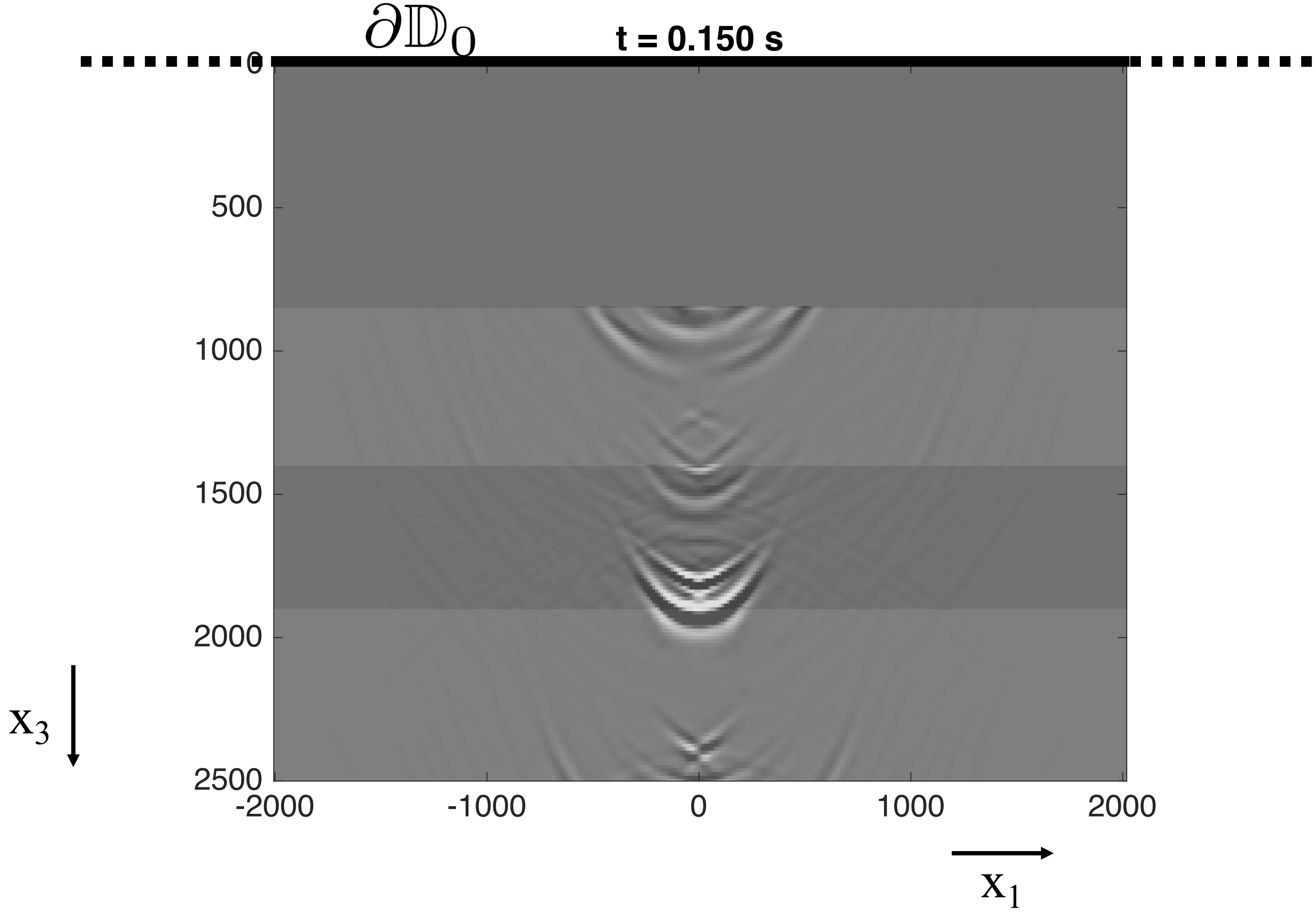
Remove bottom boundary



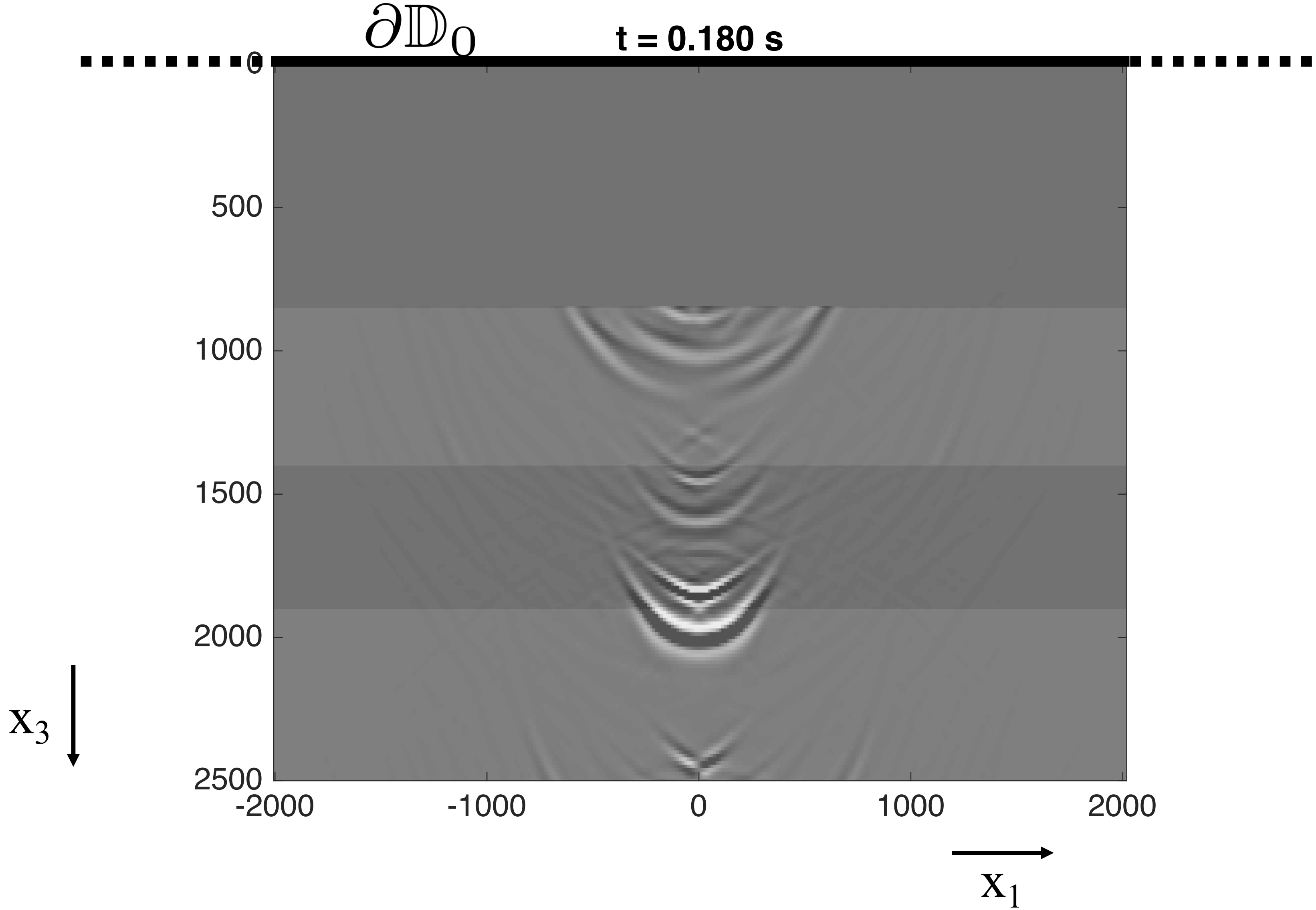
Remove bottom boundary



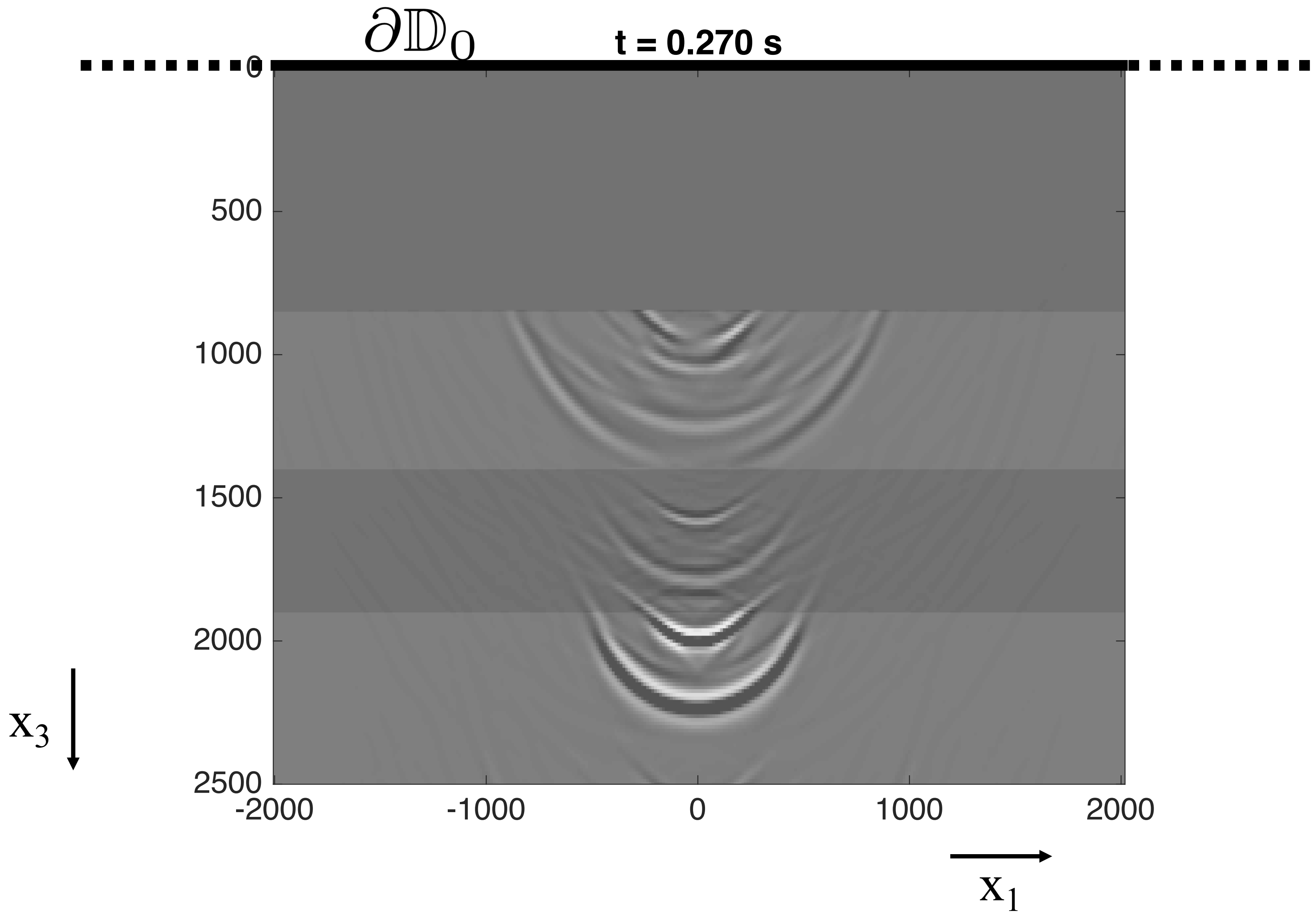
Remove bottom boundary



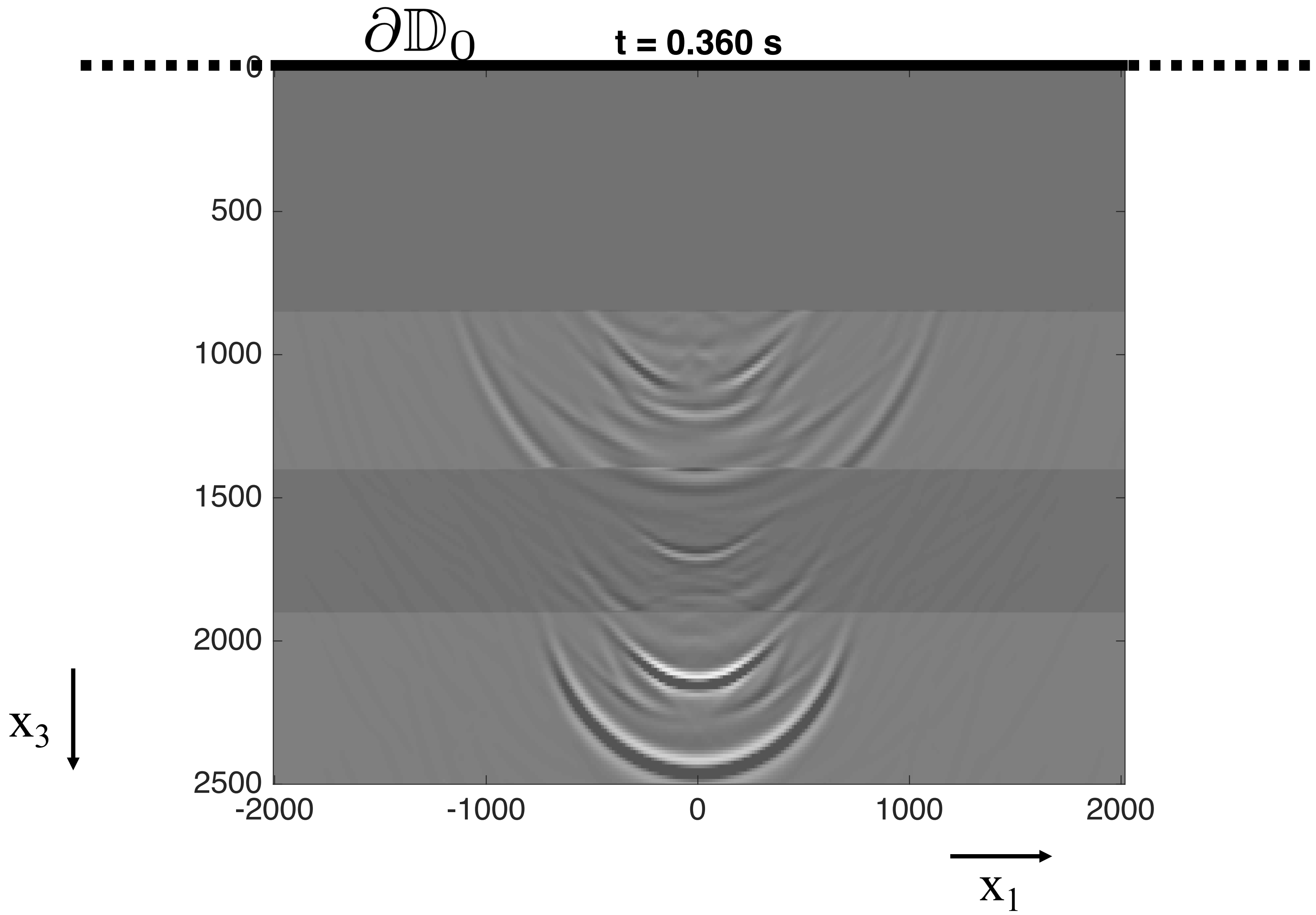
Remove bottom boundary



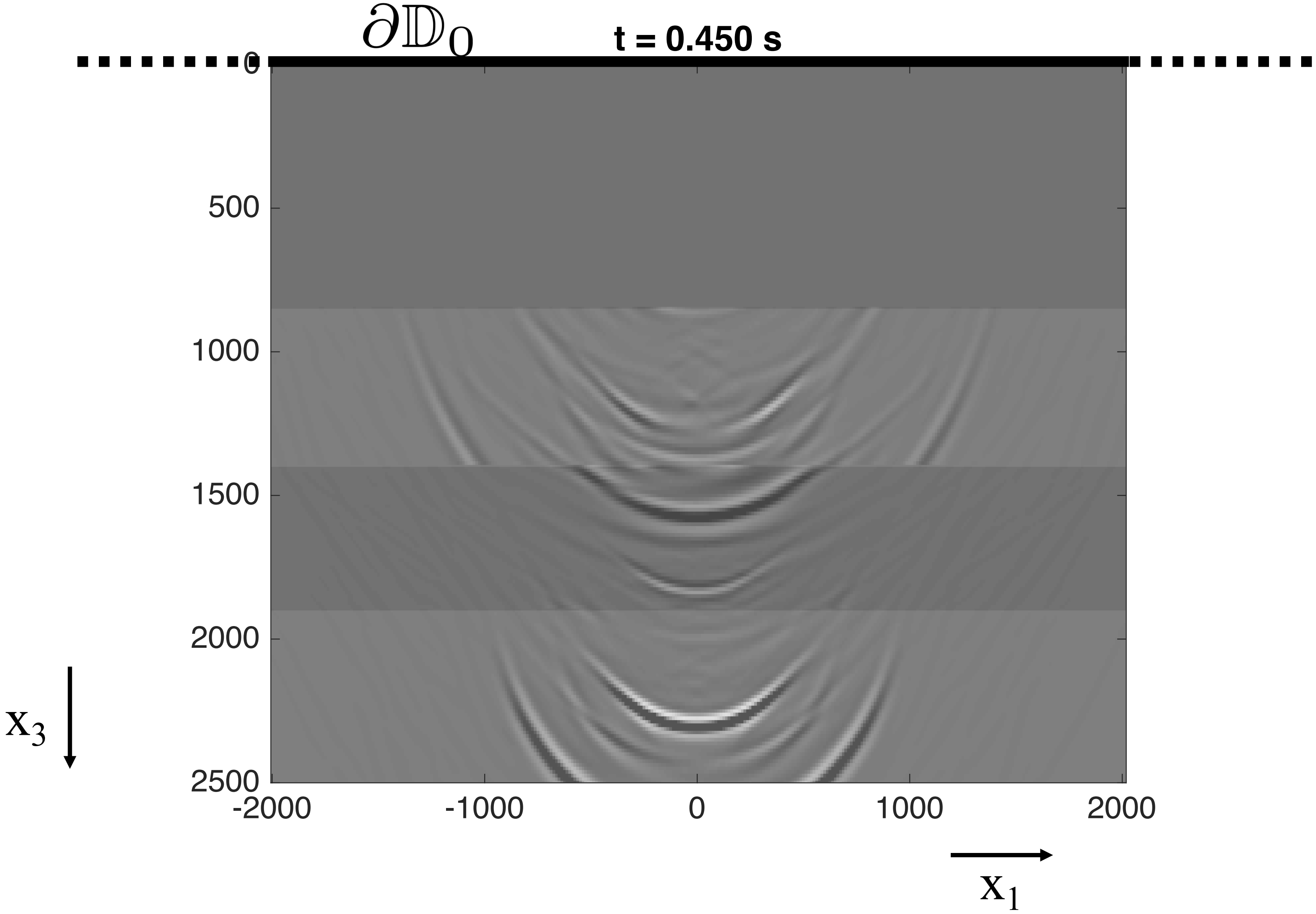
Remove bottom boundary



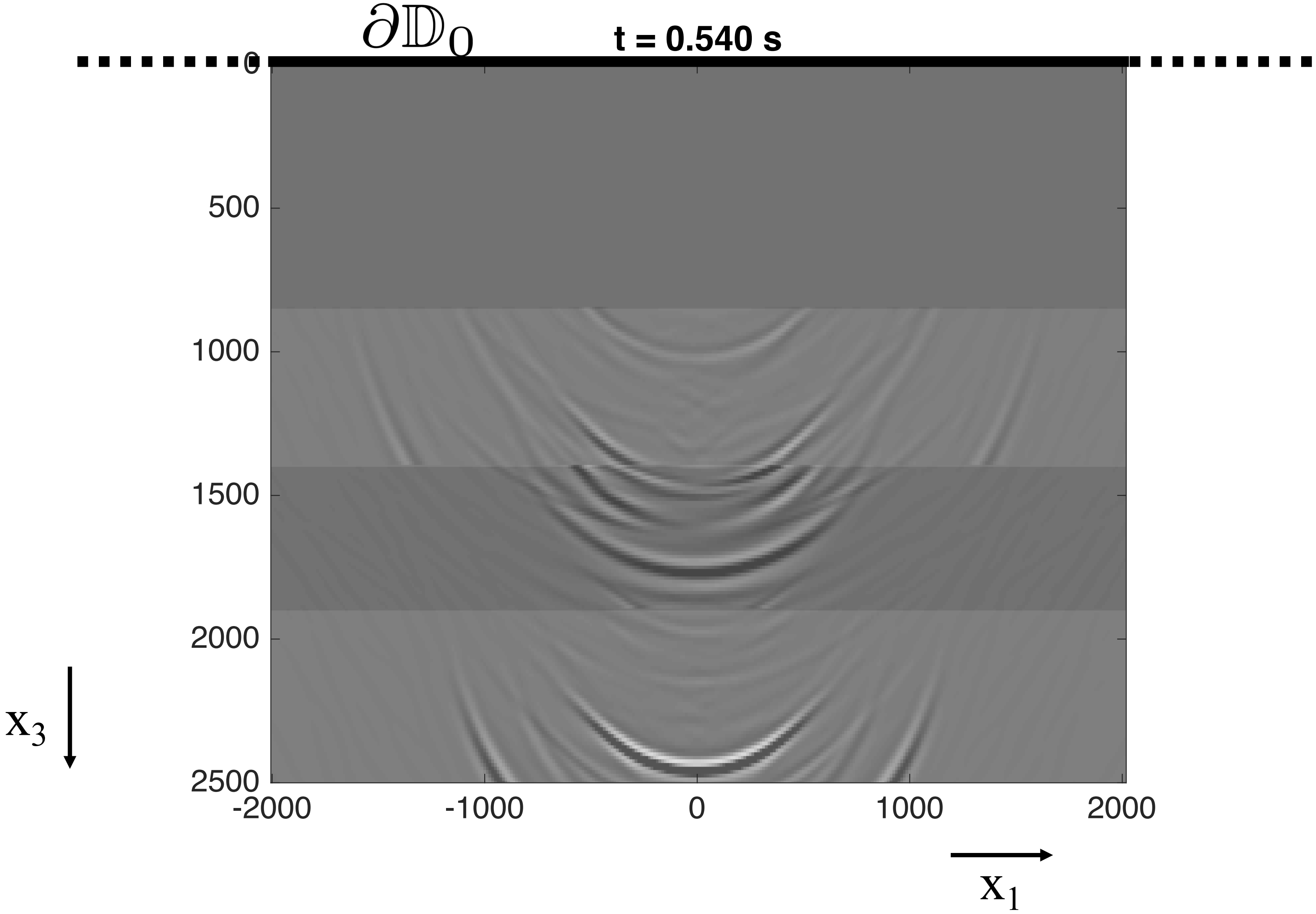
Remove bottom boundary



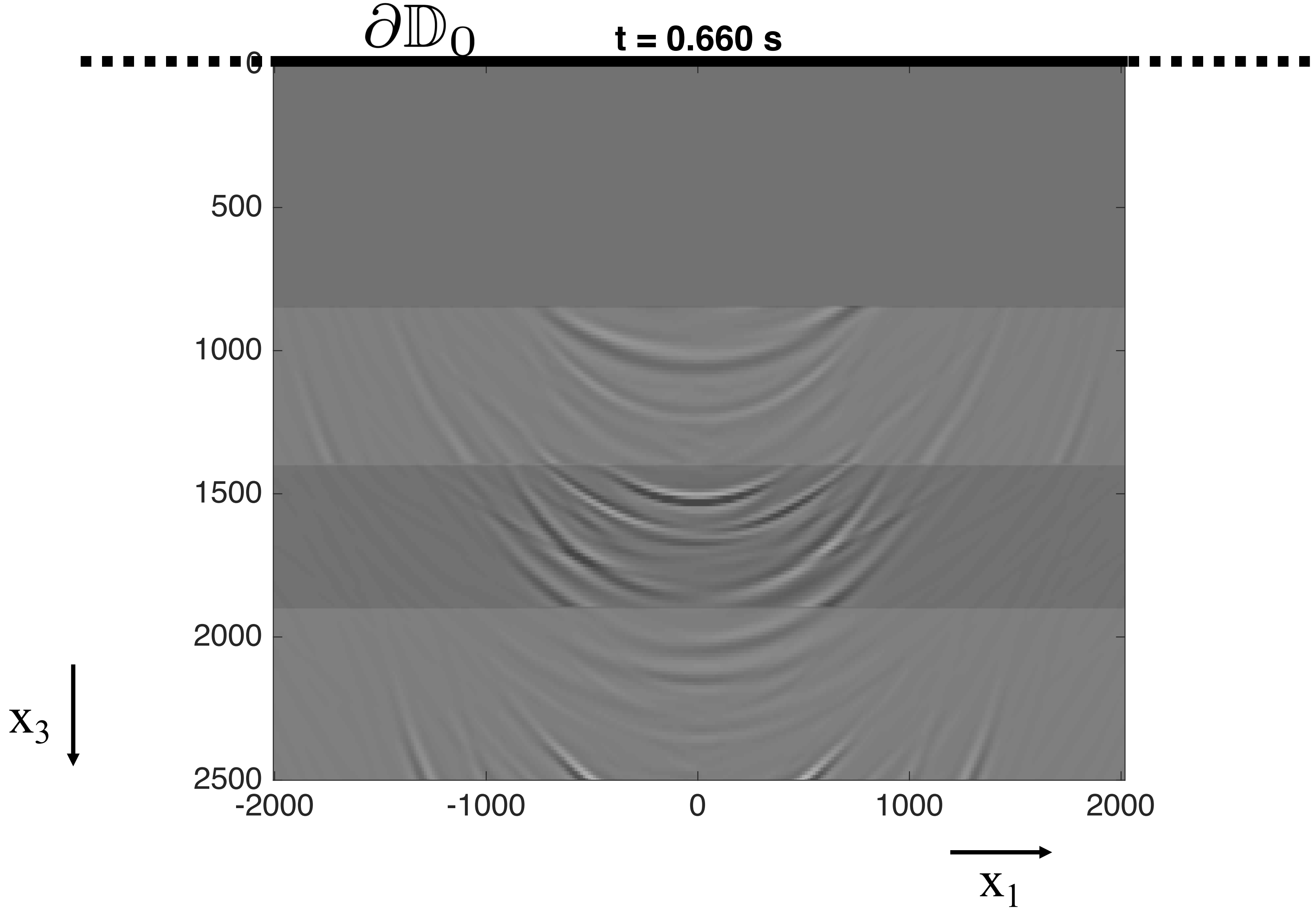
Remove bottom boundary



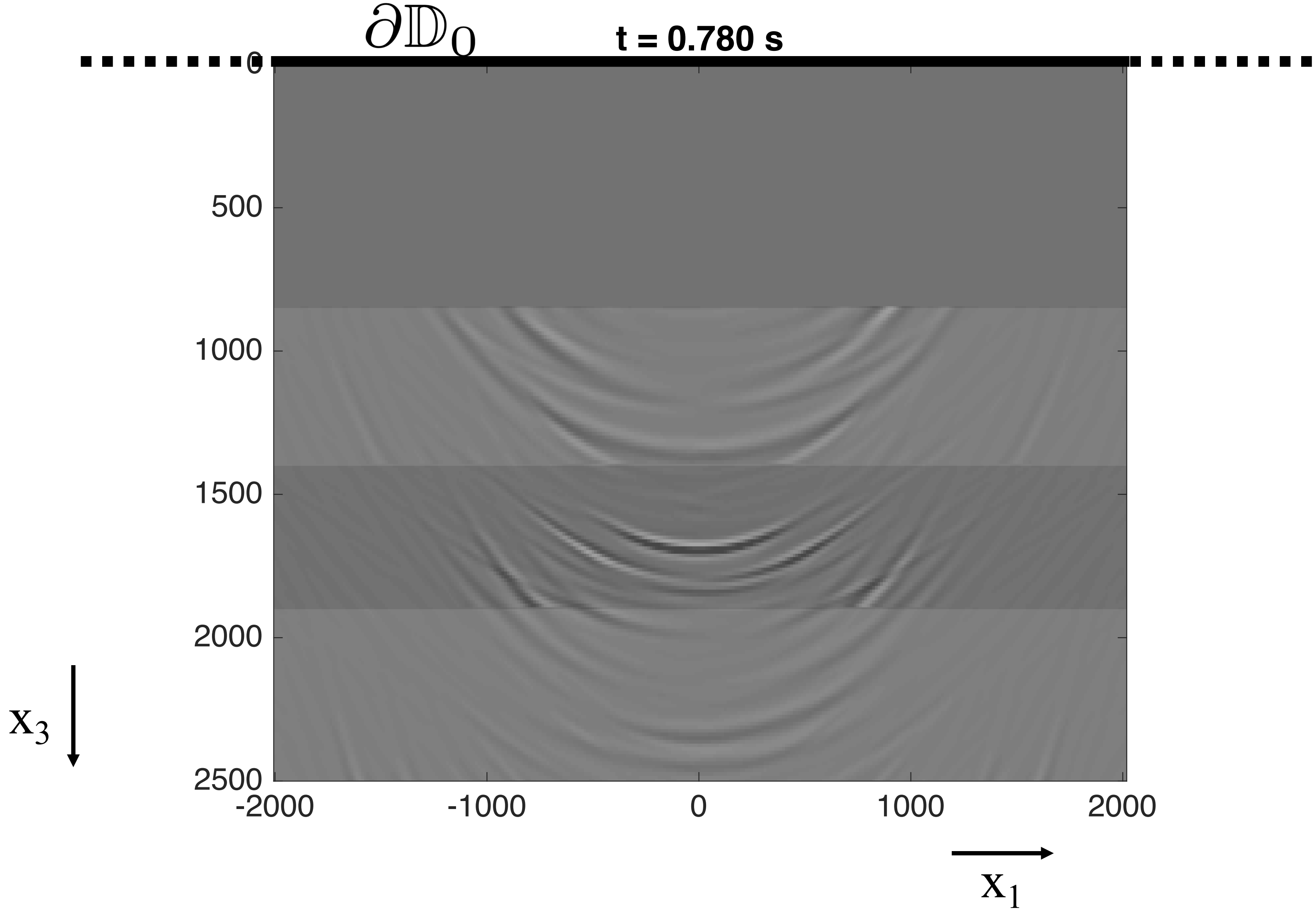
Remove bottom boundary



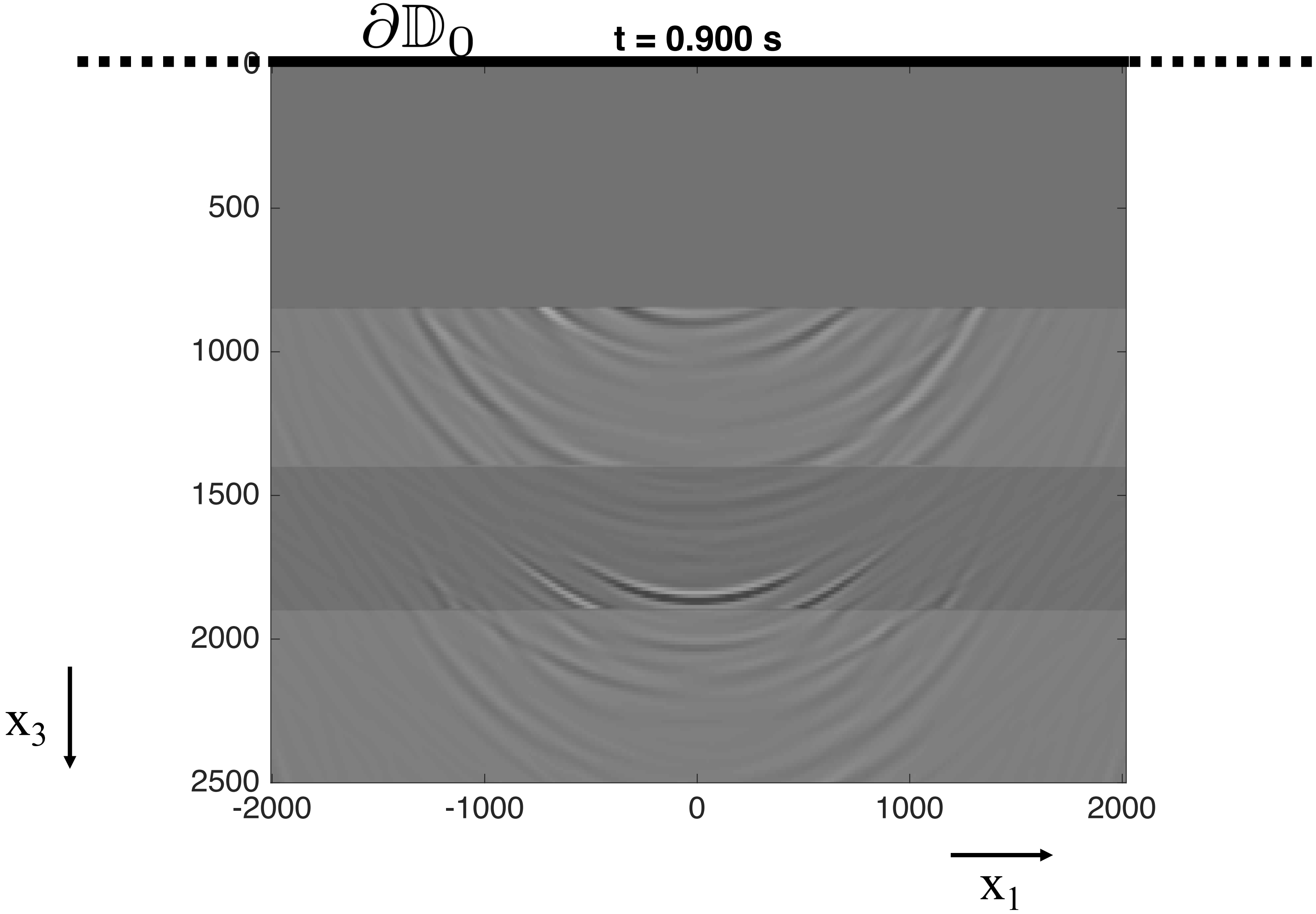
Remove bottom boundary



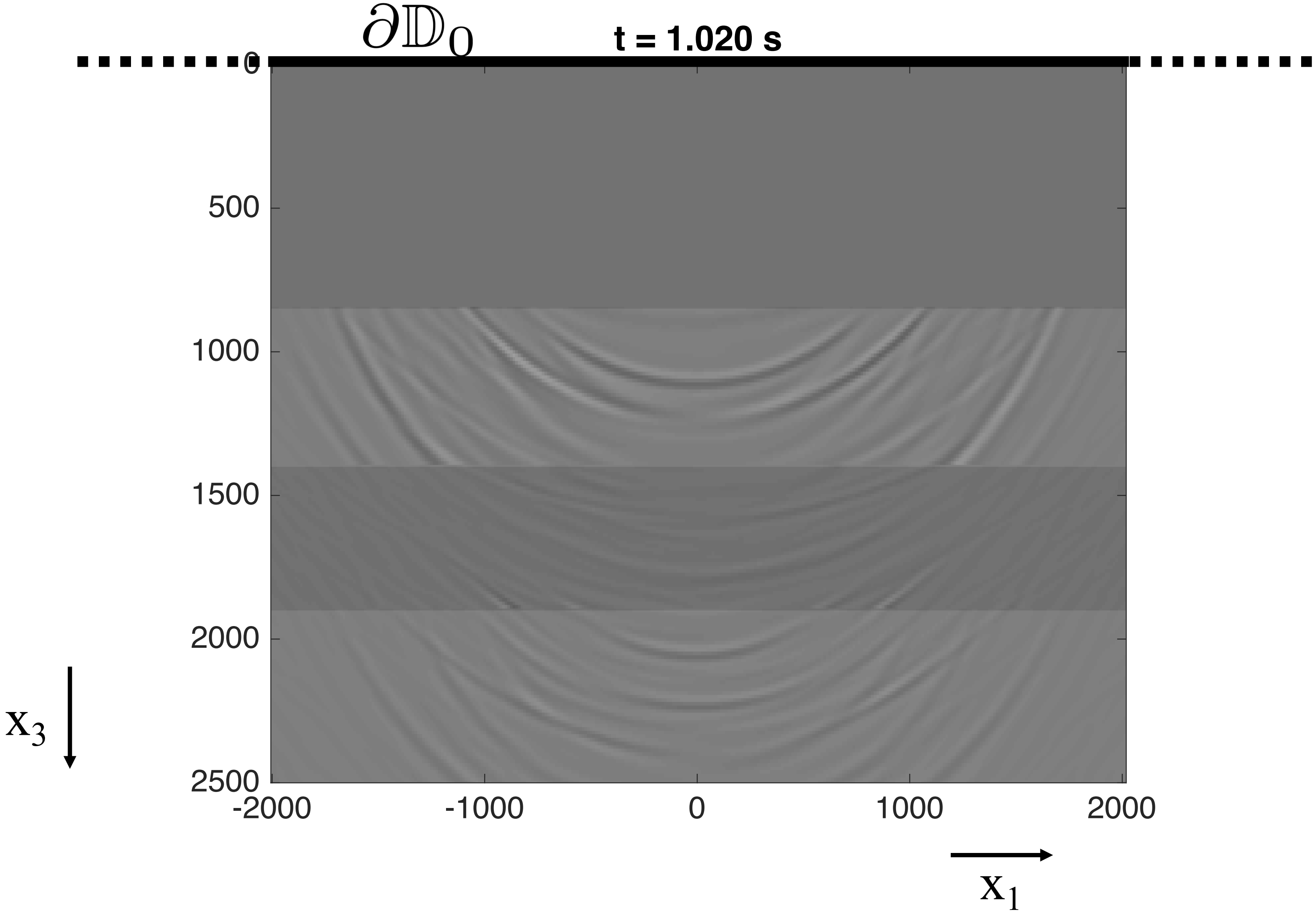
Remove bottom boundary



Remove bottom boundary



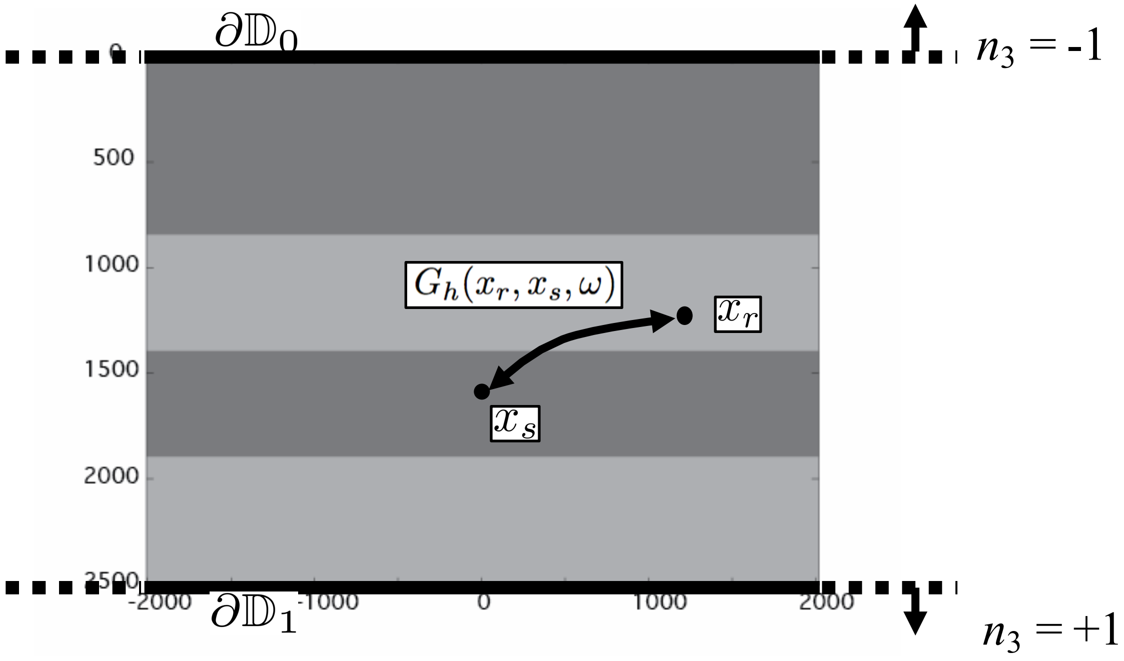
Remove bottom boundary



Acoustic single-sided homogeneous Green's function representation

<u>Derivation outline:</u>

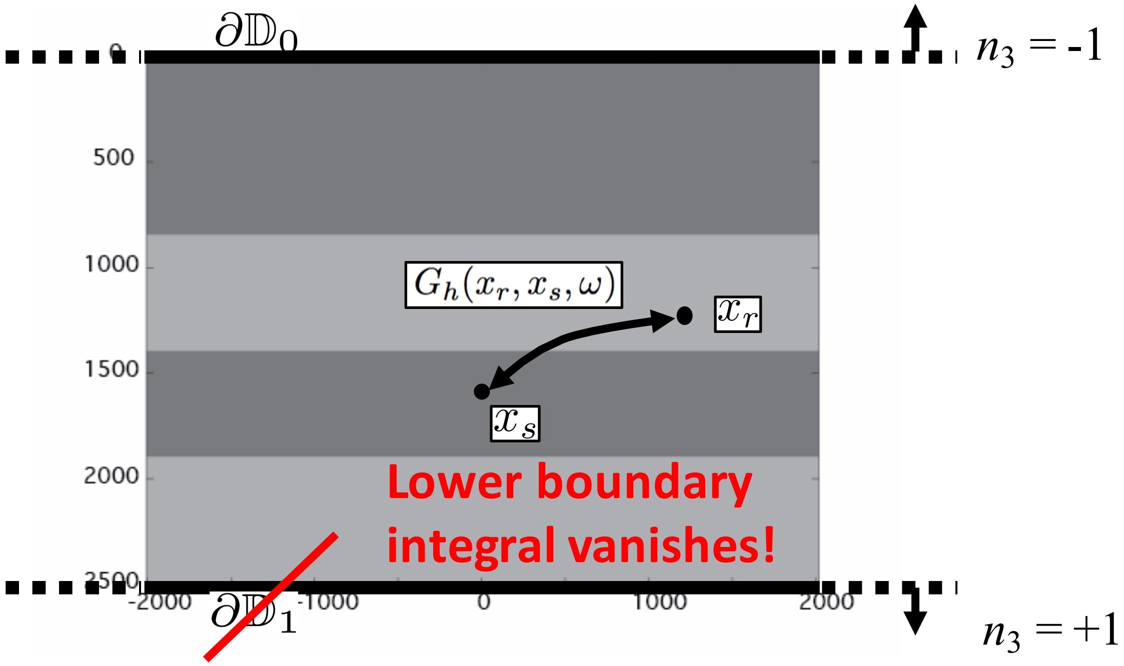
- Start with double-sided representation (Porter & Oristaglio)



Acoustic single-sided homogeneous Green's function representation

<u>Derivation outline:</u>

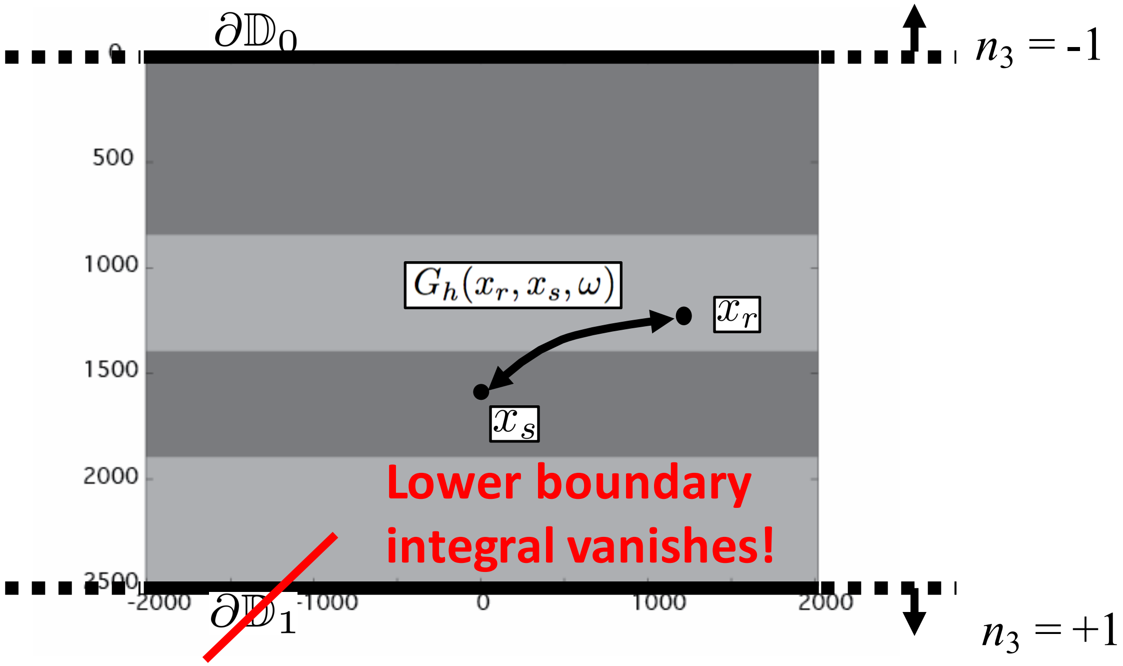
- Start with double-sided representation (Porter & Oristaglio)
- Introduce auxiliary function to cancel lower boundary integral



Acoustic single-sided homogeneous Green's function representation

<u>Derivation outline:</u>

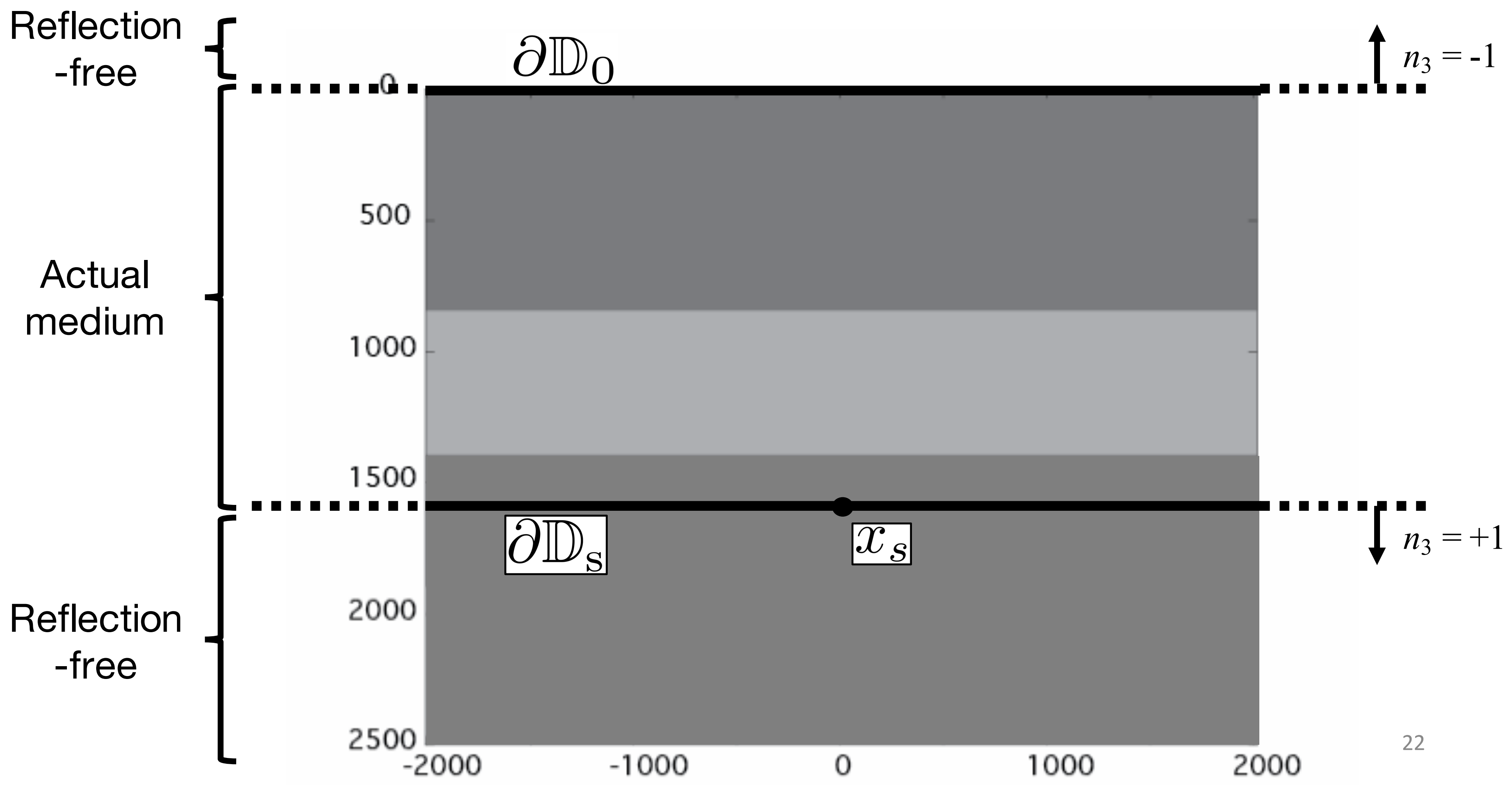
- Start with double-sided representation (Porter & Oristaglio)
- Introduce auxiliary function to cancel lower boundary integral
- The auxiliary function leads us to an expression with the focusing function $f_1(x, x_r, \omega)$



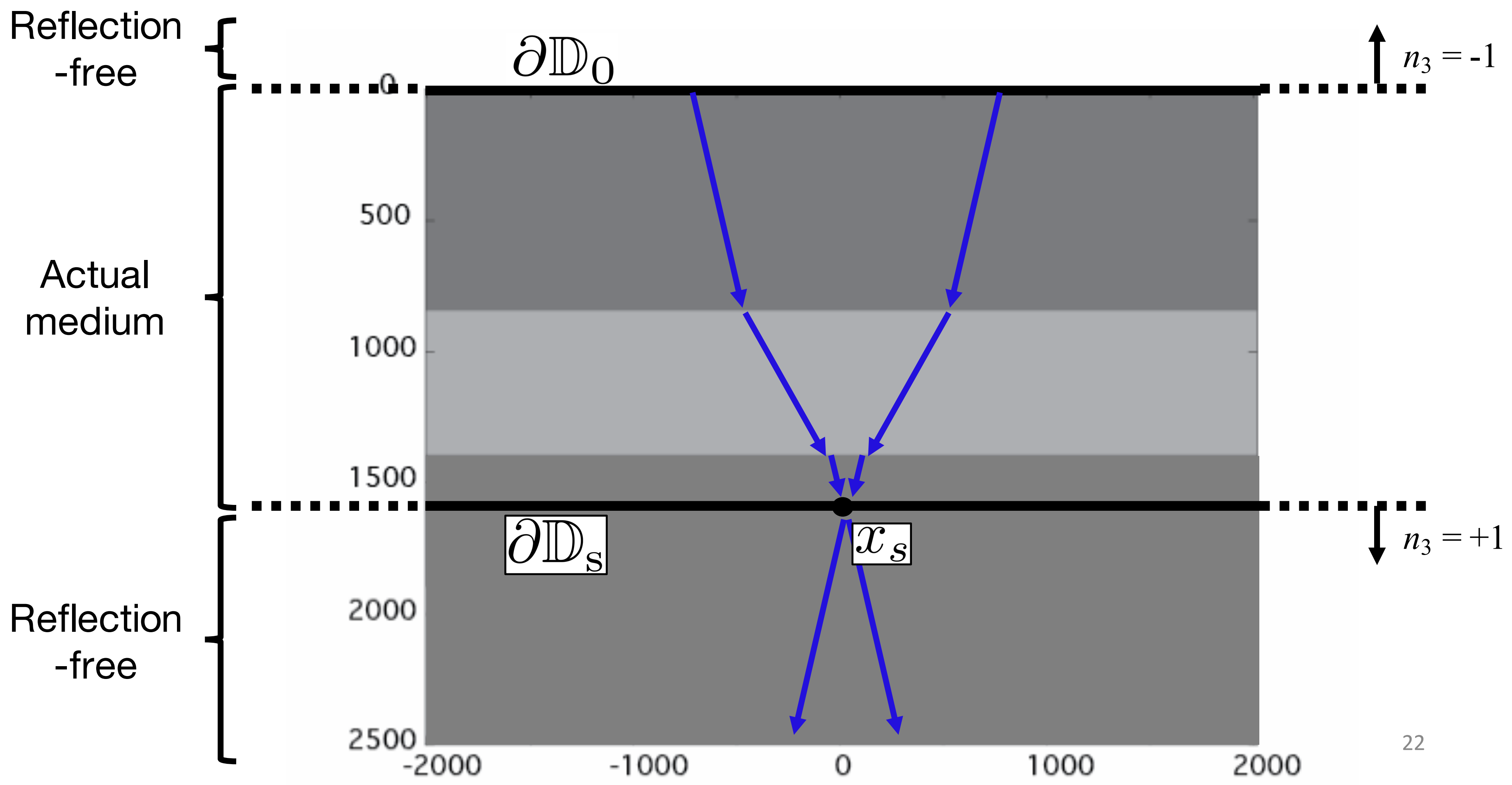
$$G_h(x_r, x_s, \omega) = \int_{\partial\mathbb{D}_0} \frac{2}{\omega\rho(x)} \{\Im(f_1(x, x_r, \omega)\partial_3 G_h(x, x_s, \omega) - \Im(\partial_3 f_1(x, x_r, \omega) G_h(x, x_s, \omega)\}\ \mathrm{d}^2 x$$

(Wapenaar, Thorbecke, van der Neut. "A single-sided homogeneous Green's function representation for holographic imaging, inverse scattering, time-reversal acoustics and interferometric Green's function retrieval." *GJI*)

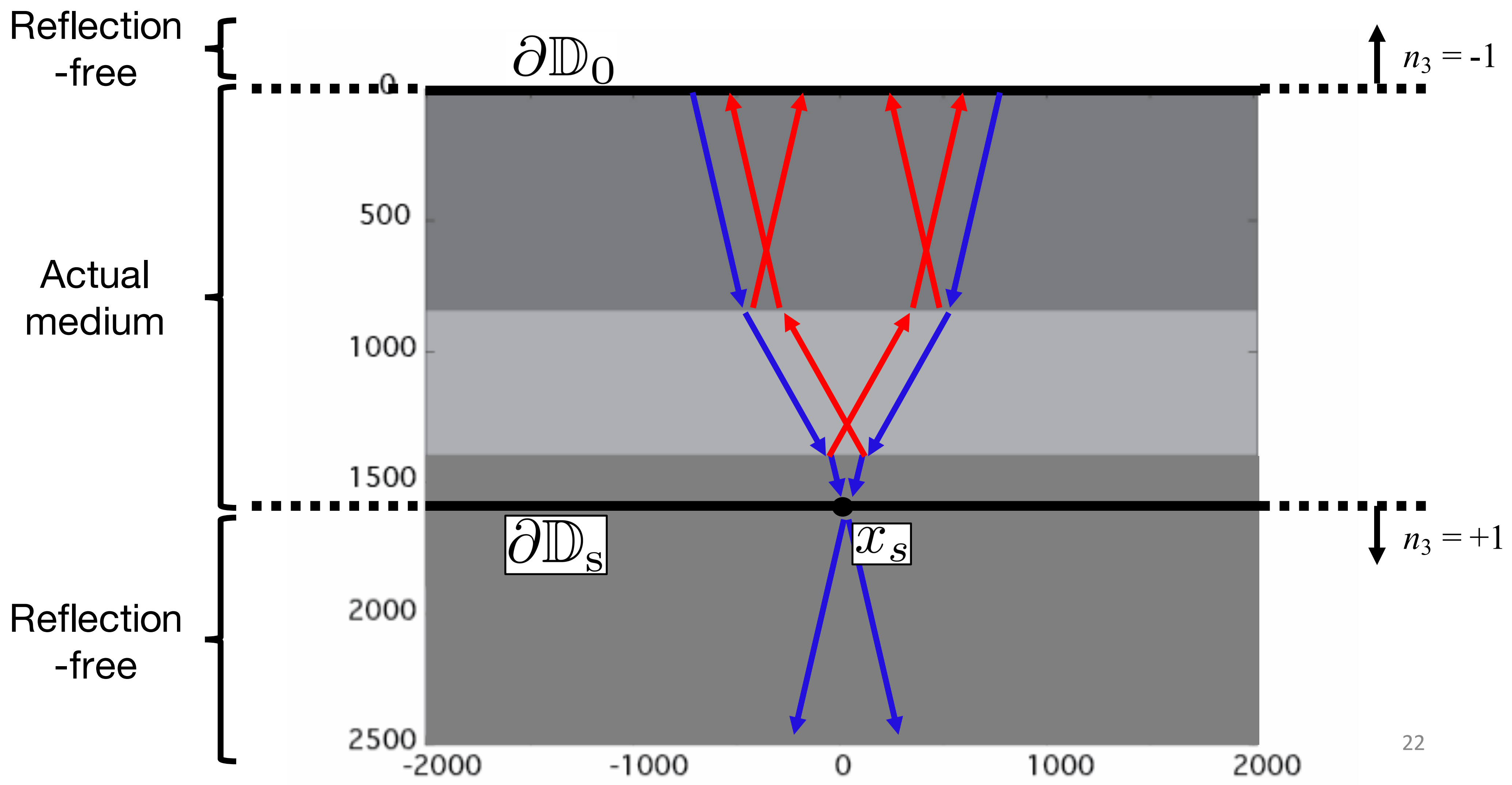
Focusing function: $f_1(x, x_s, \omega) = f_1^+(x, x_s, \omega) + f_1^-(x, x_s, \omega)$



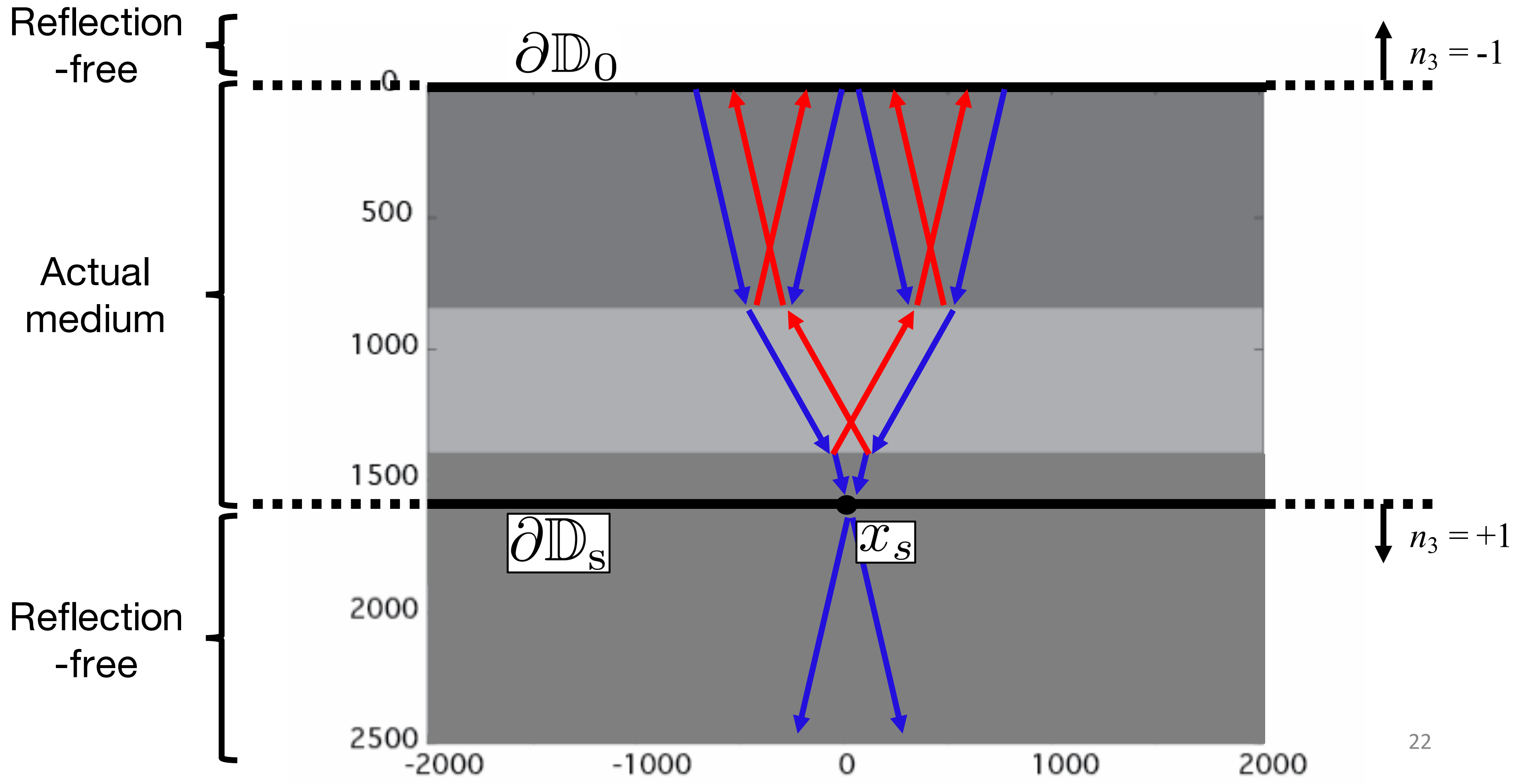
Focusing function: $f_1(x, x_s, \omega) = f_1^+(x, x_s, \omega) + f_1^-(x, x_s, \omega)$



Focusing function: $f_1(x, x_s, \omega) = f_1^+(x, x_s, \omega) + f_1^-(x, x_s, \omega)$

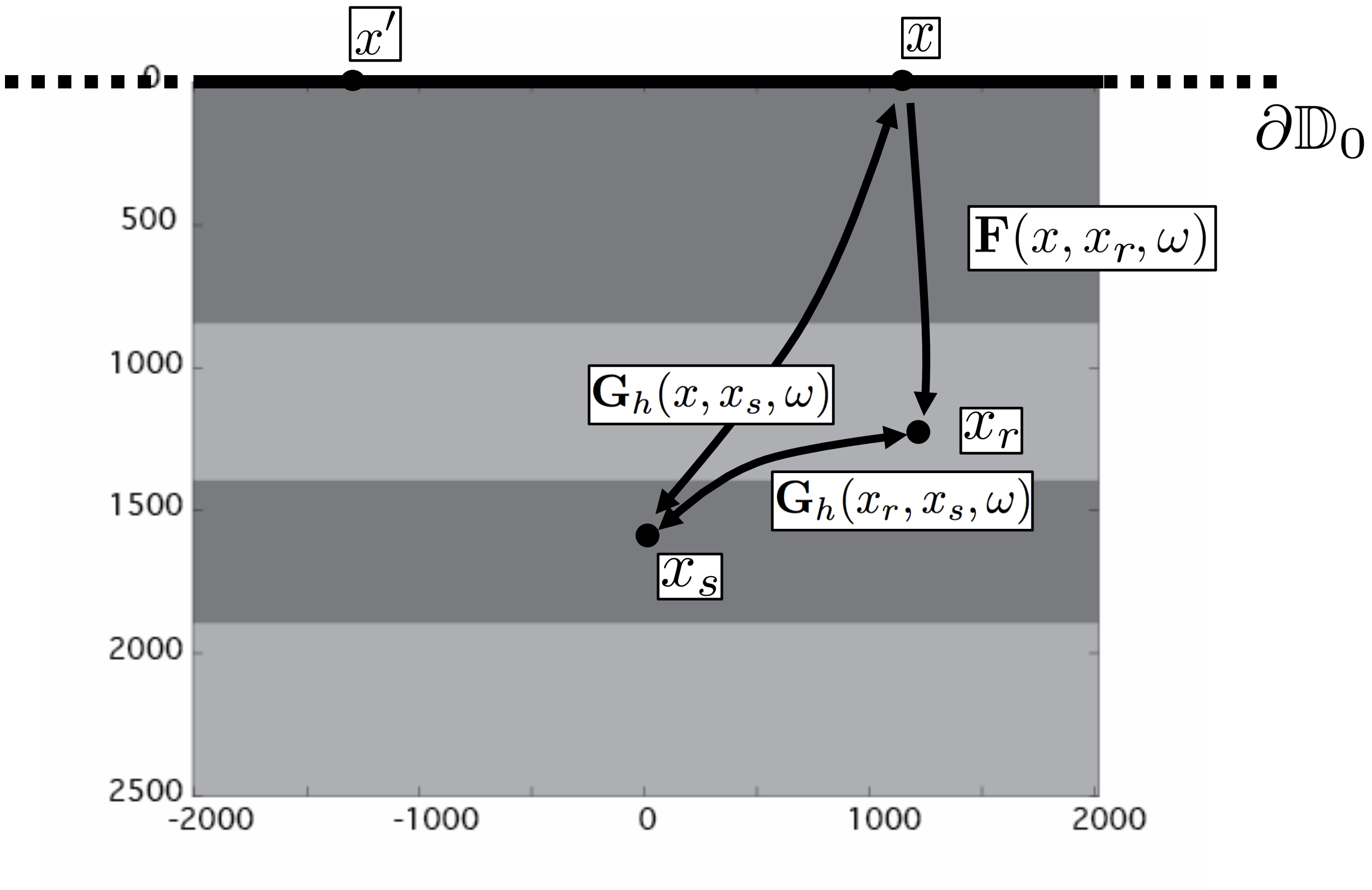


Focusing function: $f_1(x, x_s, \omega) = f_1^+(x, x_s, \omega) + f_1^-(x, x_s, \omega)$



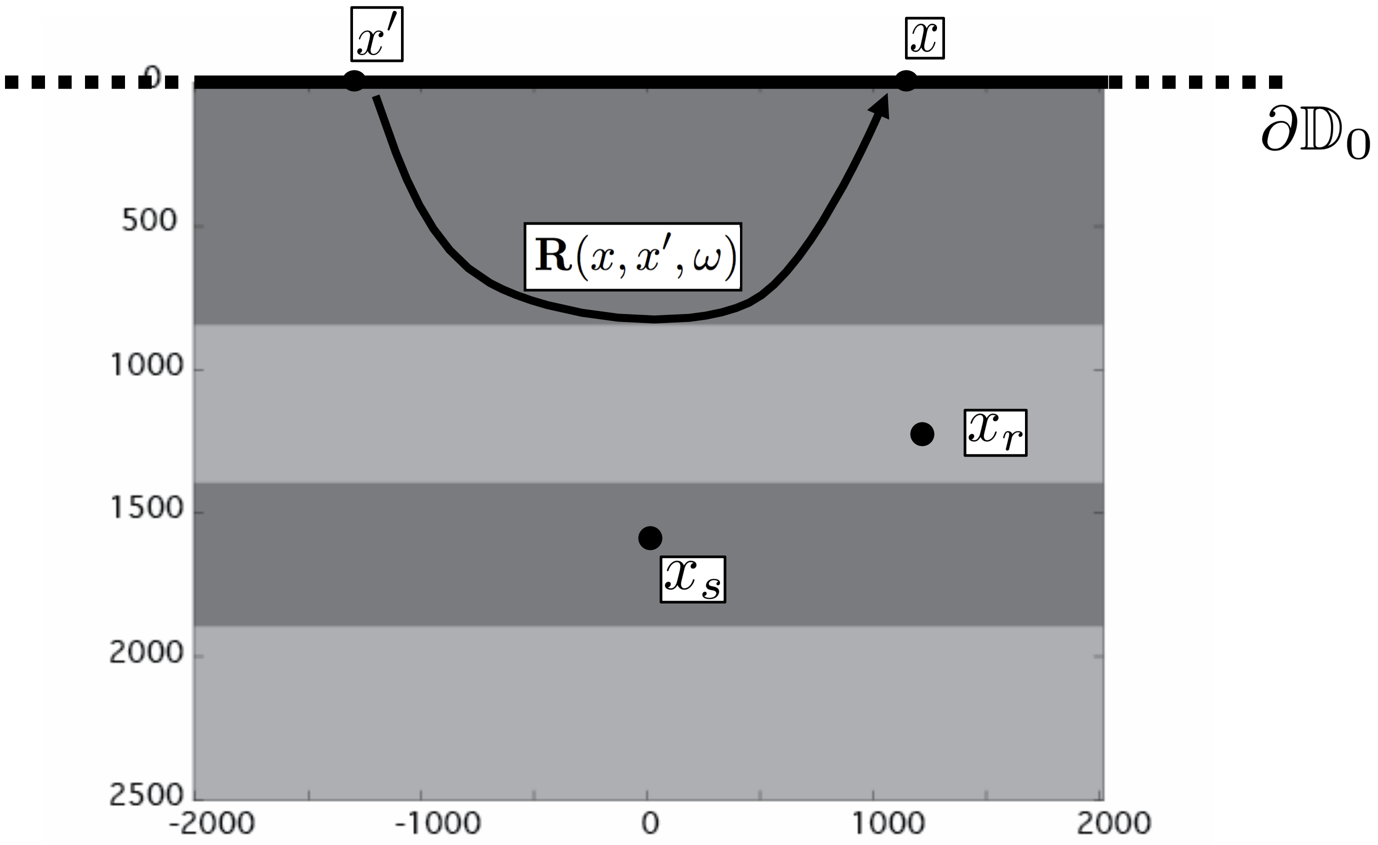
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



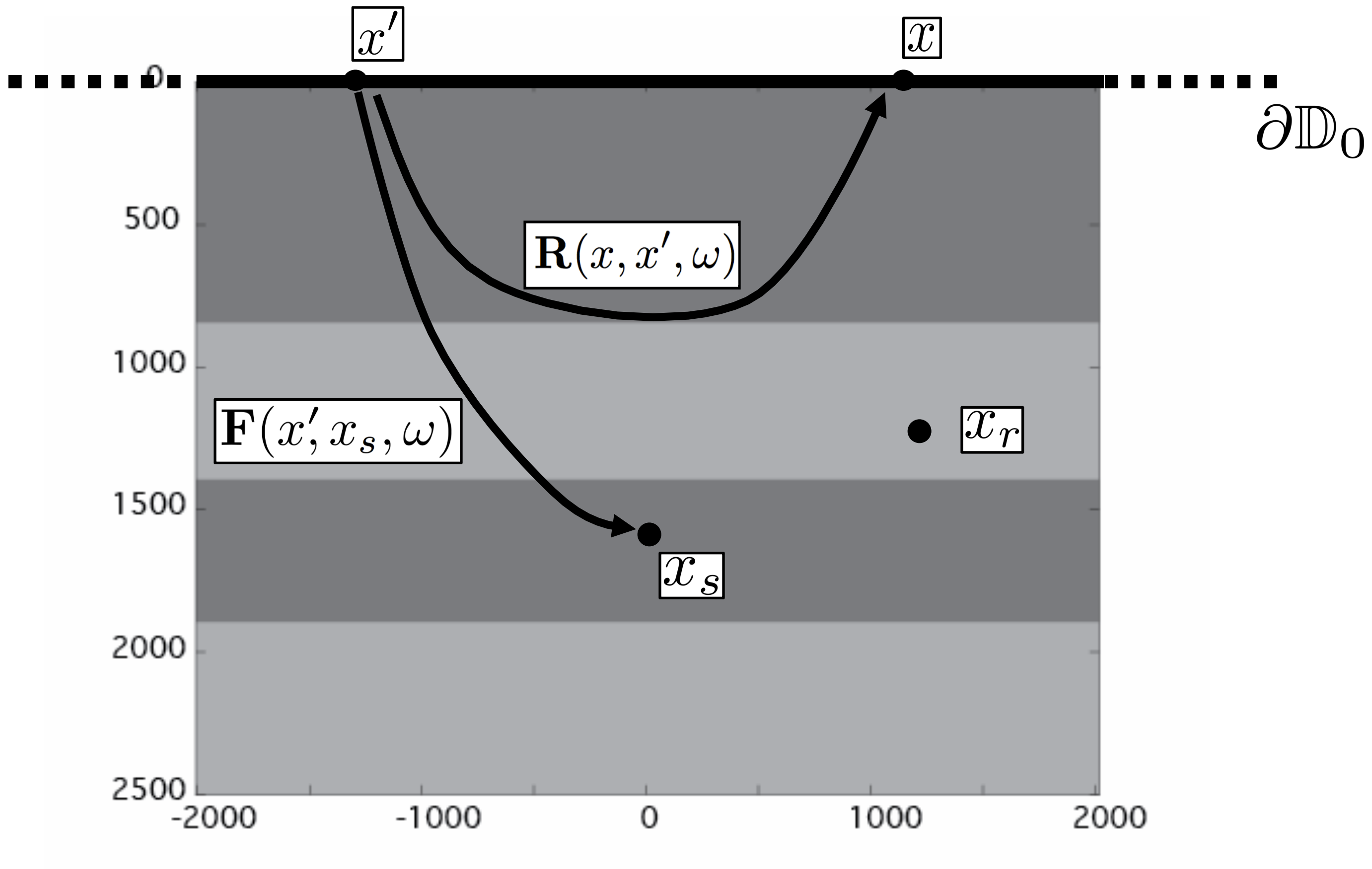
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



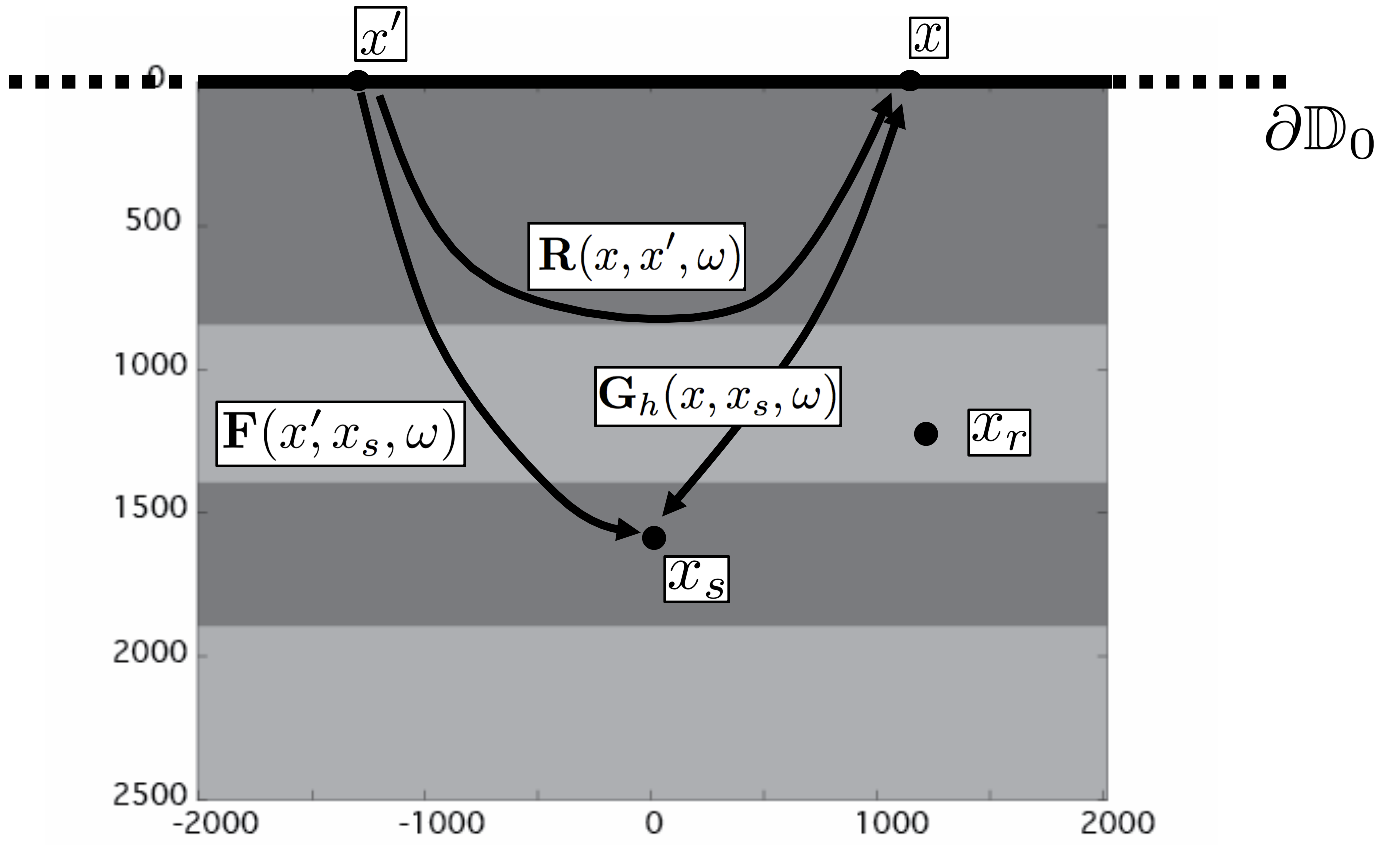
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



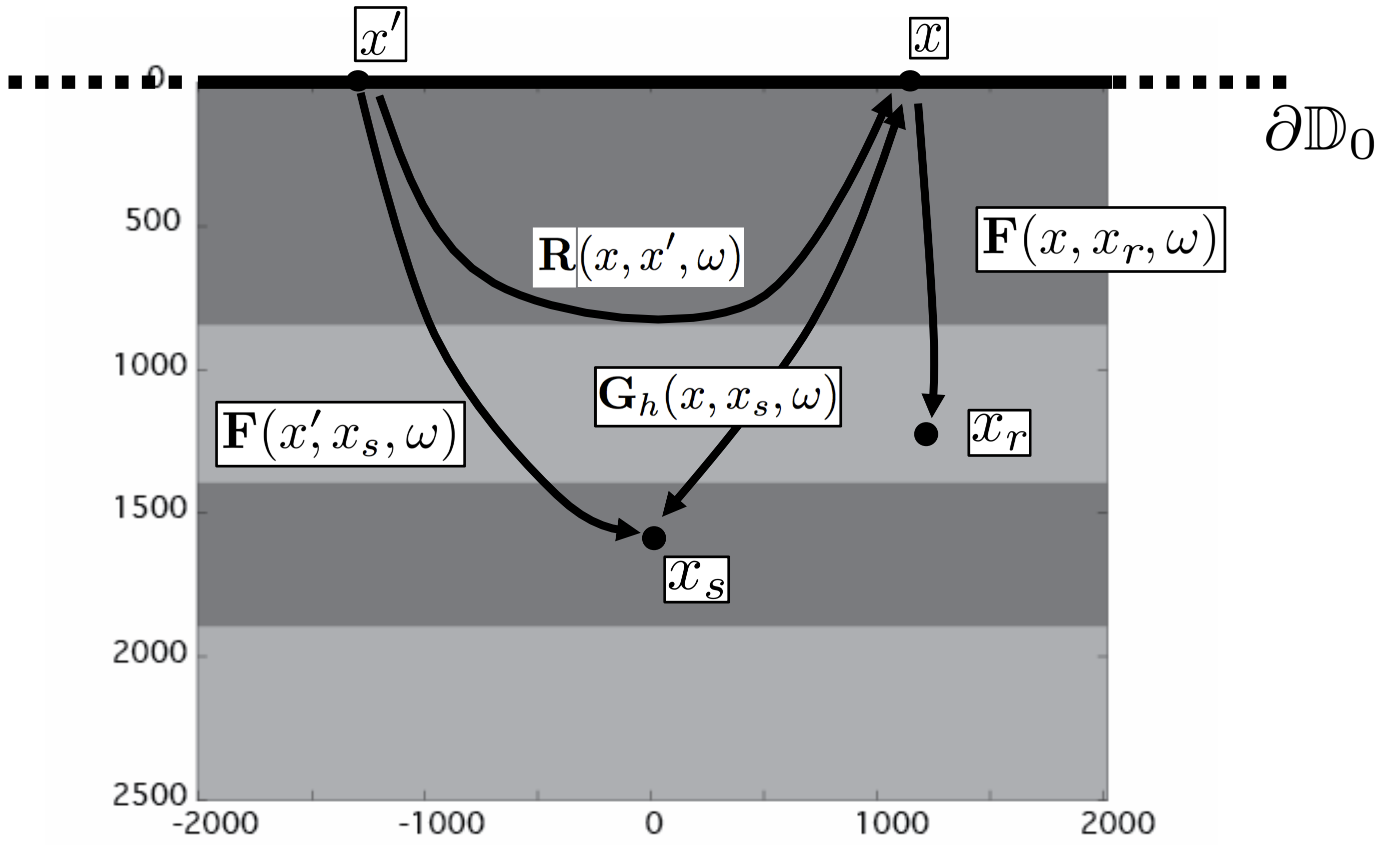
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



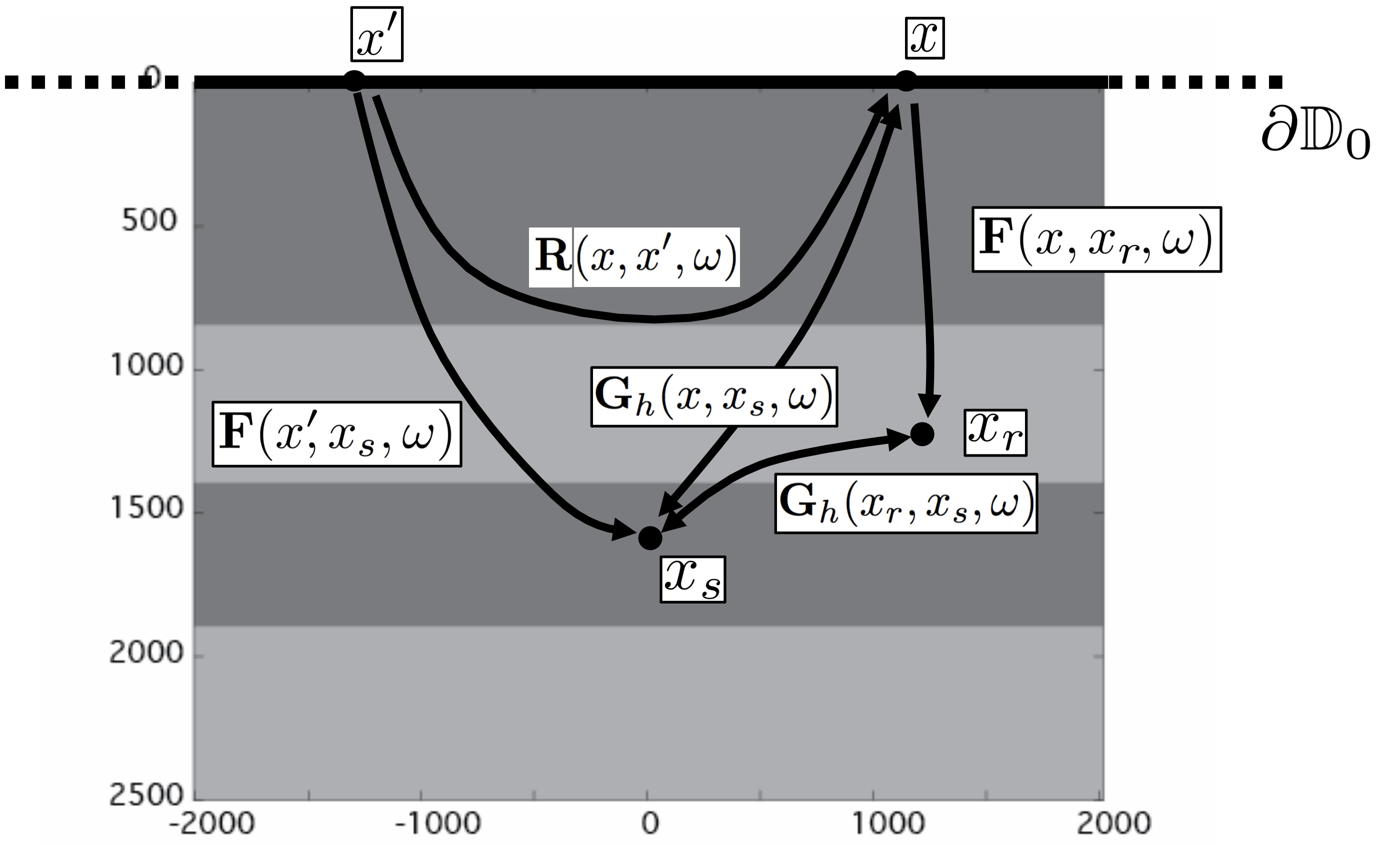
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



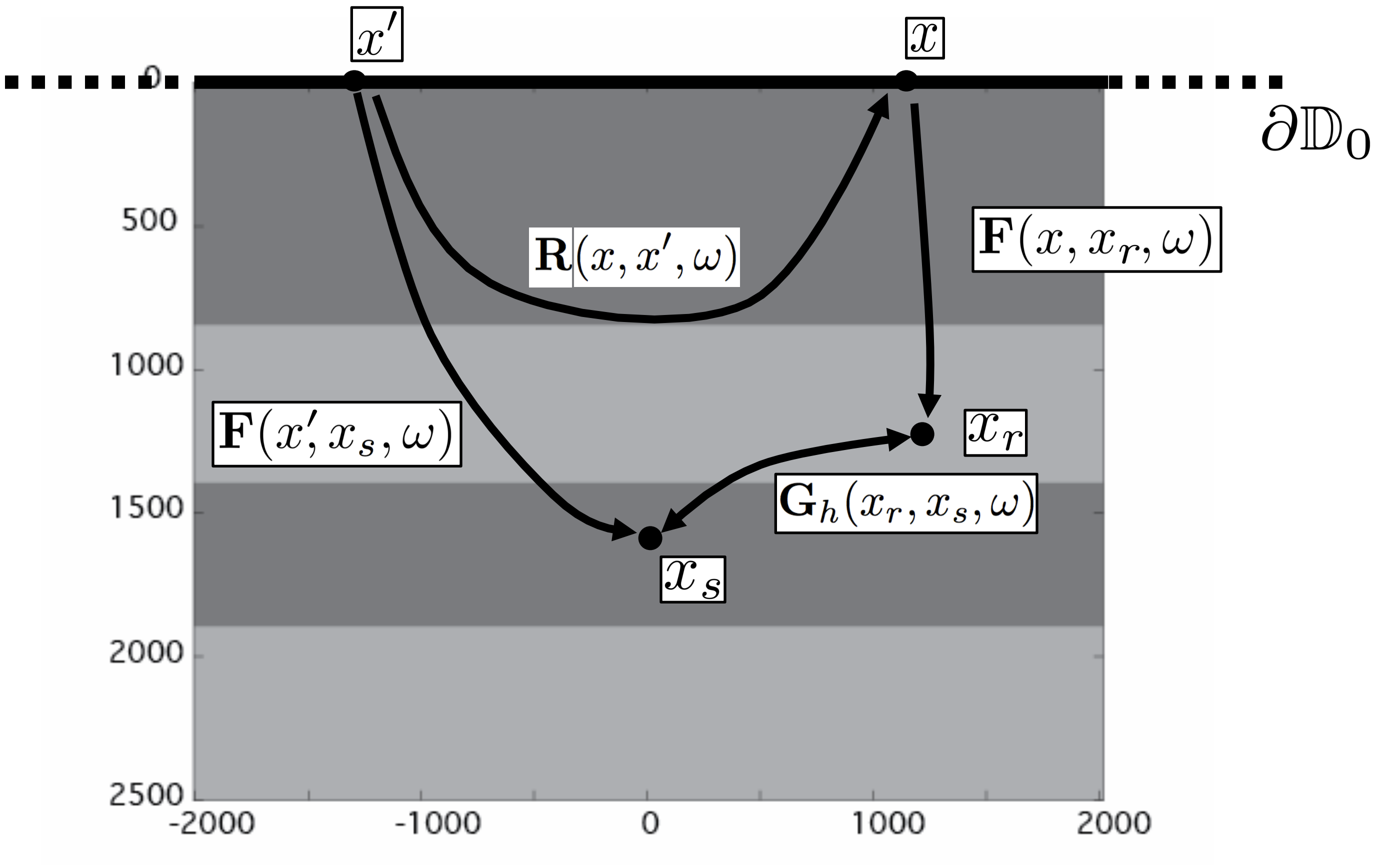
Elastodynamic single-sided homogeneous Green's function representation

$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$

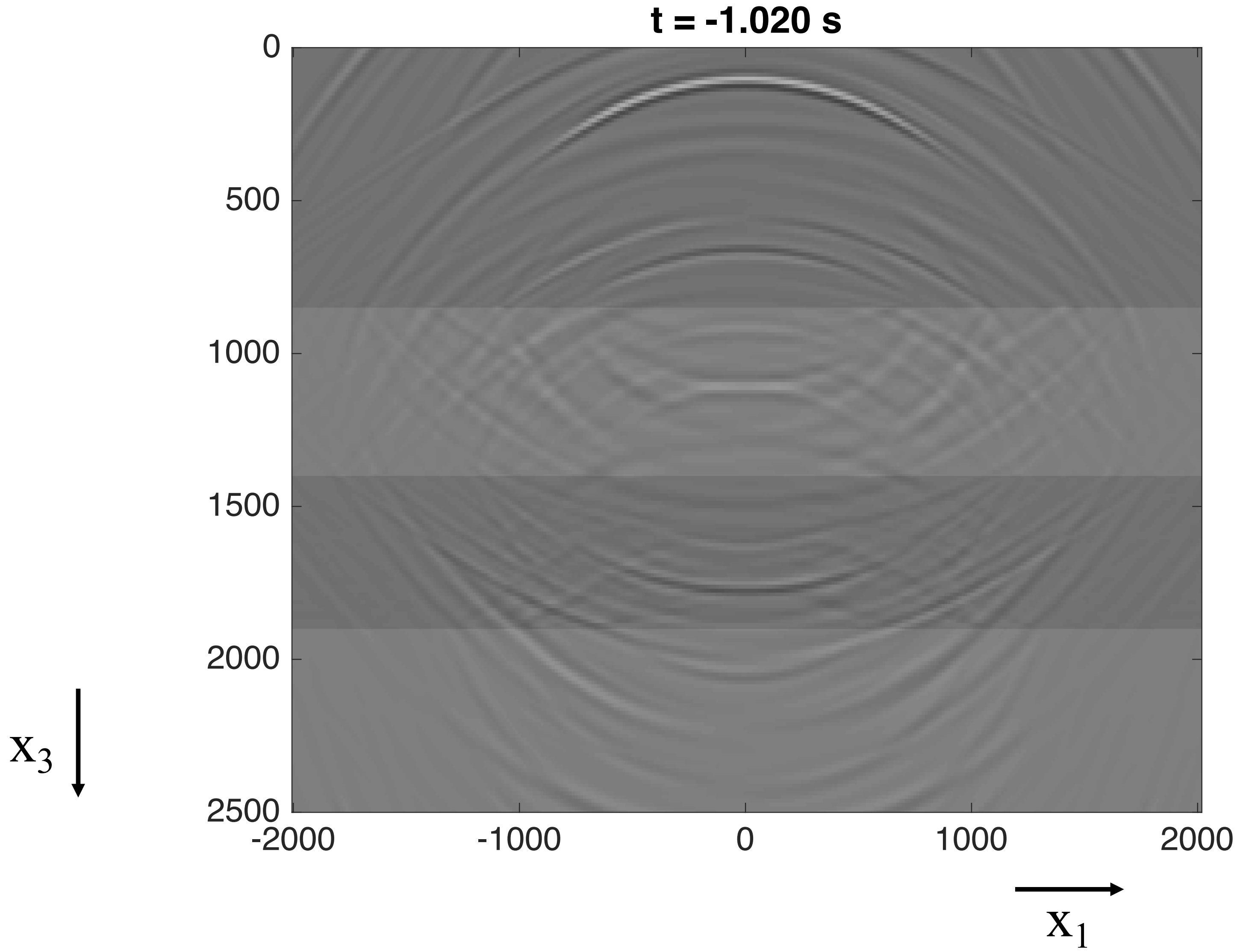


Elastodynamic single-sided homogeneous Green's function representation

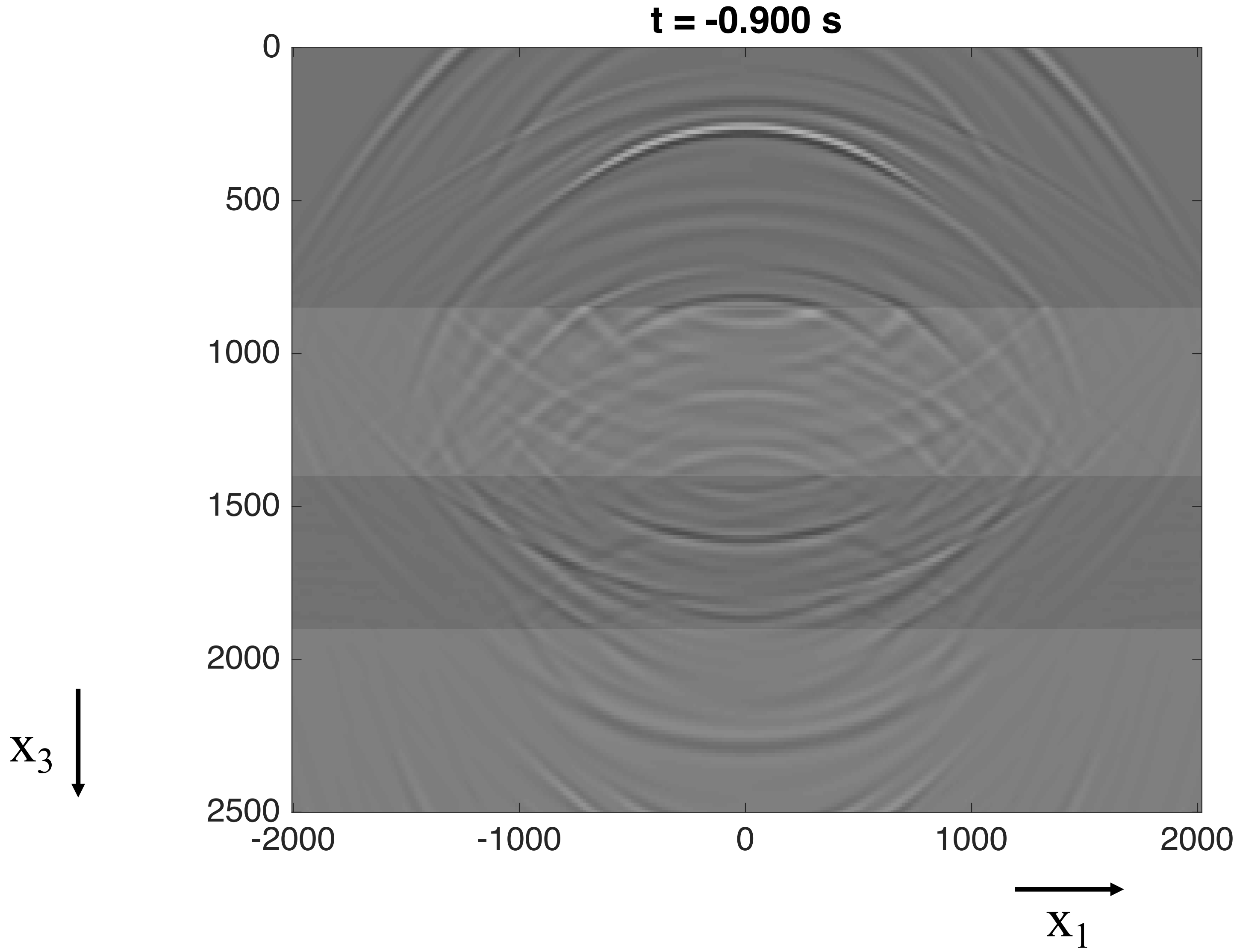
$$\mathbf{R}(x, x', \omega) \xrightarrow{\mathbf{F}(x', x_s, \omega)} \mathbf{G}_h(x, x_s, \omega) \xrightarrow{\mathbf{F}(x, x_r, \omega)} \mathbf{G}_h(x_r, x_s, \omega)$$



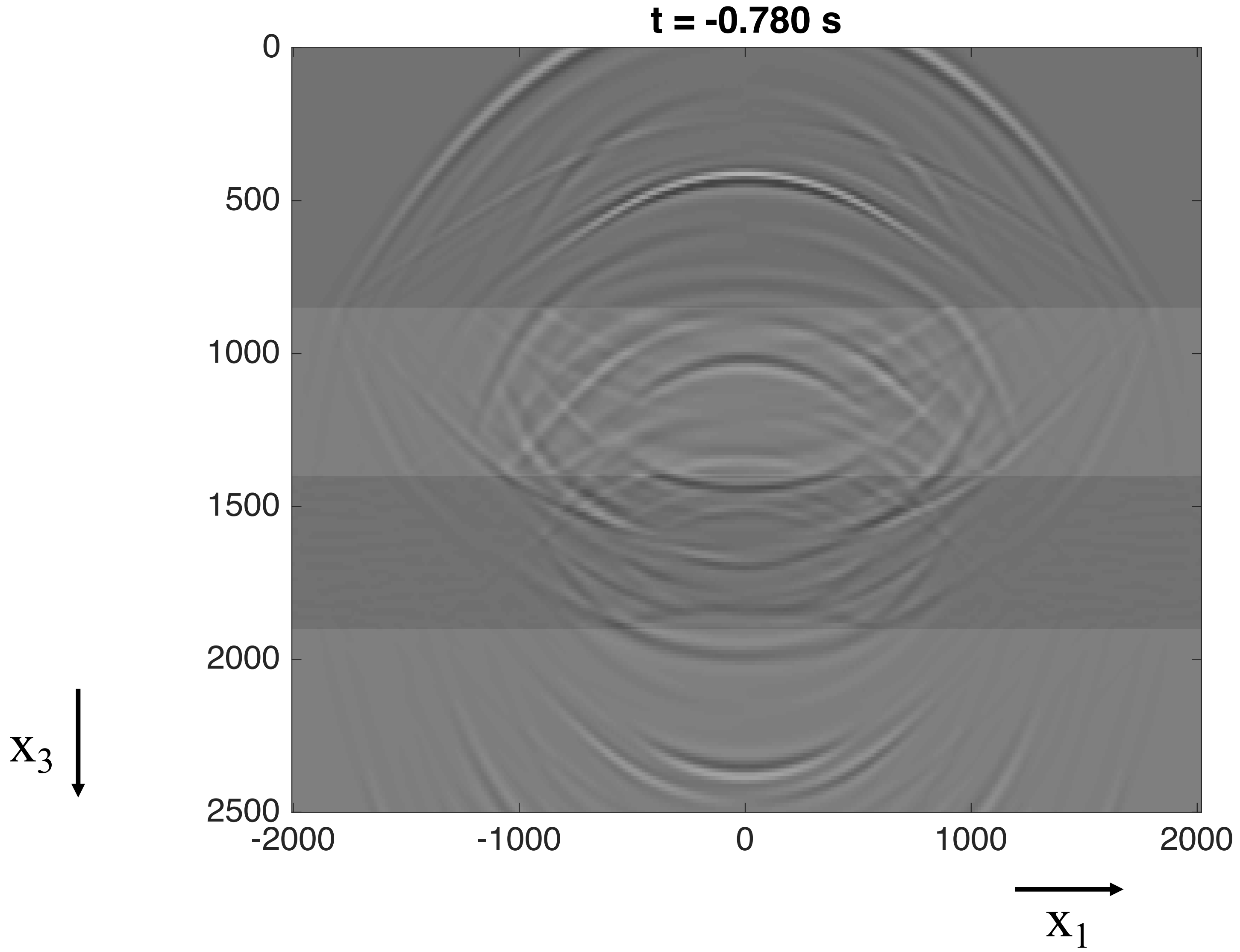
Numerical example: Single-sided homogeneous Green's function representation



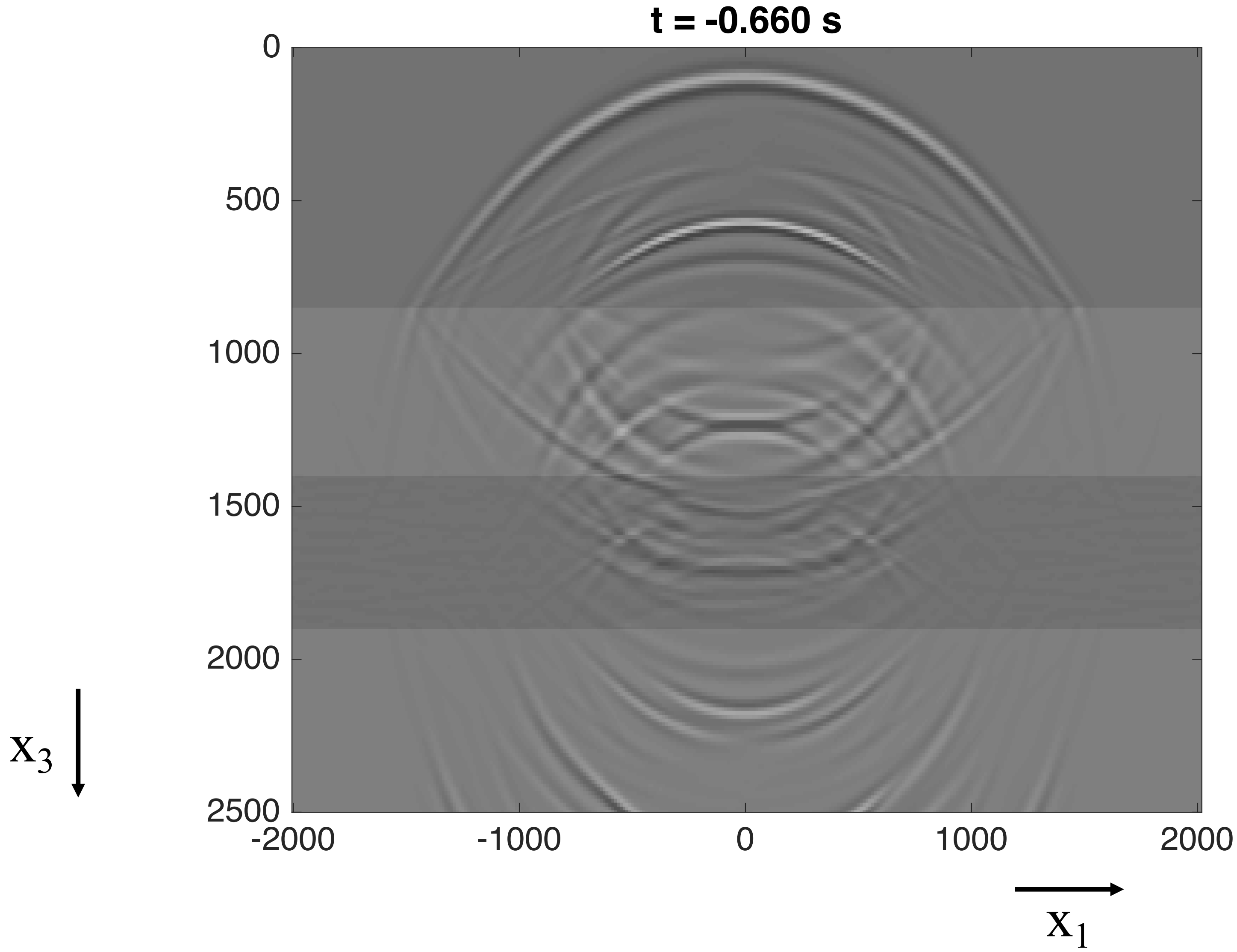
Numerical example: Single-sided homogeneous Green's function representation



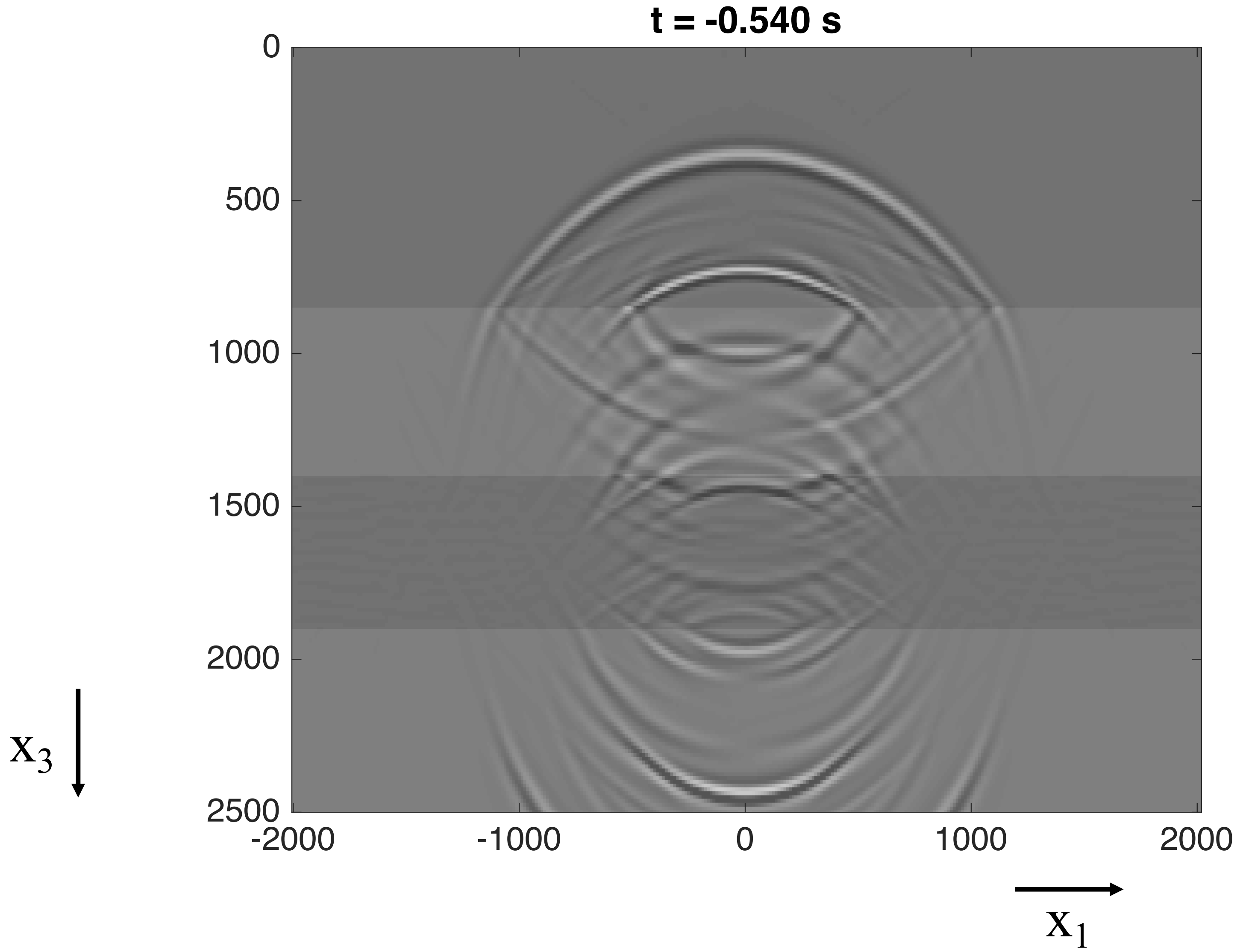
Numerical example: Single-sided homogeneous Green's function representation



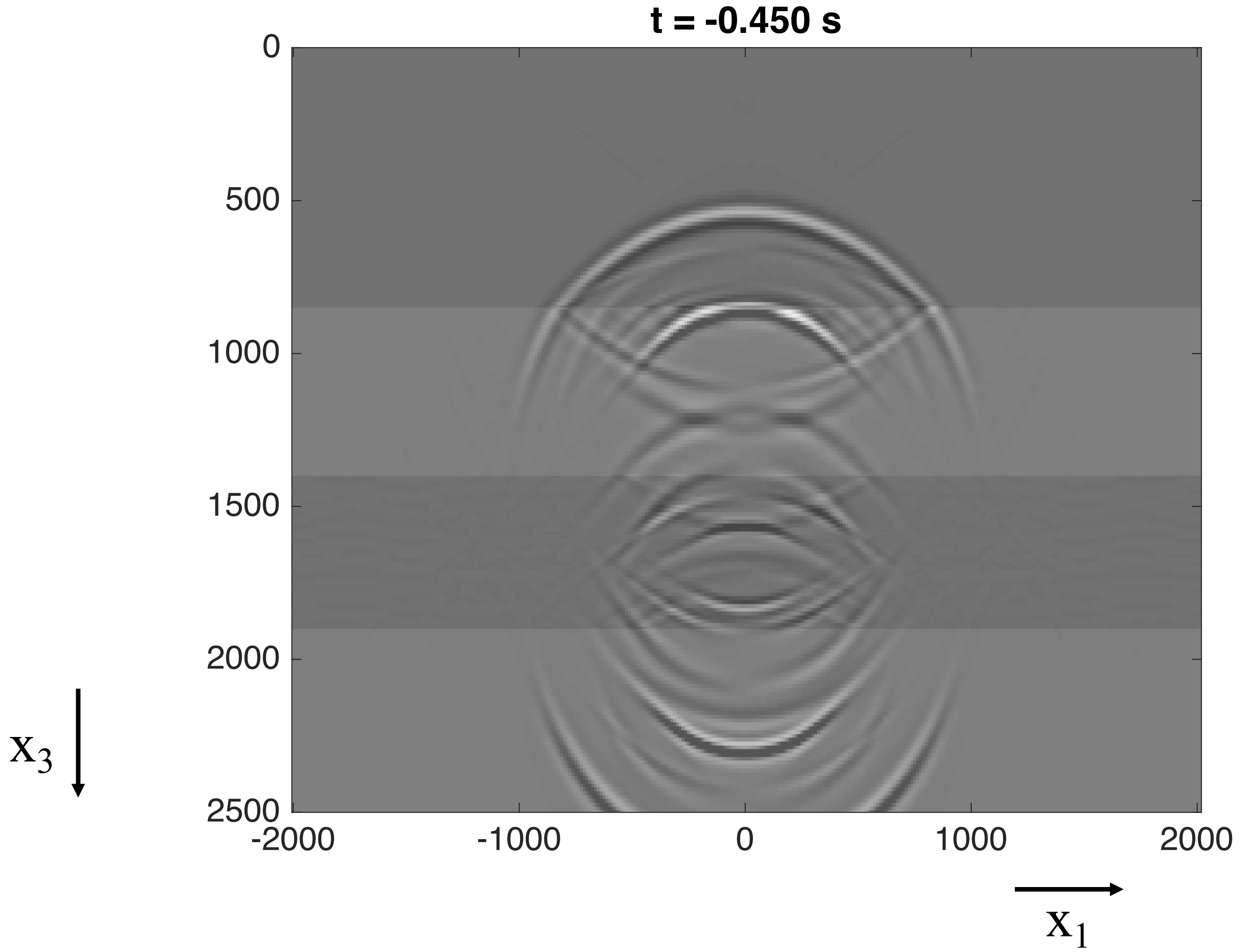
Numerical example: Single-sided homogeneous Green's function representation



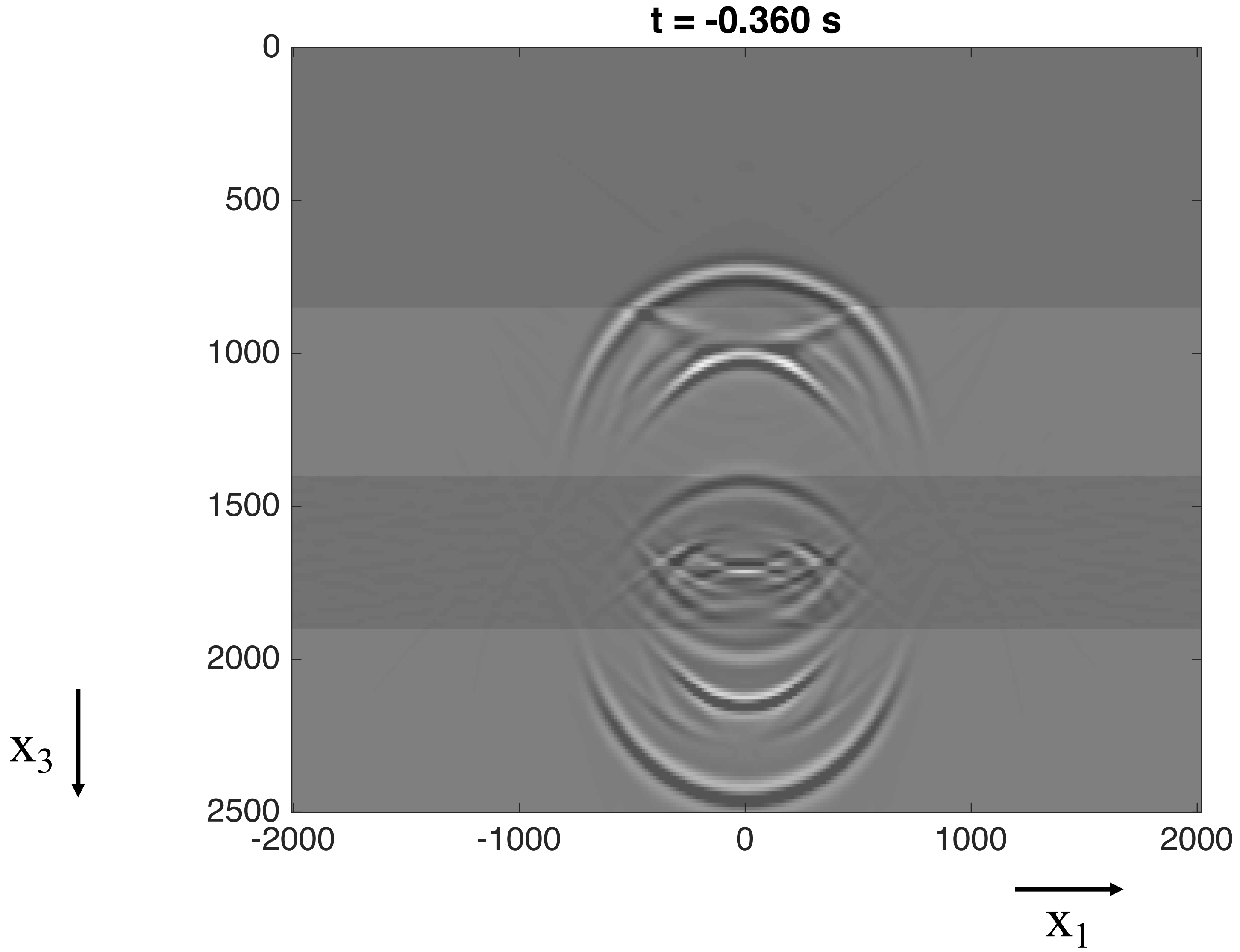
Numerical example: Single-sided homogeneous Green's function representation



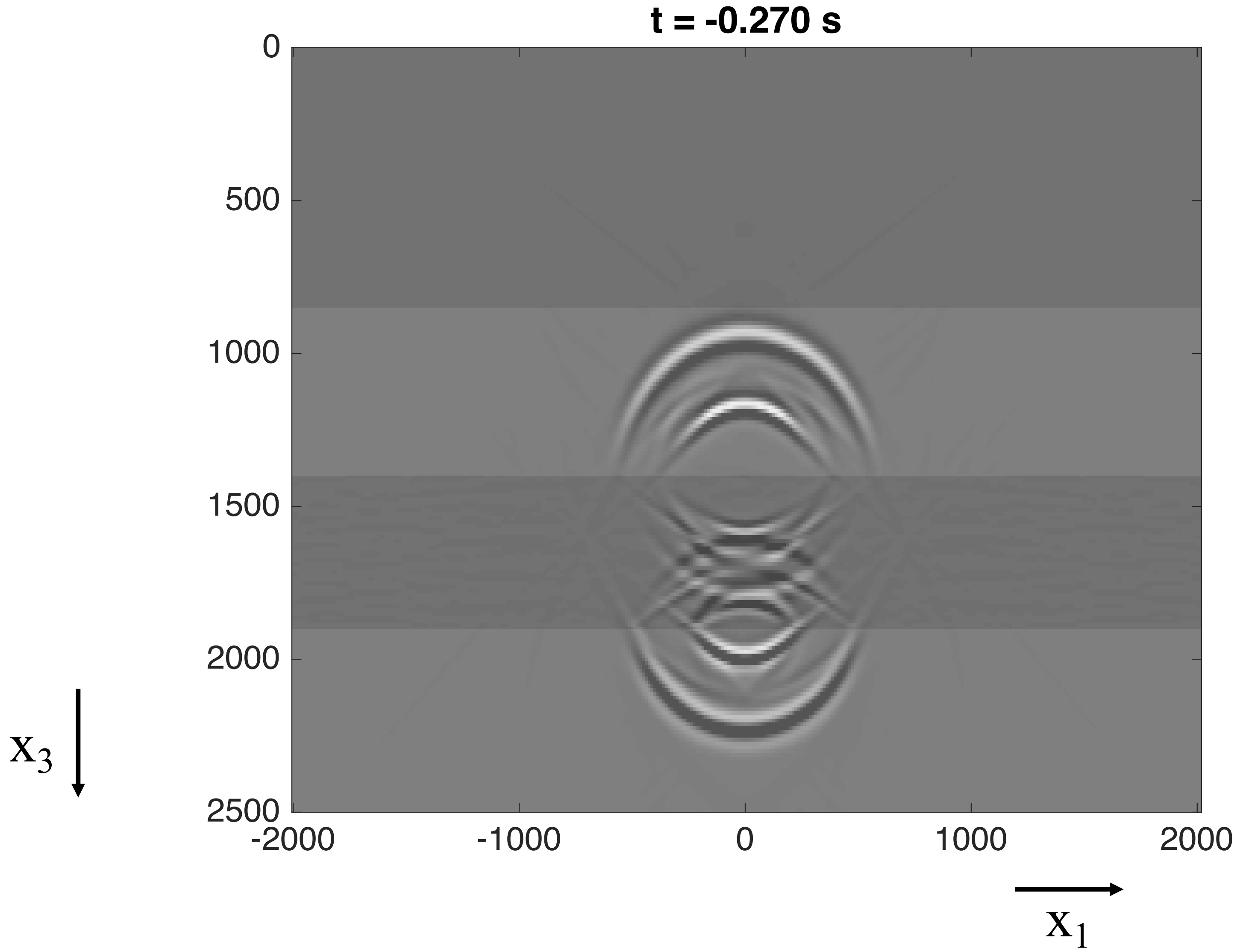
Numerical example: Single-sided homogeneous Green's function representation



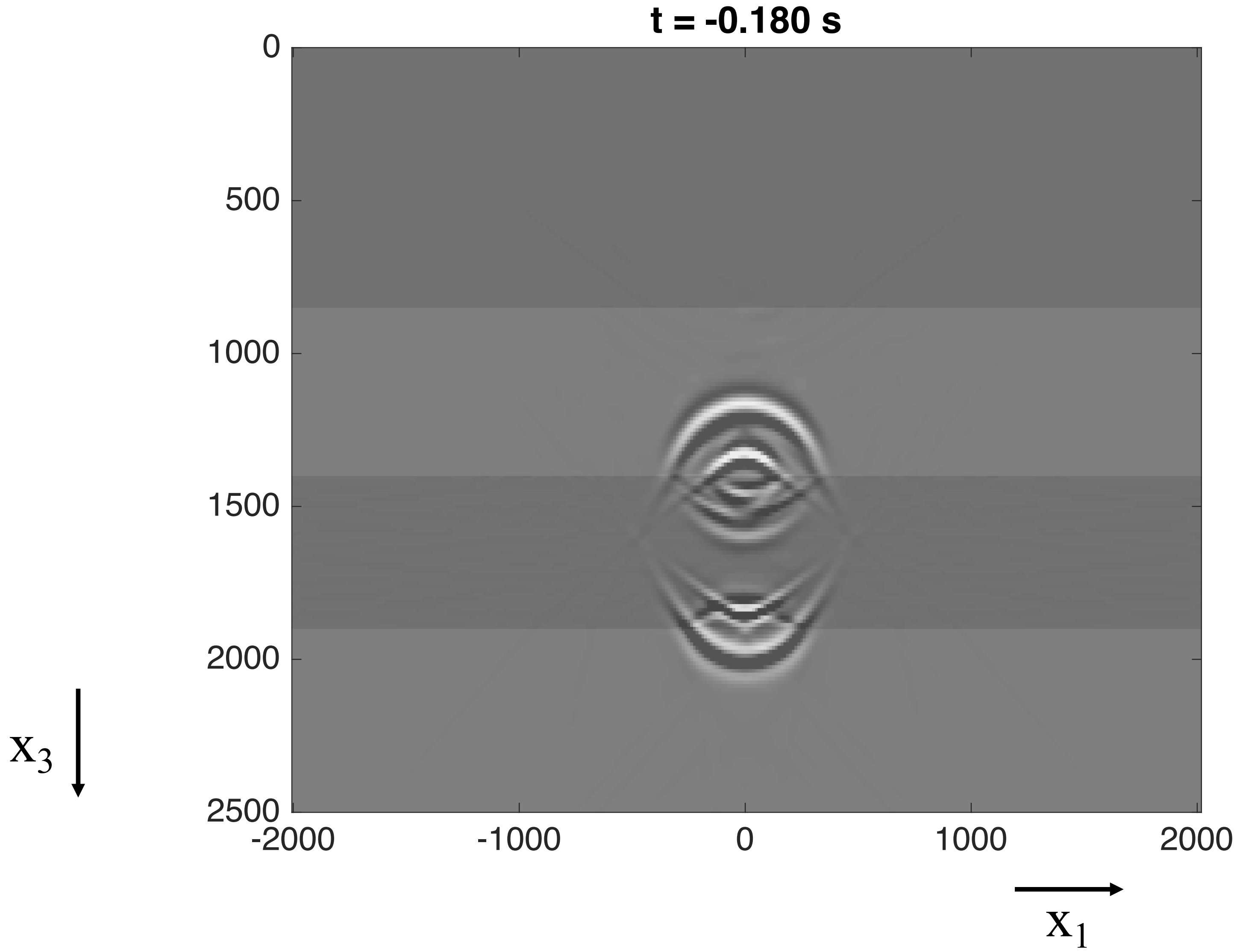
Numerical example: Single-sided homogeneous Green's function representation



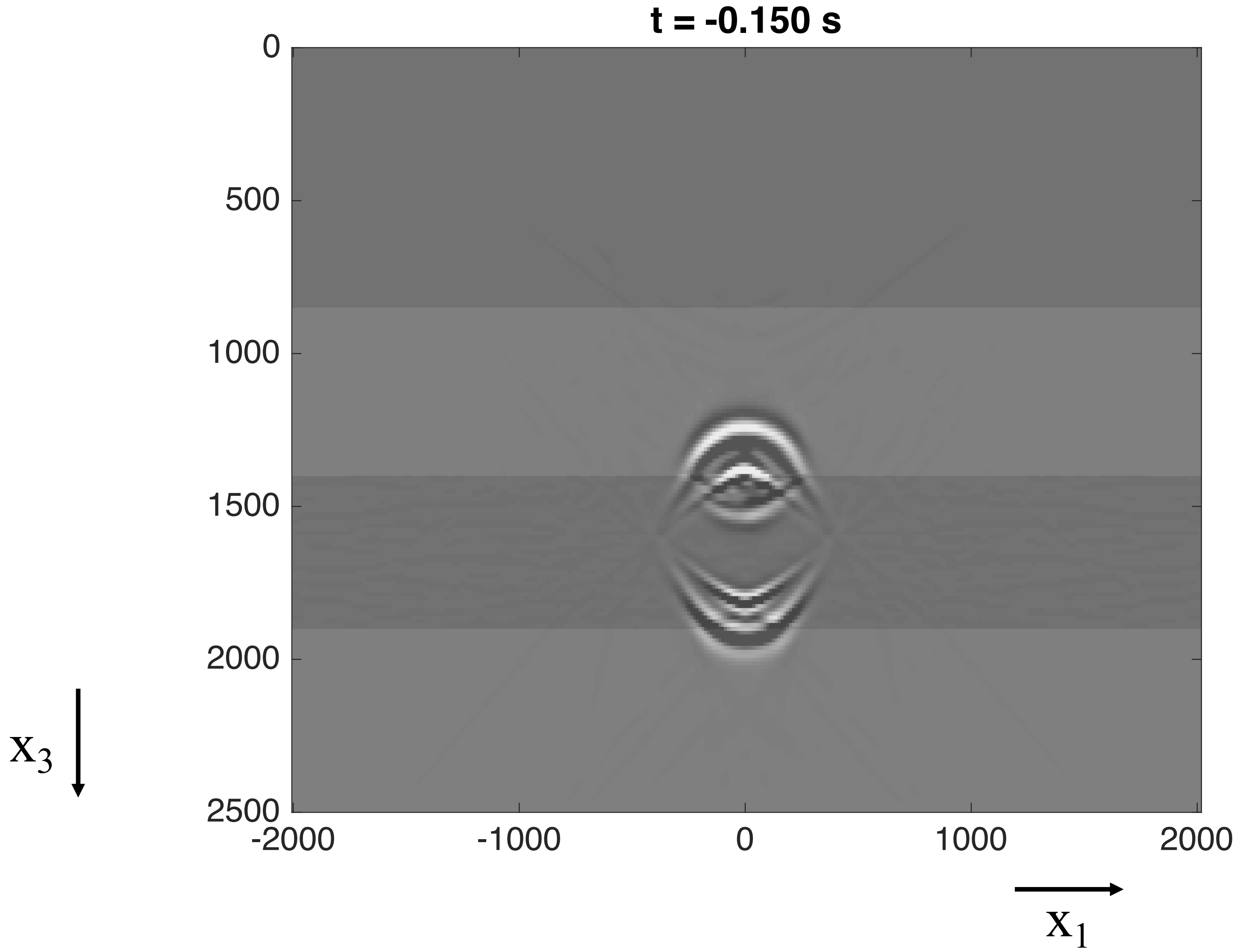
Numerical example: Single-sided homogeneous Green's function representation



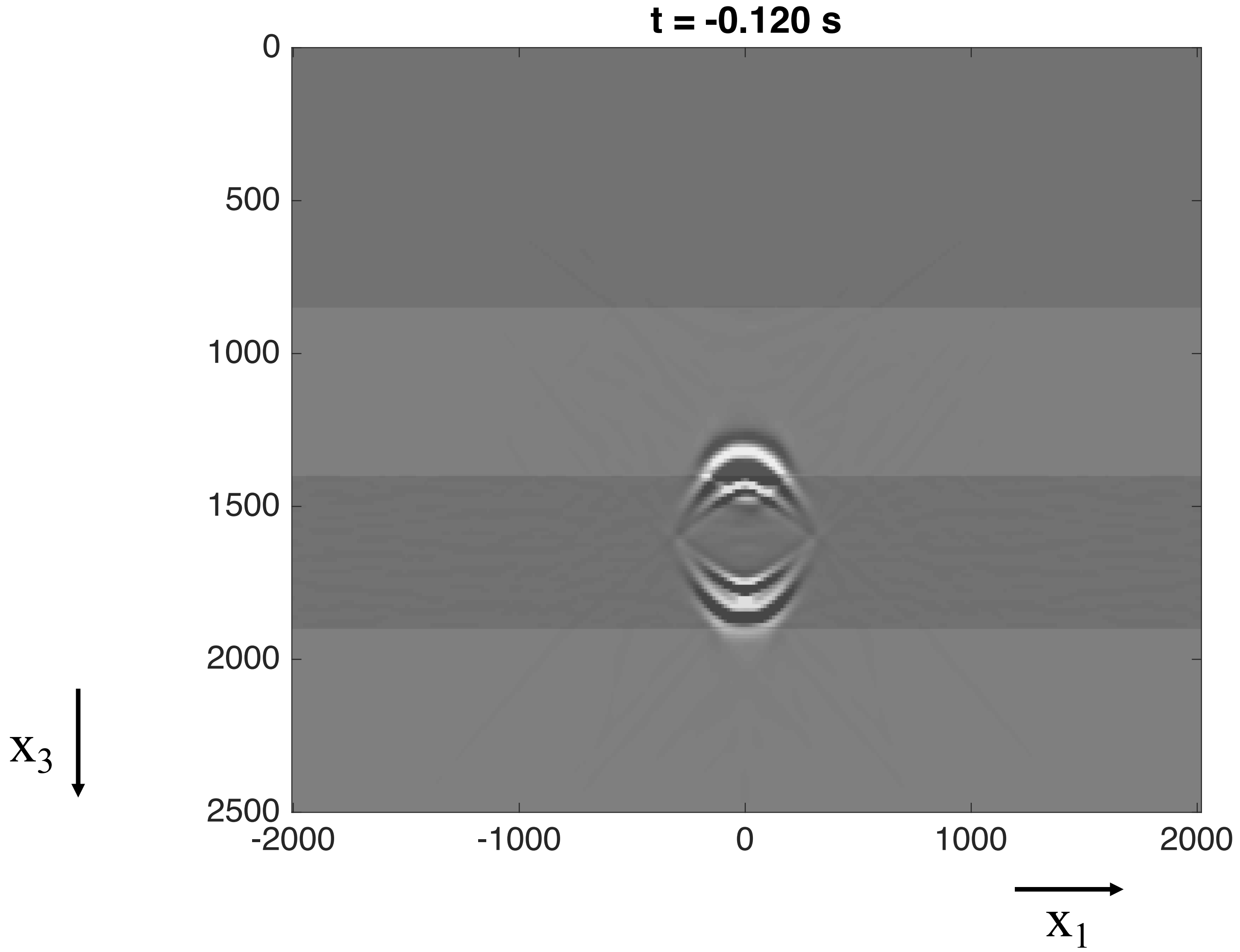
Numerical example: Single-sided homogeneous Green's function representation



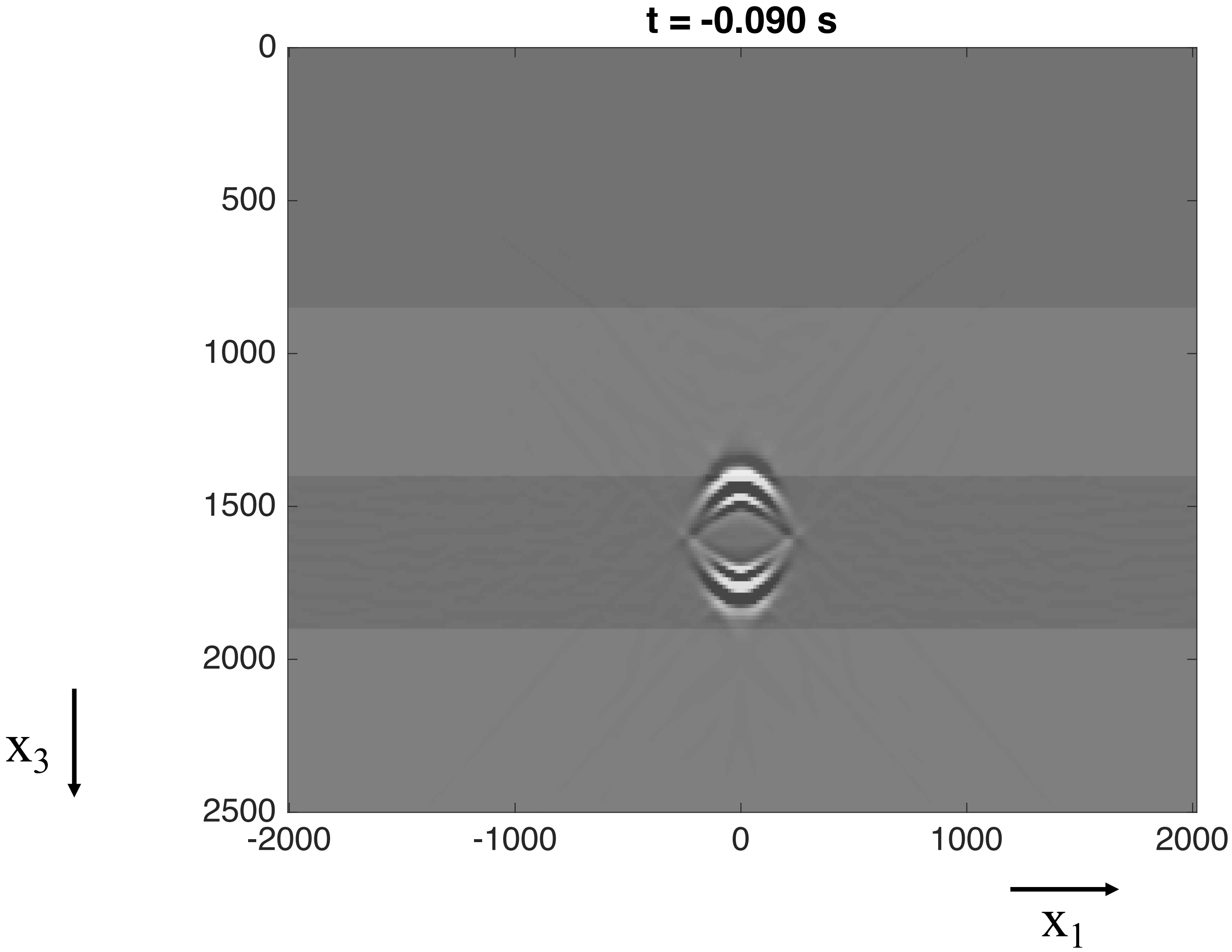
Numerical example: Single-sided homogeneous Green's function representation



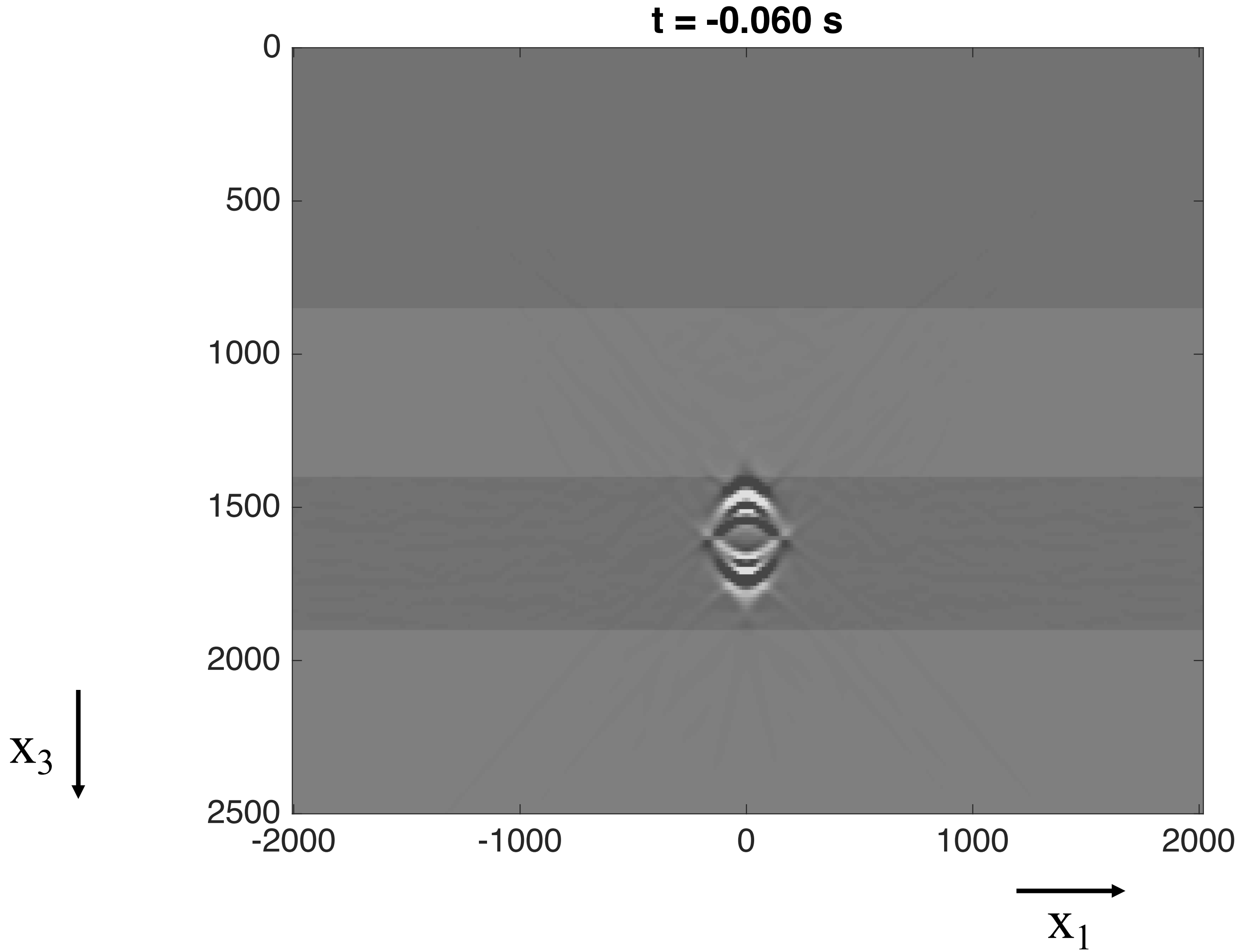
Numerical example: Single-sided homogeneous Green's function representation



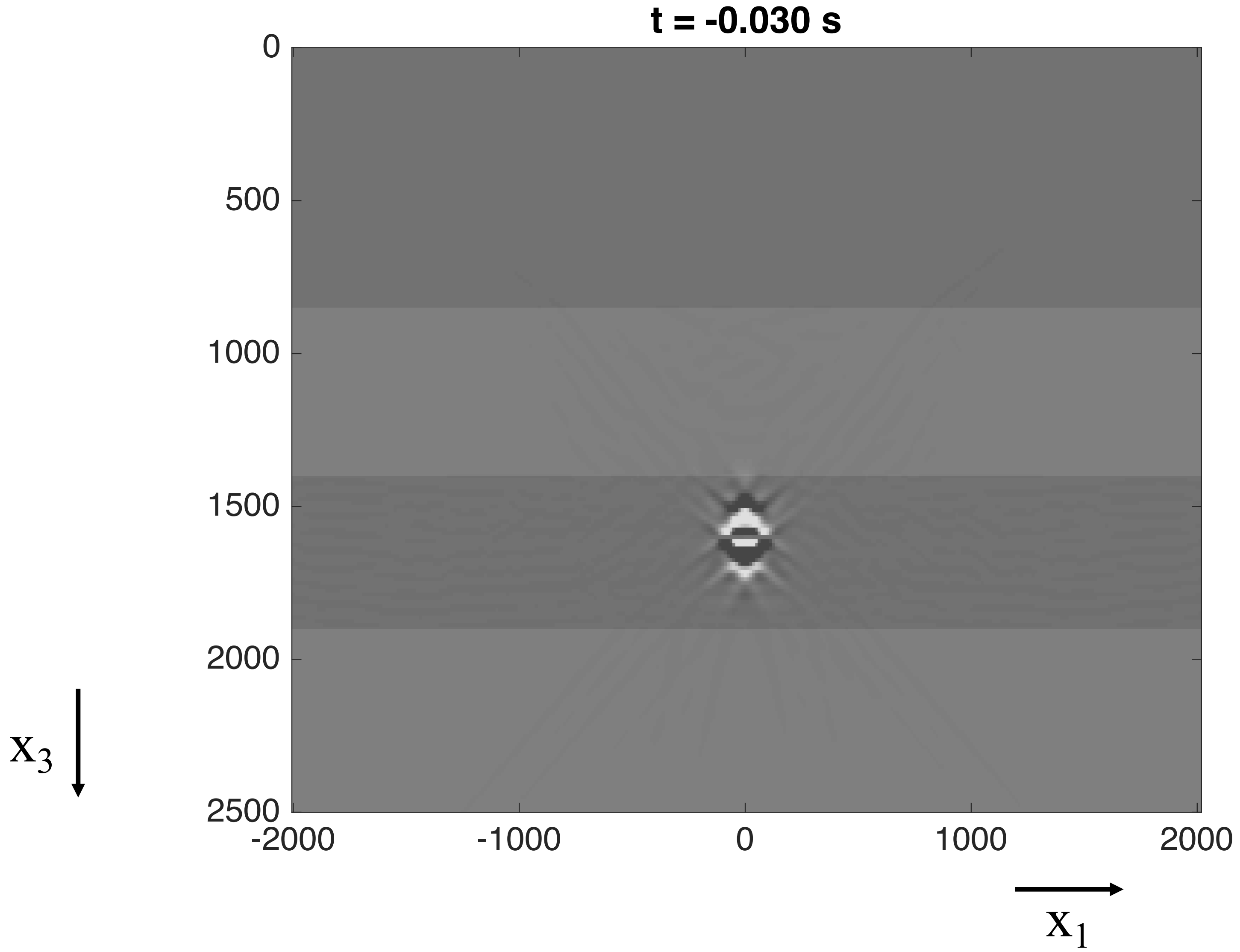
Numerical example: Single-sided homogeneous Green's function representation



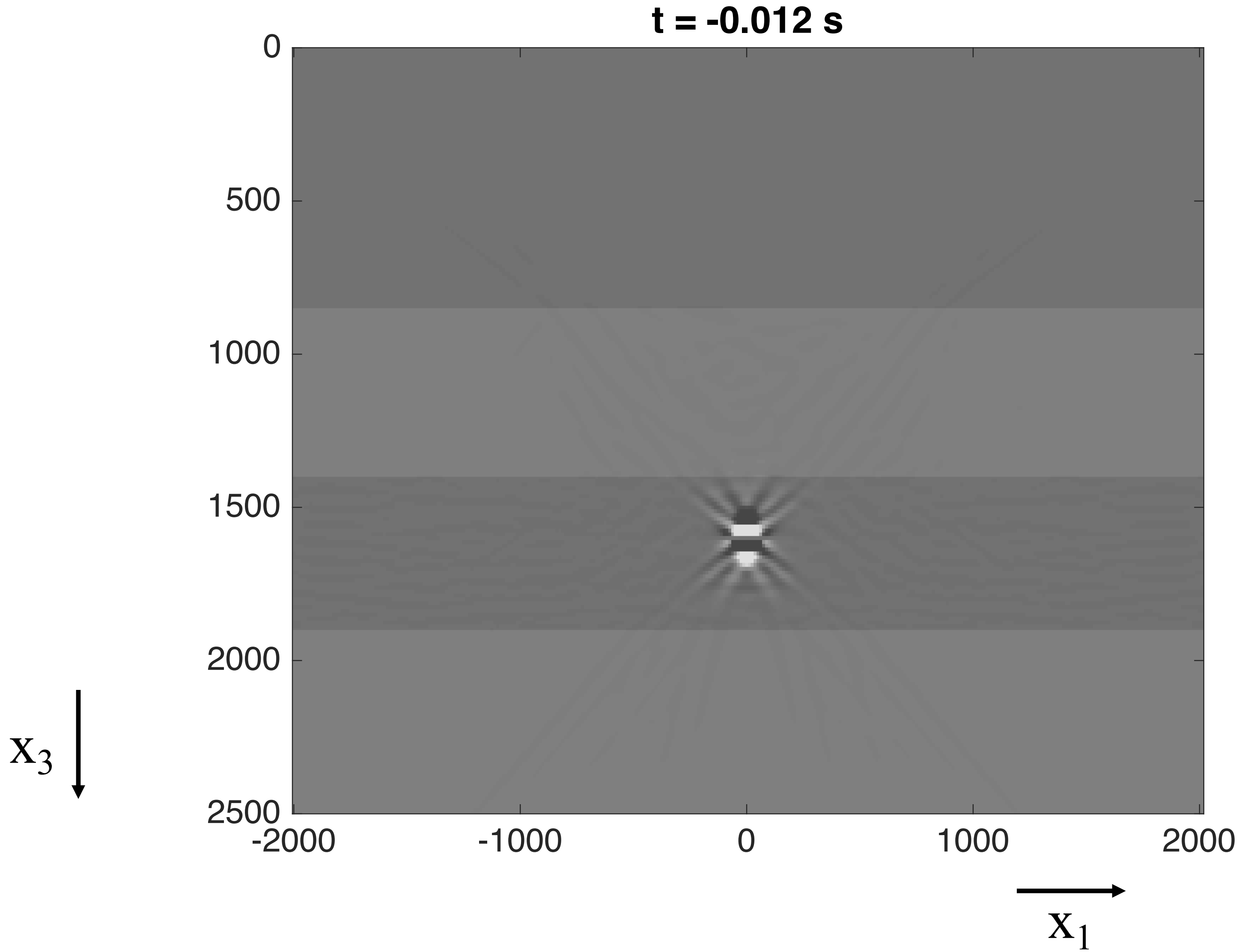
Numerical example: Single-sided homogeneous Green's function representation



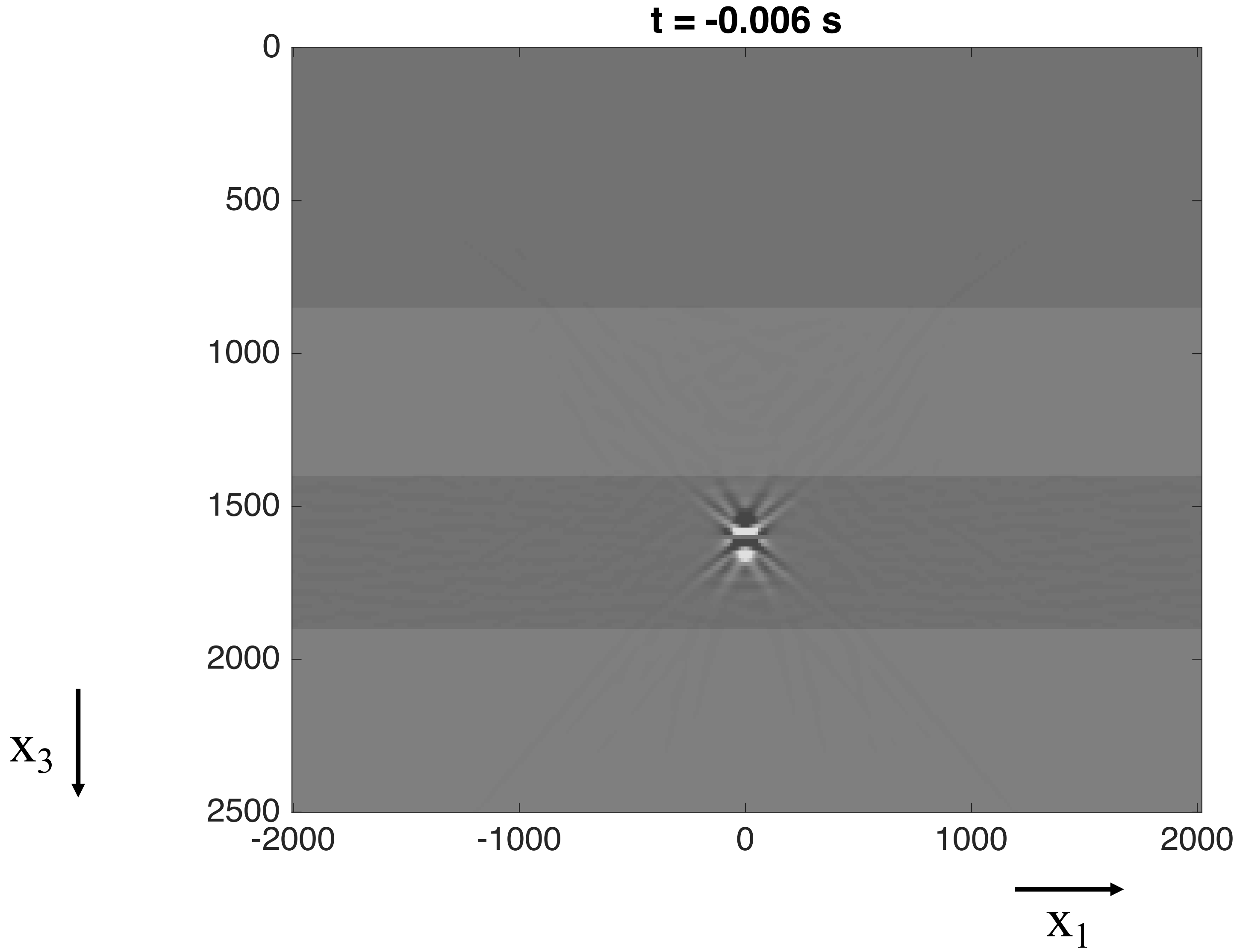
Numerical example: Single-sided homogeneous Green's function representation



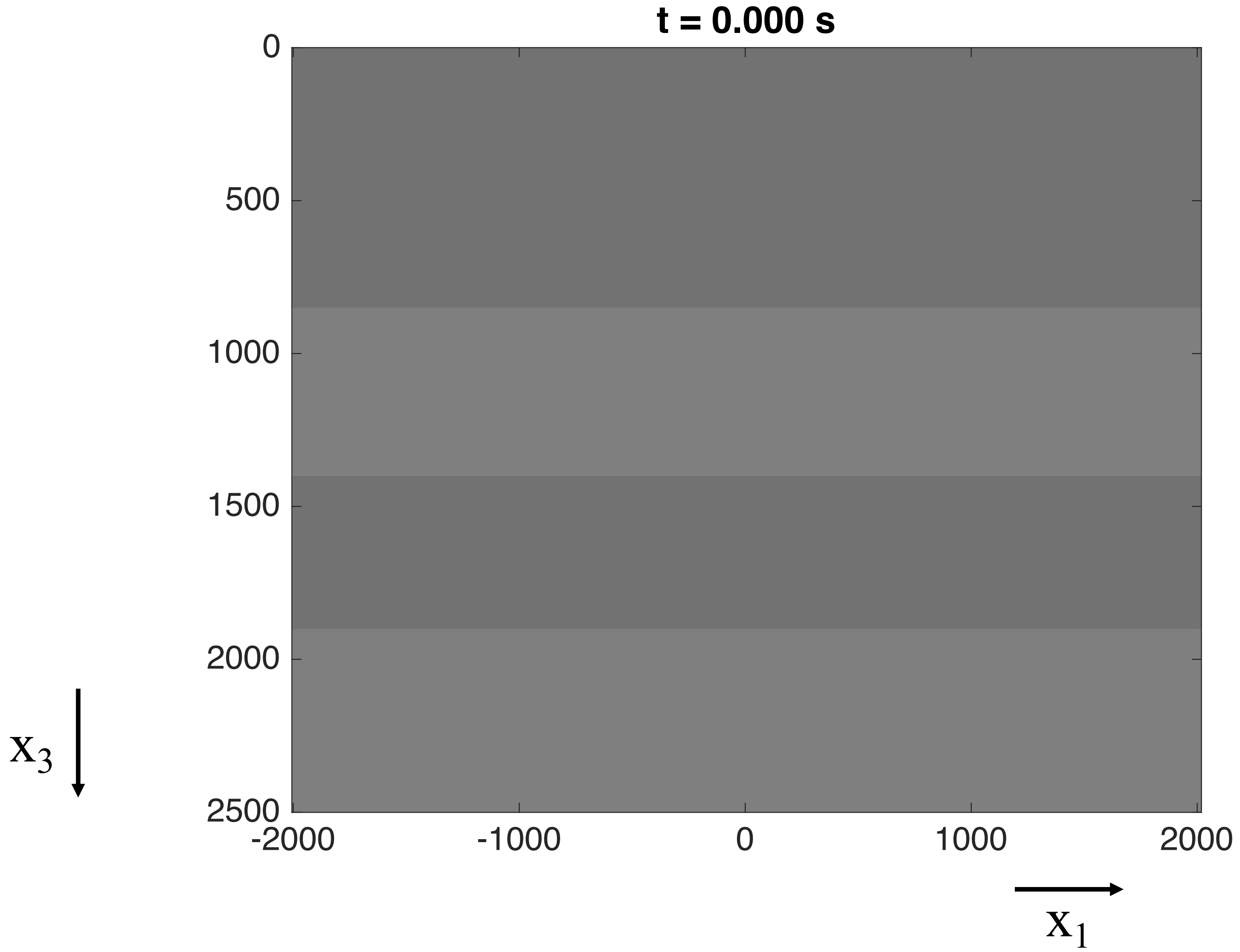
Numerical example: Single-sided homogeneous Green's function representation



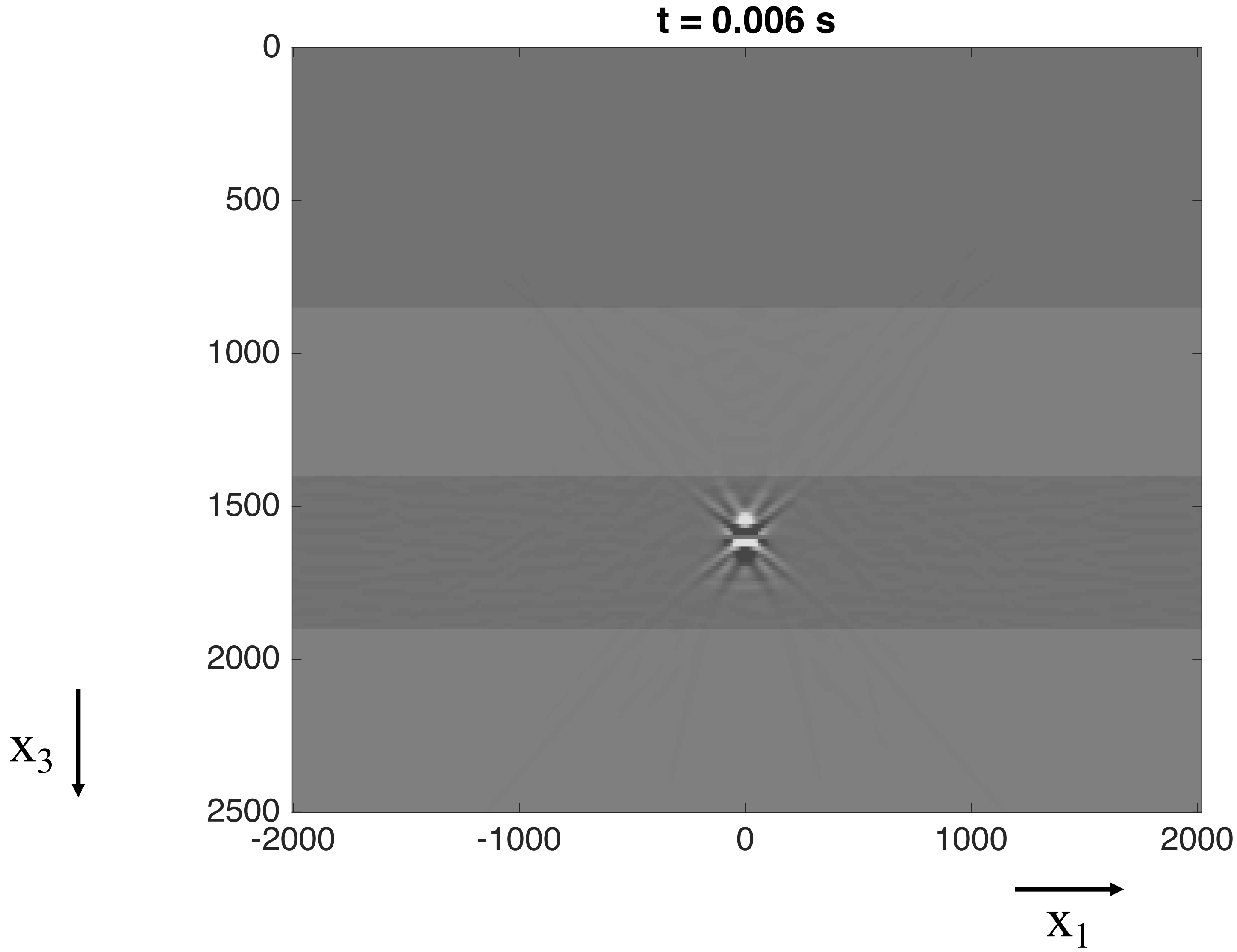
Numerical example: Single-sided homogeneous Green's function representation



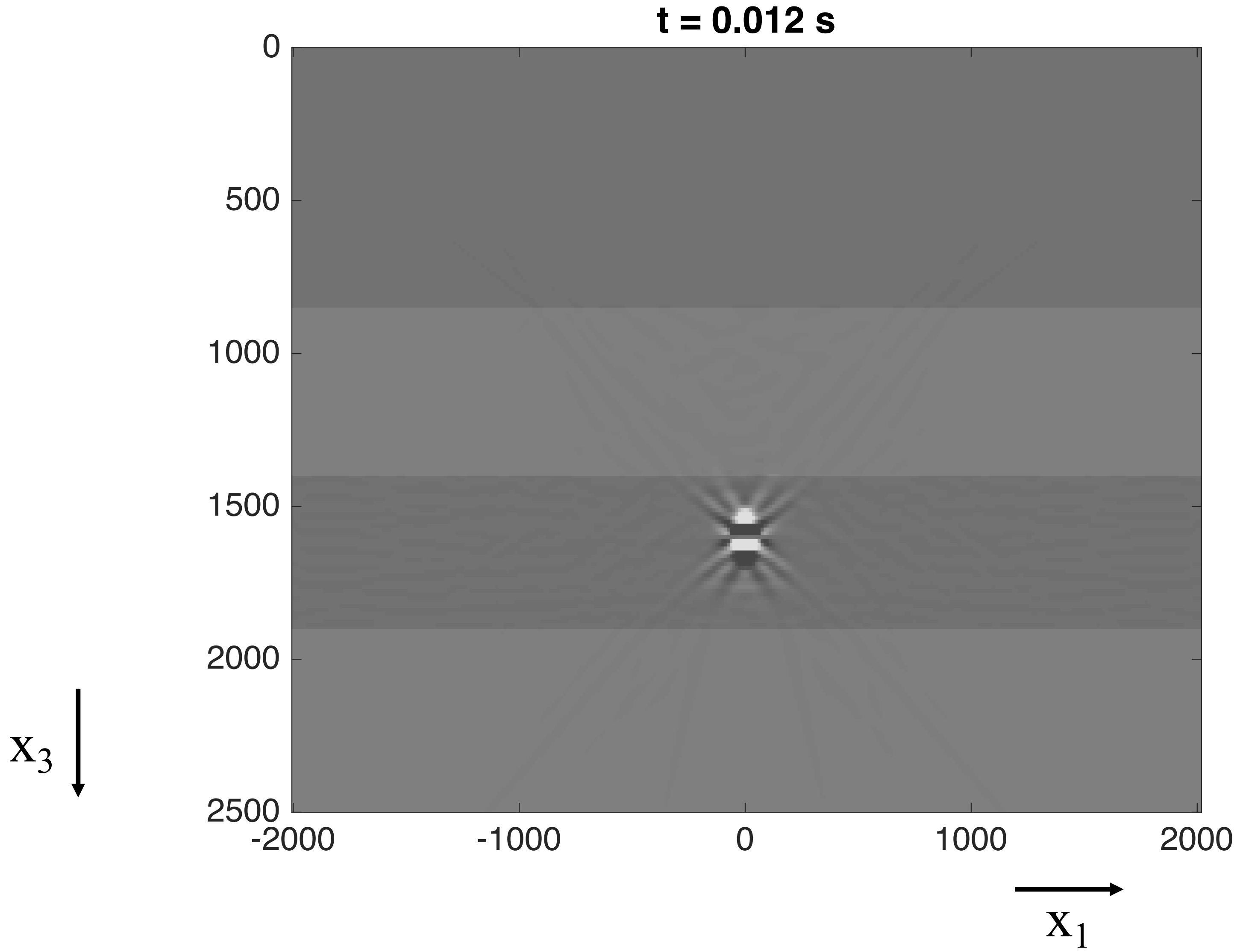
Numerical example: Single-sided homogeneous Green's function representation



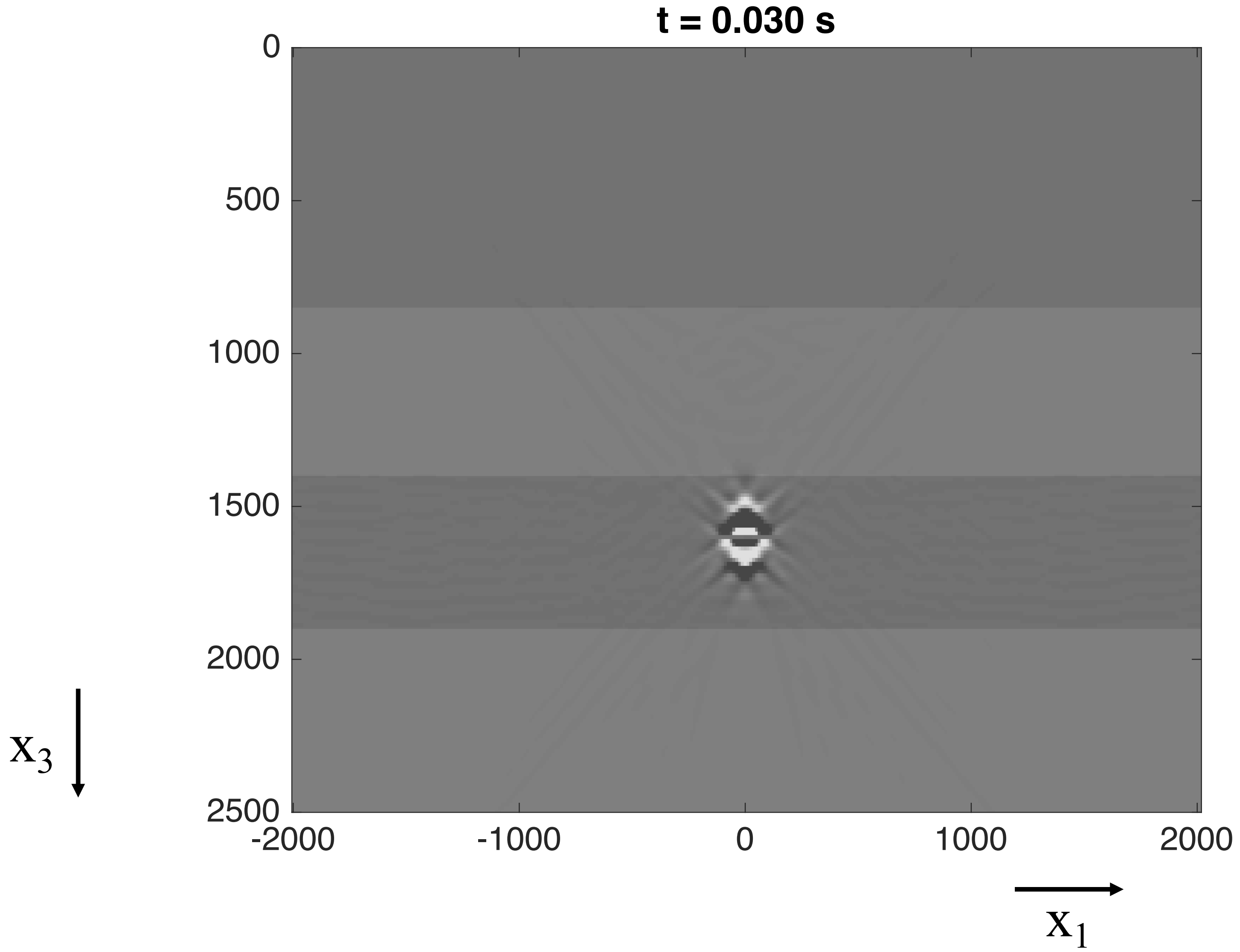
Numerical example: Single-sided homogeneous Green's function representation



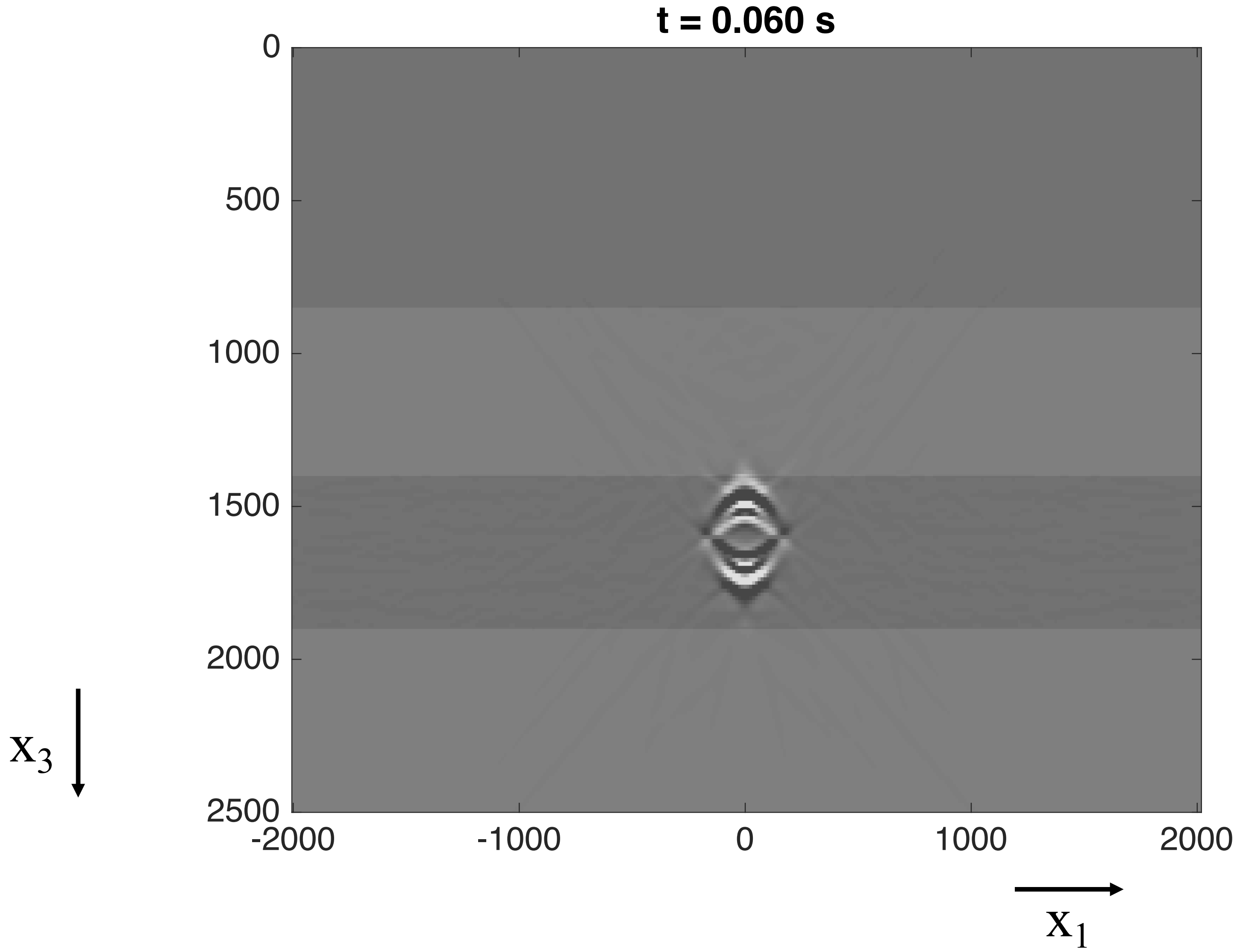
Numerical example: Single-sided homogeneous Green's function representation



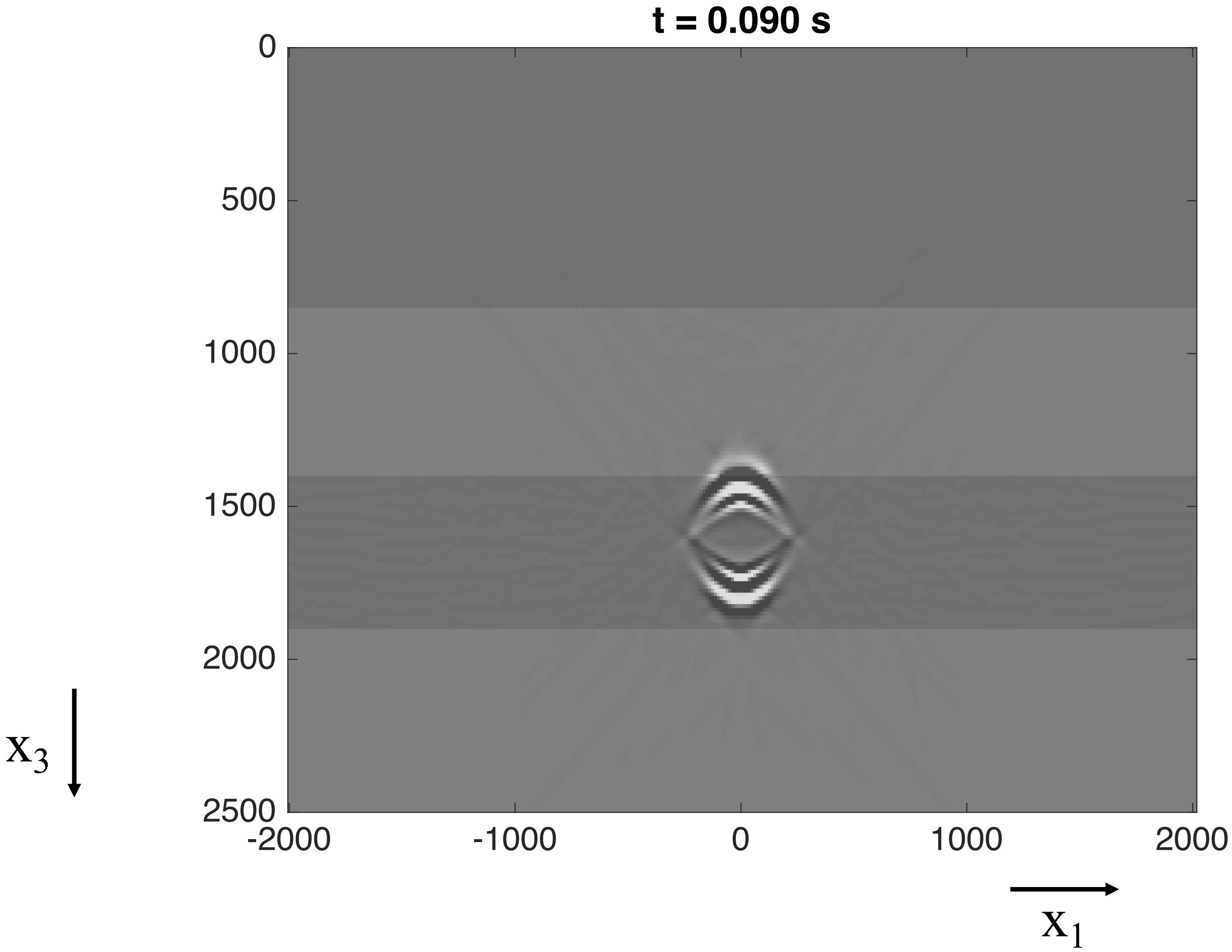
Numerical example: Single-sided homogeneous Green's function representation



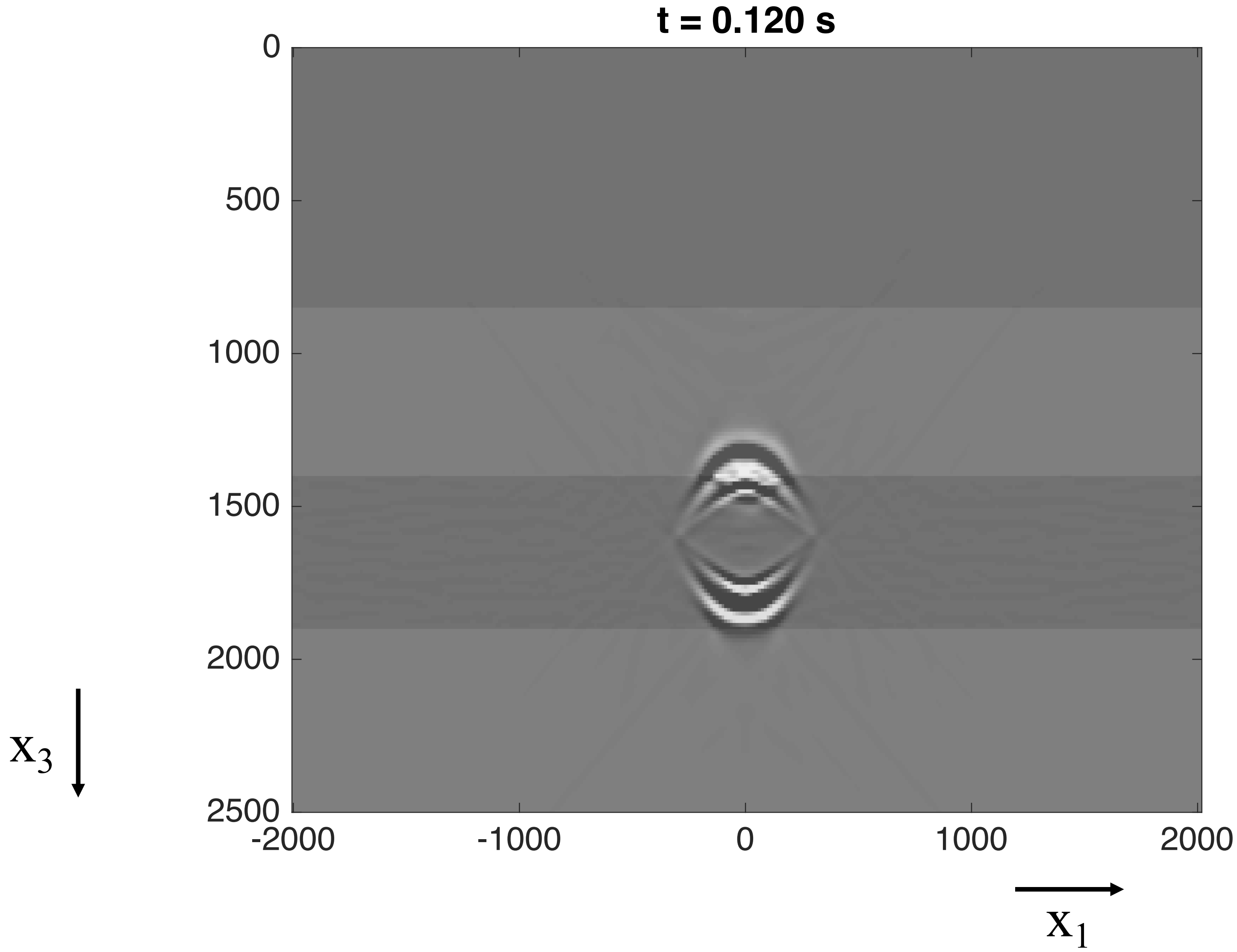
Numerical example: Single-sided homogeneous Green's function representation



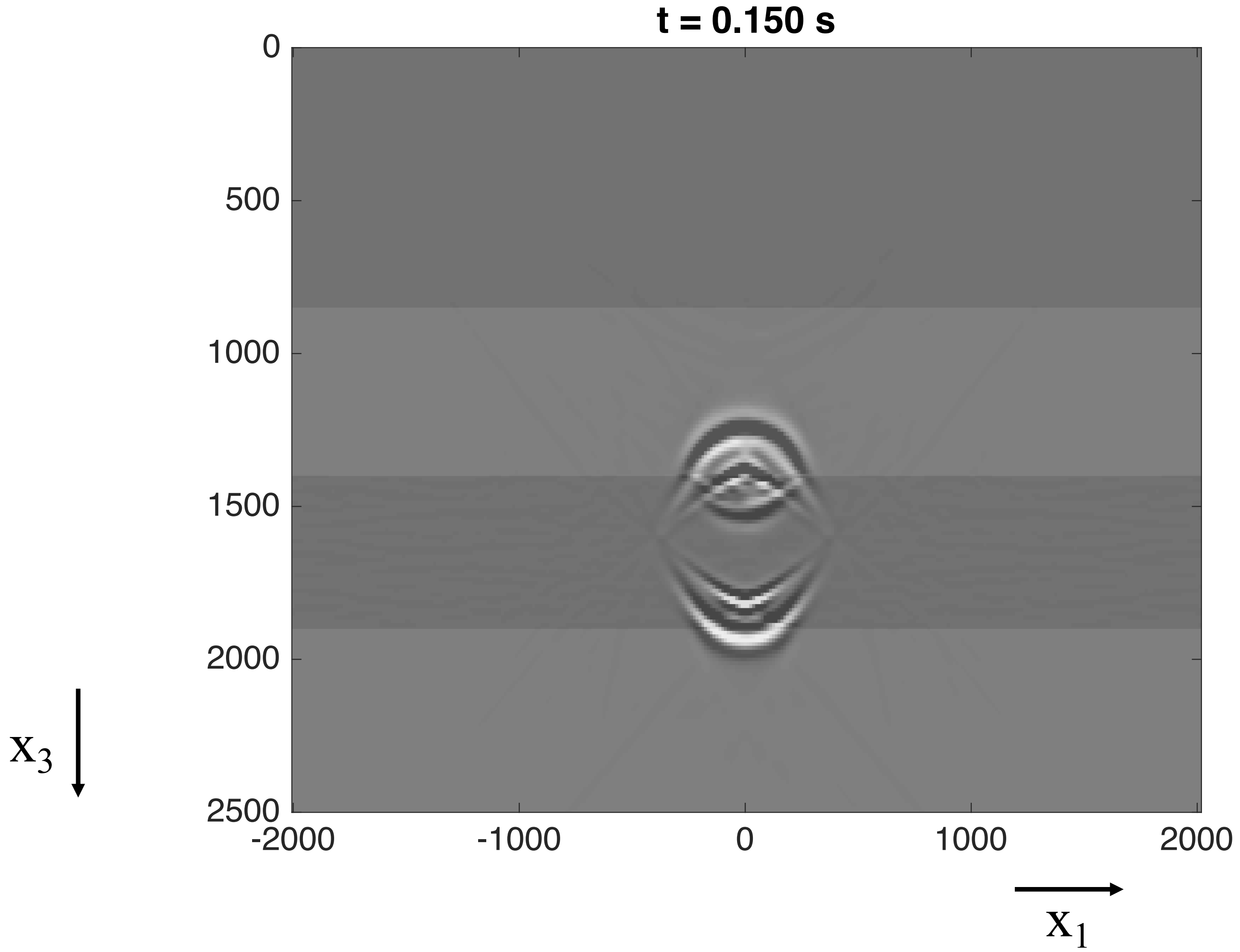
Numerical example: Single-sided homogeneous Green's function representation



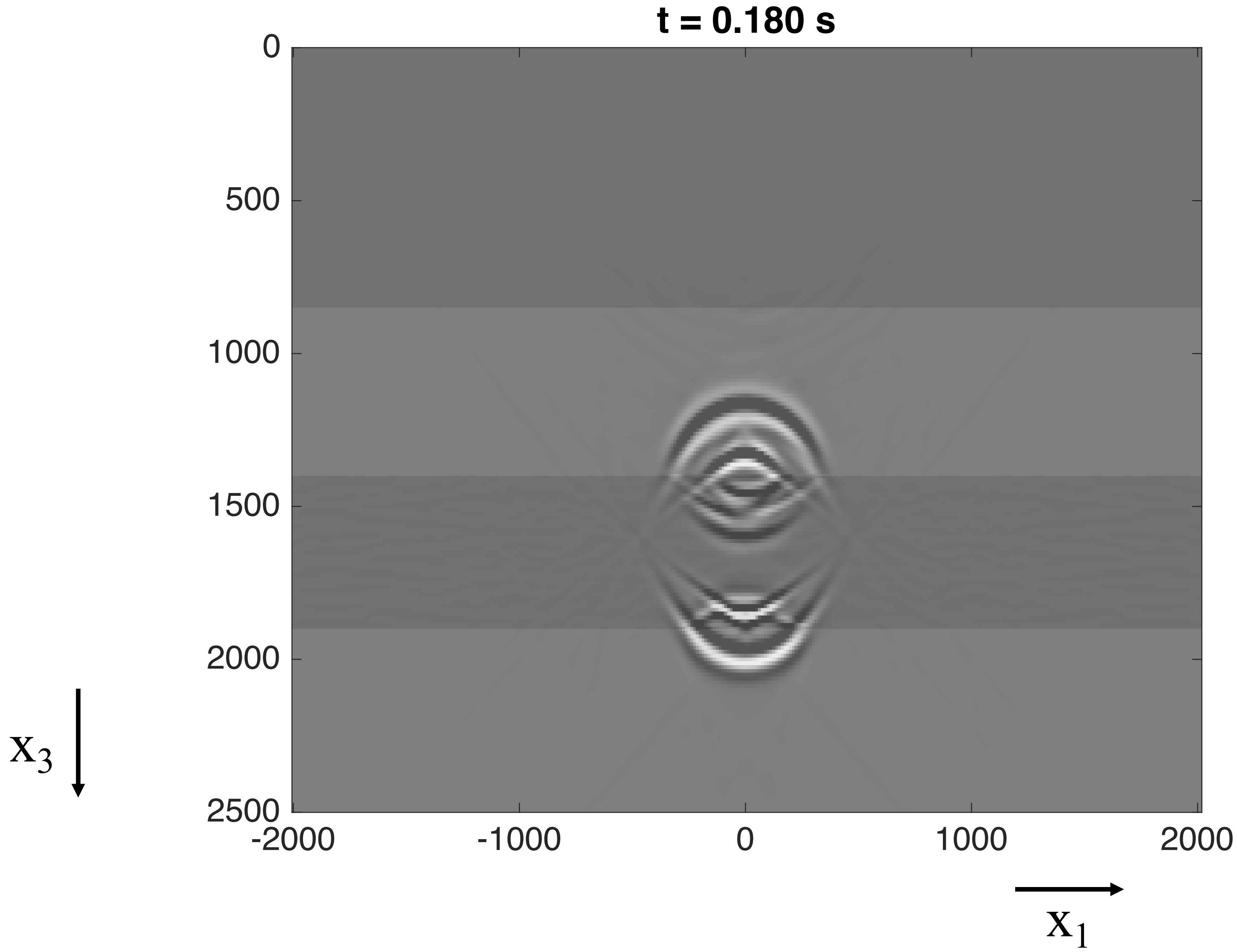
Numerical example: Single-sided homogeneous Green's function representation



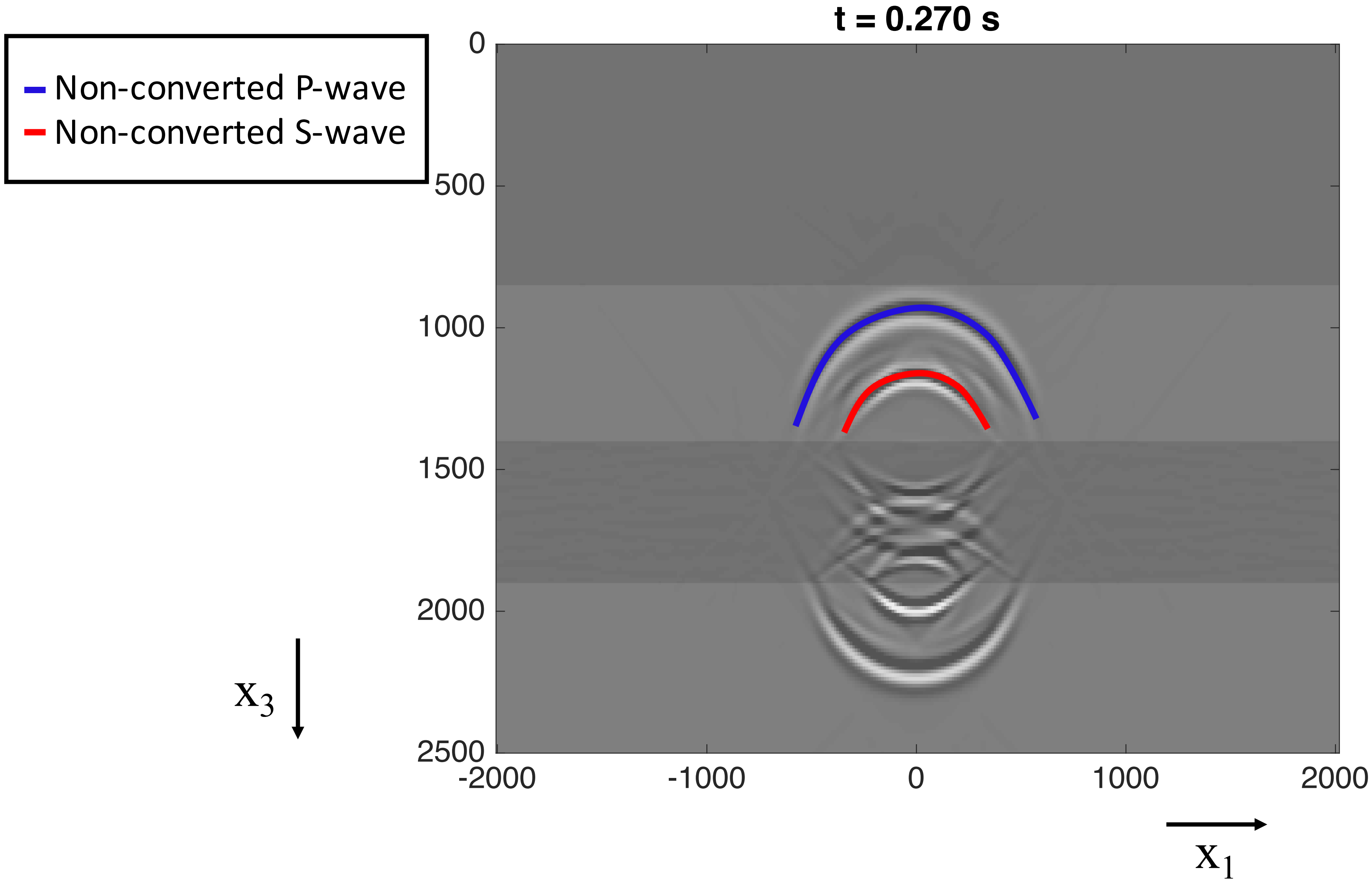
Numerical example: Single-sided homogeneous Green's function representation



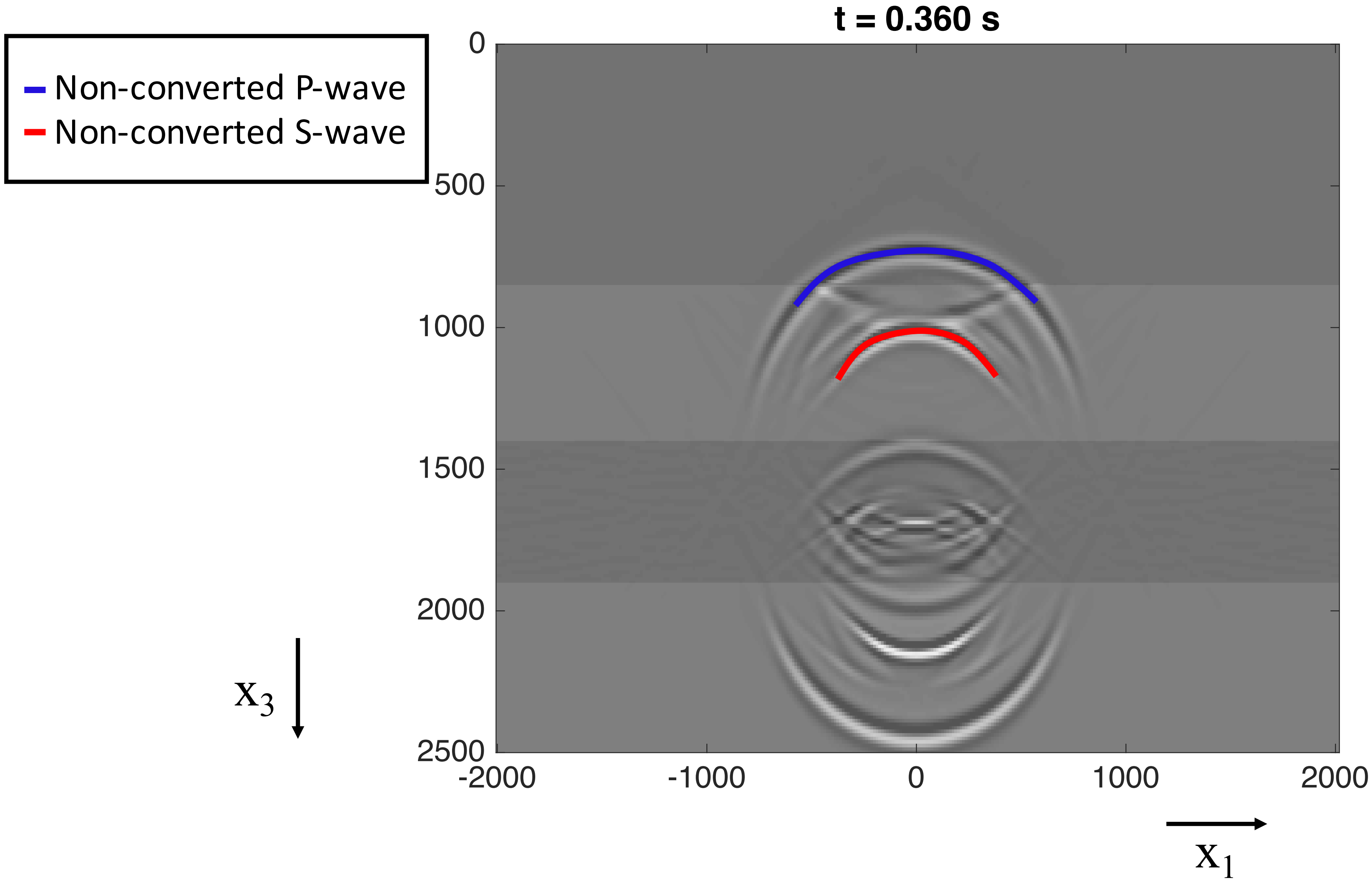
Numerical example: Single-sided homogeneous Green's function representation



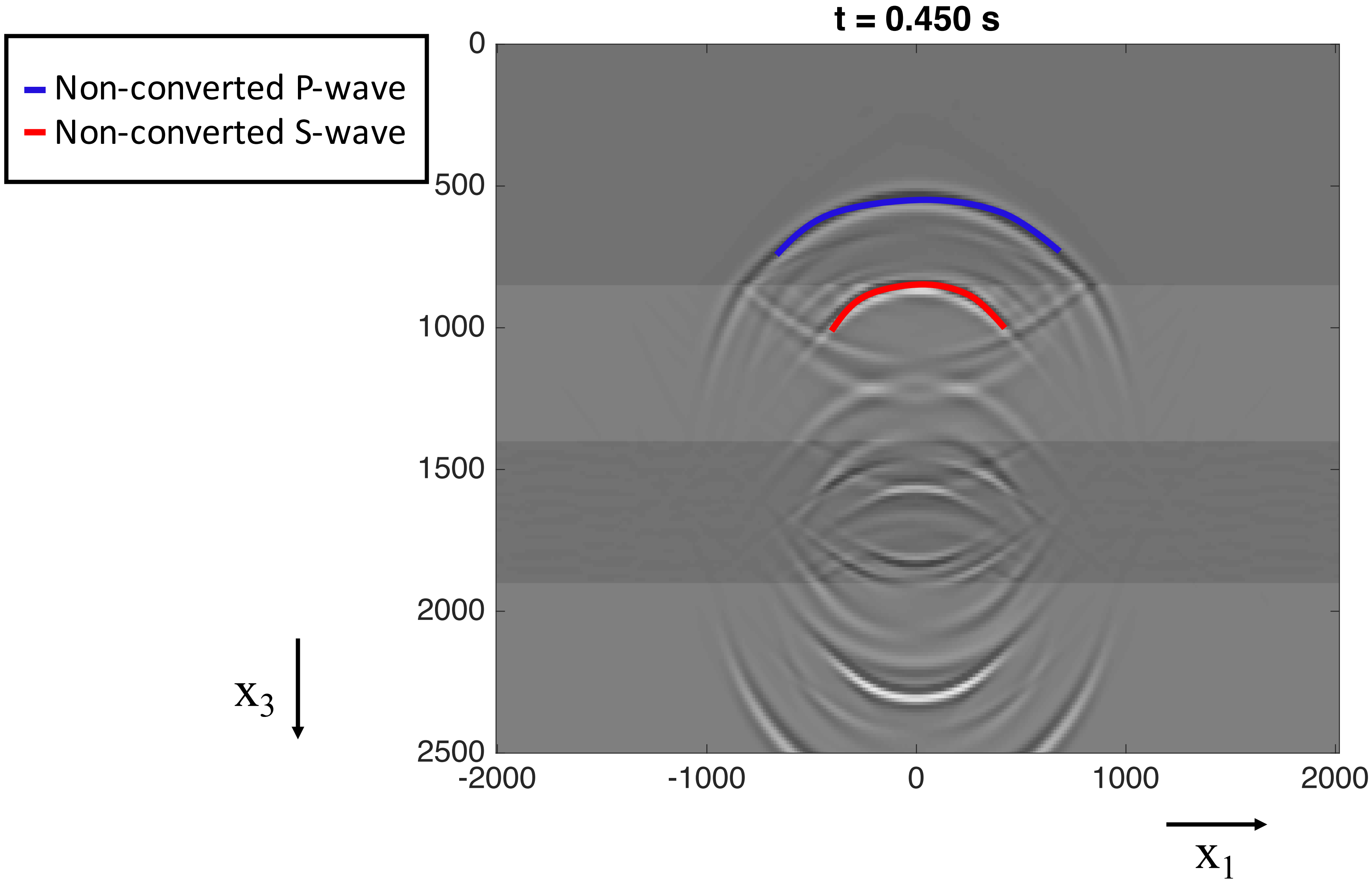
Numerical example: Single-sided homogeneous Green's function representation



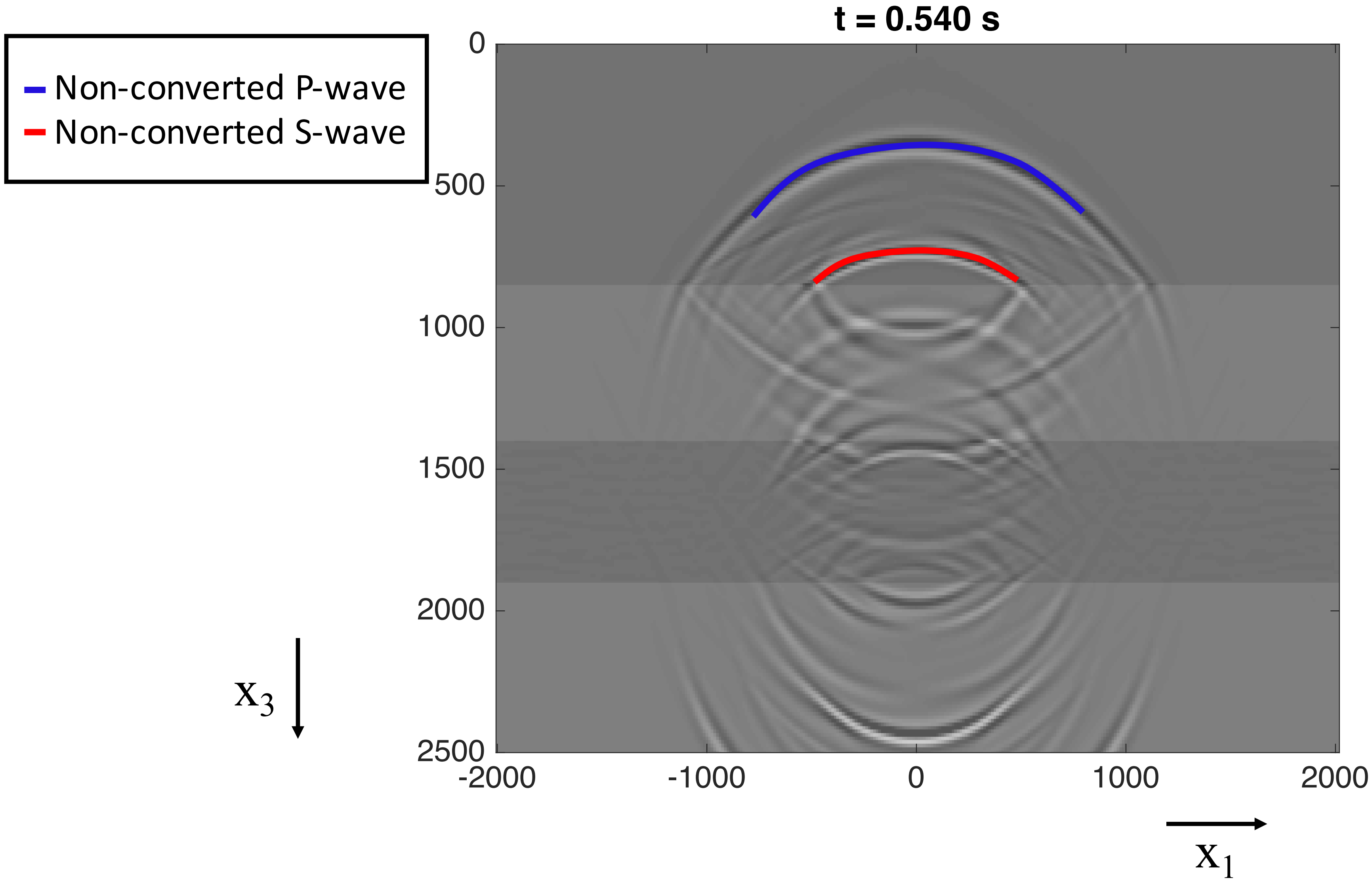
Numerical example: Single-sided homogeneous Green's function representation



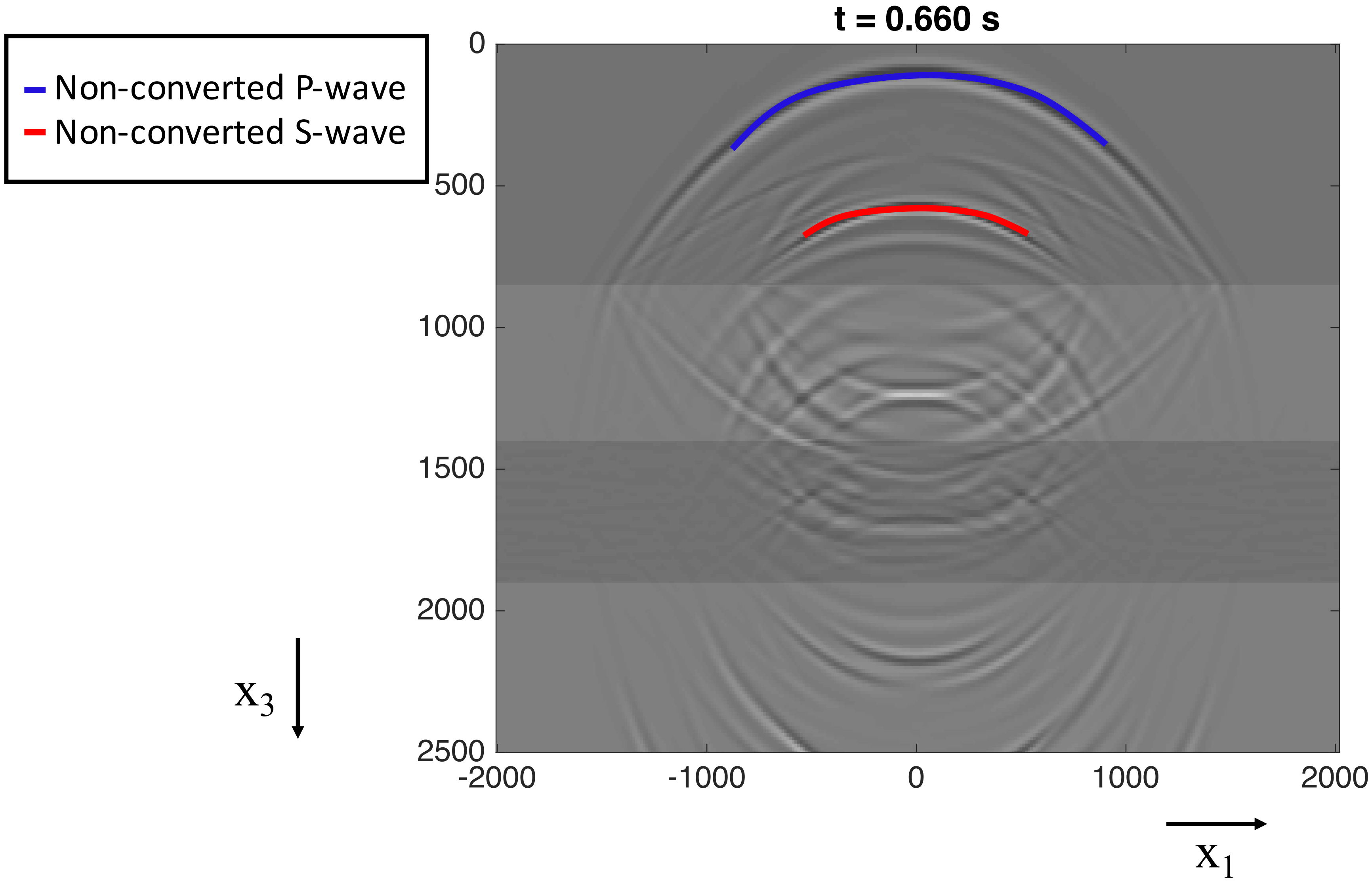
Numerical example: Single-sided homogeneous Green's function representation



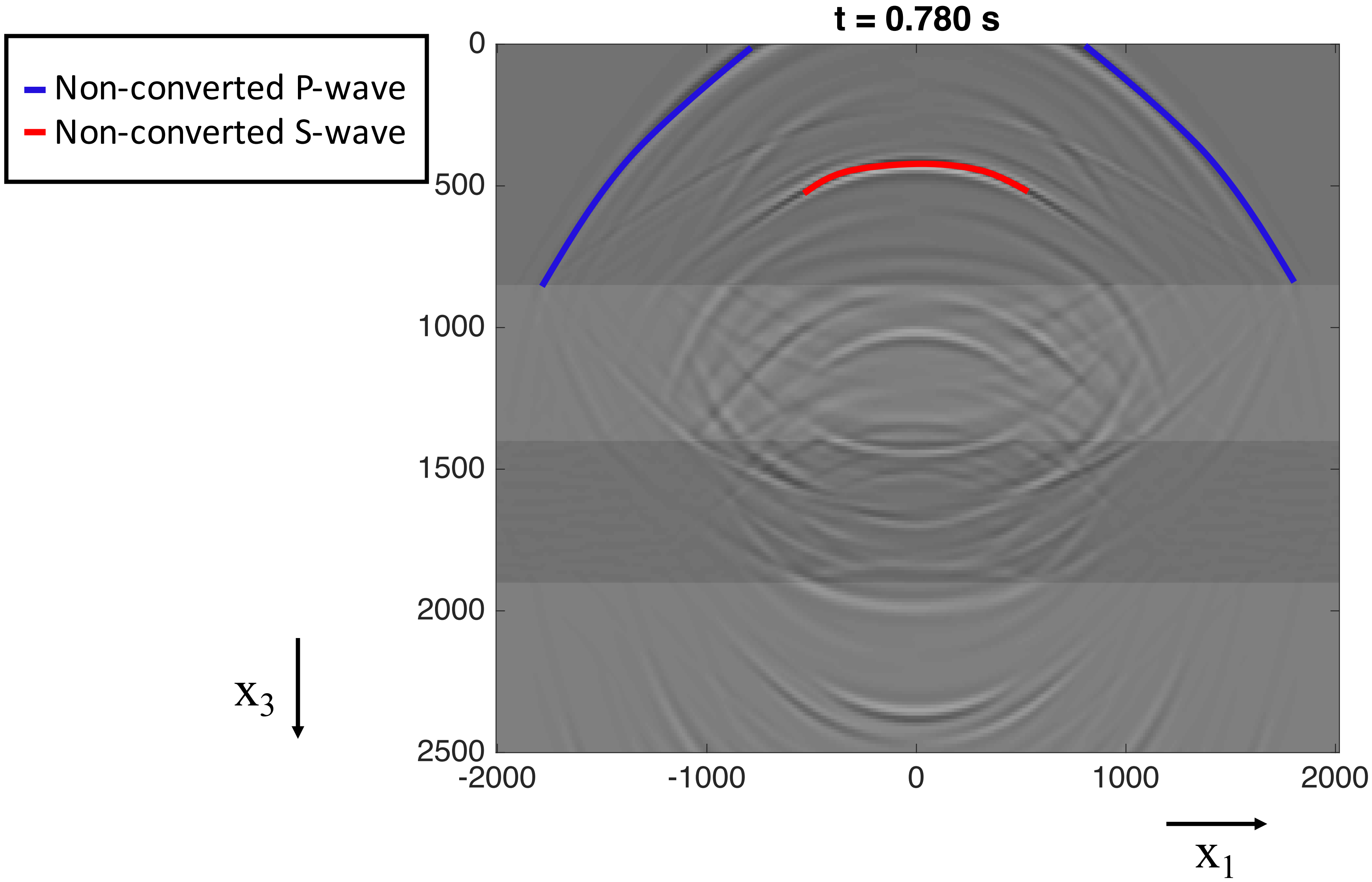
Numerical example: Single-sided homogeneous Green's function representation



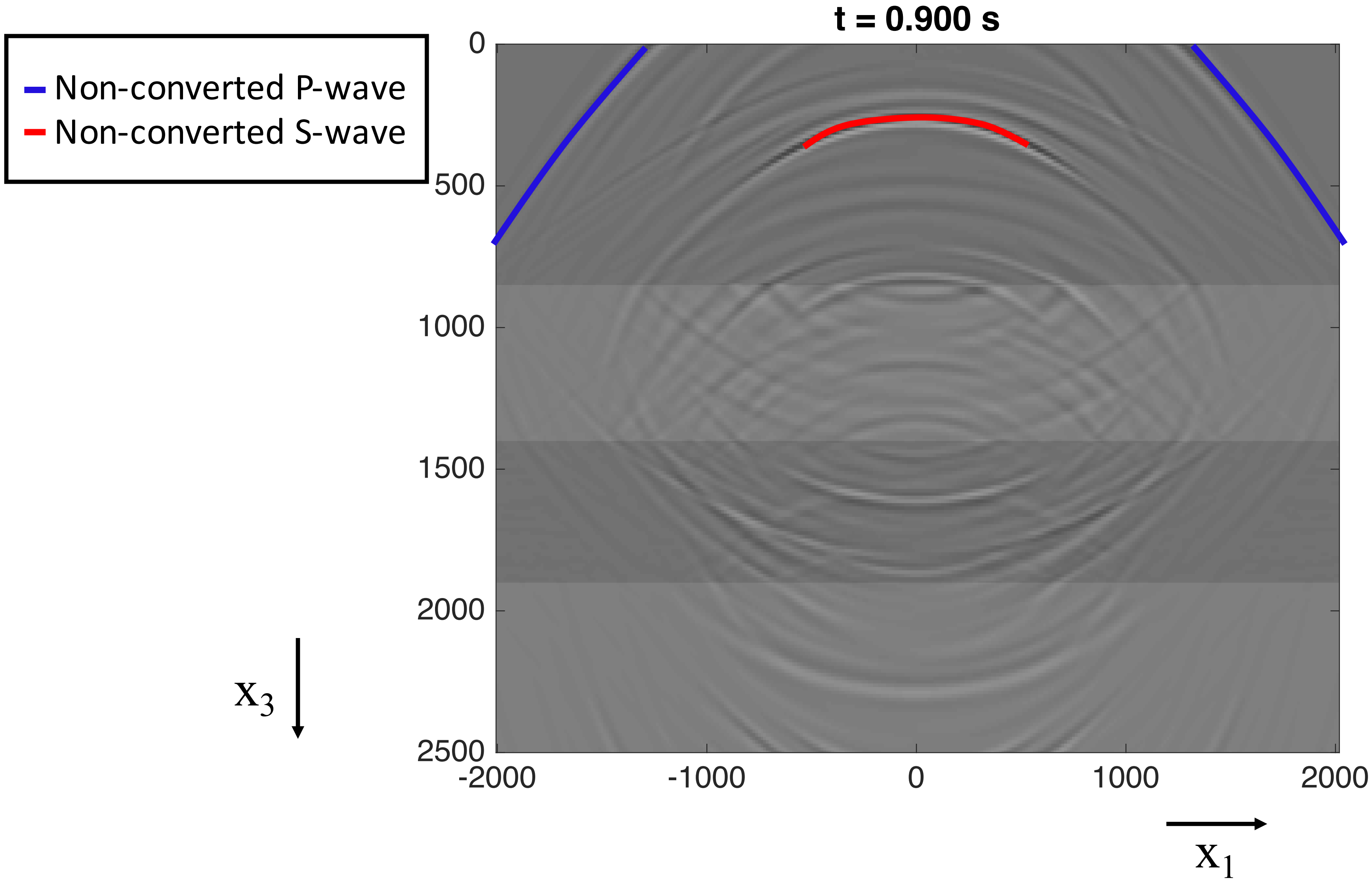
Numerical example: Single-sided homogeneous Green's function representation



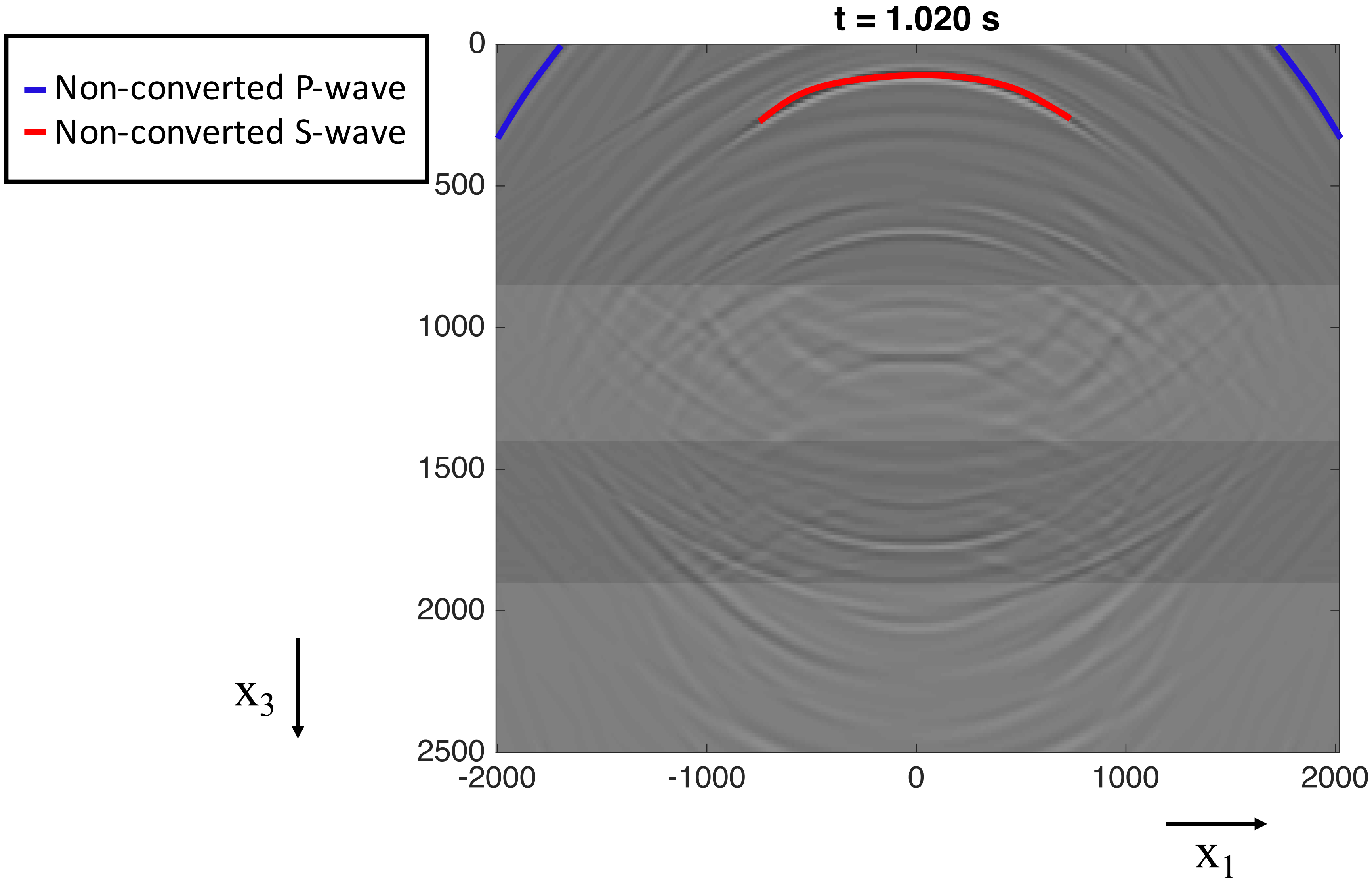
Numerical example: Single-sided homogeneous Green's function representation



Numerical example: Single-sided homogeneous Green's function representation



Numerical example: Single-sided homogeneous Green’s function representation



Contents

1. Introduction
2. Homogeneous Green's function
3. Elastodynamic double-sided homogeneous Green's function representation
4. Elastodynamic single-sided homogeneous Green's function representation
5. **Conclusions**

TU Delft

WAVES

Conclusions: The presented numerical example

Conclusions: The presented numerical example

- Numerical demonstration of single-sided homogeneous Green's function representation for elastic waves

Conclusions: The presented numerical example

- Numerical demonstration of single-sided homogeneous Green's function representation for elastic waves
- Single-sided access of the medium suffices

Conclusions: The presented numerical example

- Numerical demonstration of single-sided homogeneous Green's function representation for elastic waves
- Single-sided access of the medium suffices
- Converted as well as multiply scattered waves are represented correctly

TU Delft

WAVES

Conclusions: The presented numerical example

- Numerical demonstration of single-sided homogeneous Green's function representation for elastic waves
- Single-sided access of the medium suffices
- Converted as well as multiply scattered waves are represented correctly
- Layered medium was used in the presented example

Conclusions: The presented numerical example

- Numerical demonstration of single-sided homogeneous Green's function representation for elastic waves
- Single-sided access of the medium suffices
- Converted as well as multiply scattered waves are represented correctly
- Layered medium was used in the presented example
- Focusing functions were modelled in the presented example

Conclusions: Outlook

Conclusions: Outlook

- The theory is valid for laterally varying media

  ➡ Extension of our code

Conclusions: Outlook

- The theory is valid for laterally varying media

  ➡ Extension of our code

- The focusing functions can be retrieved from reflection data using limited knowledge of the medium via the Marchenko method (Wapenaar, 2014 & da Costa Filho et al., 2014)

  ➡ We are developing the Marchenko method to reduce the required prior knowledge of the medium

TU Delft

WAVES

Acknowledgements

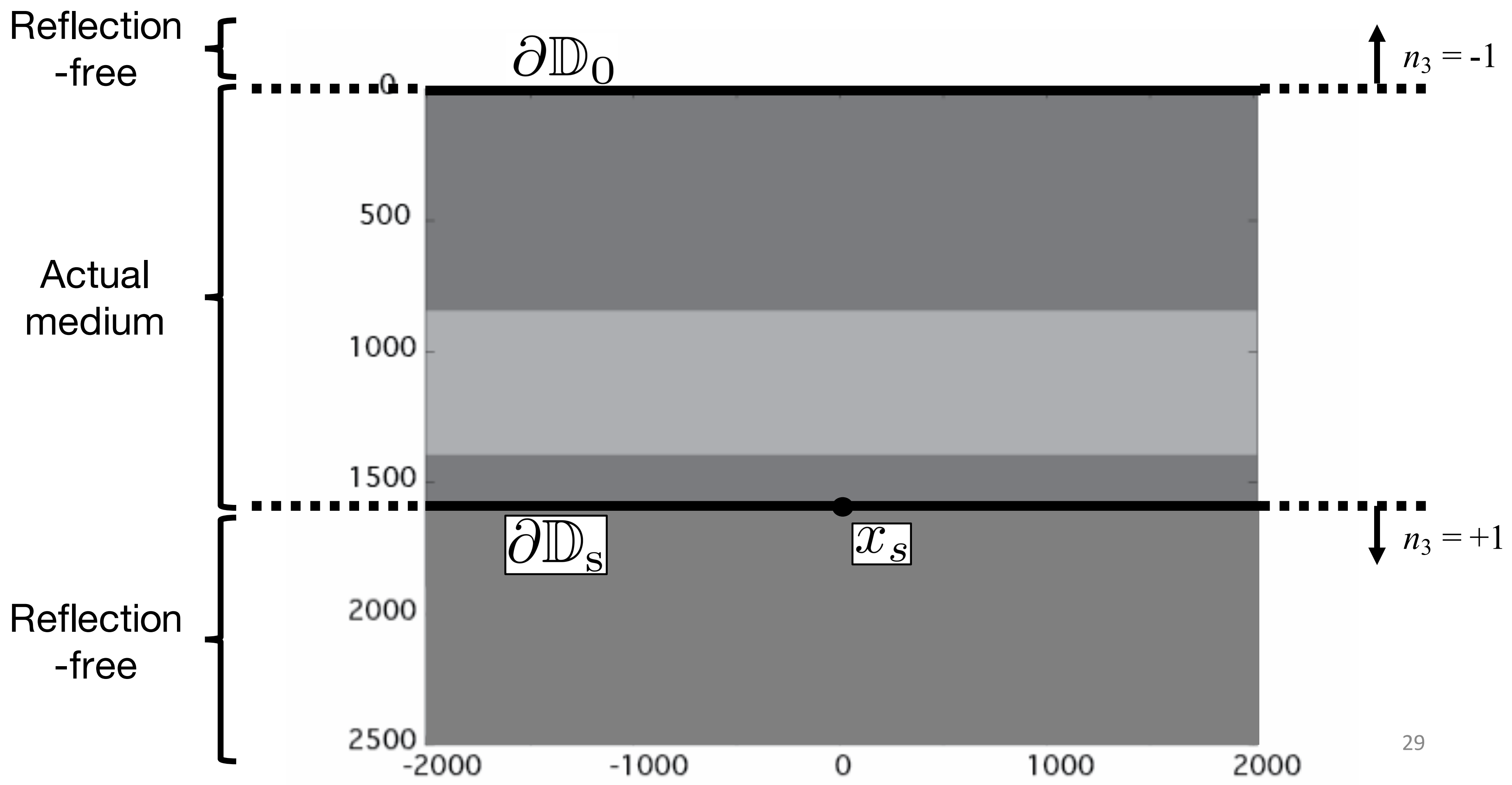
- Our colleagues in Delft
- Funding:
  - European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement

TU Delft

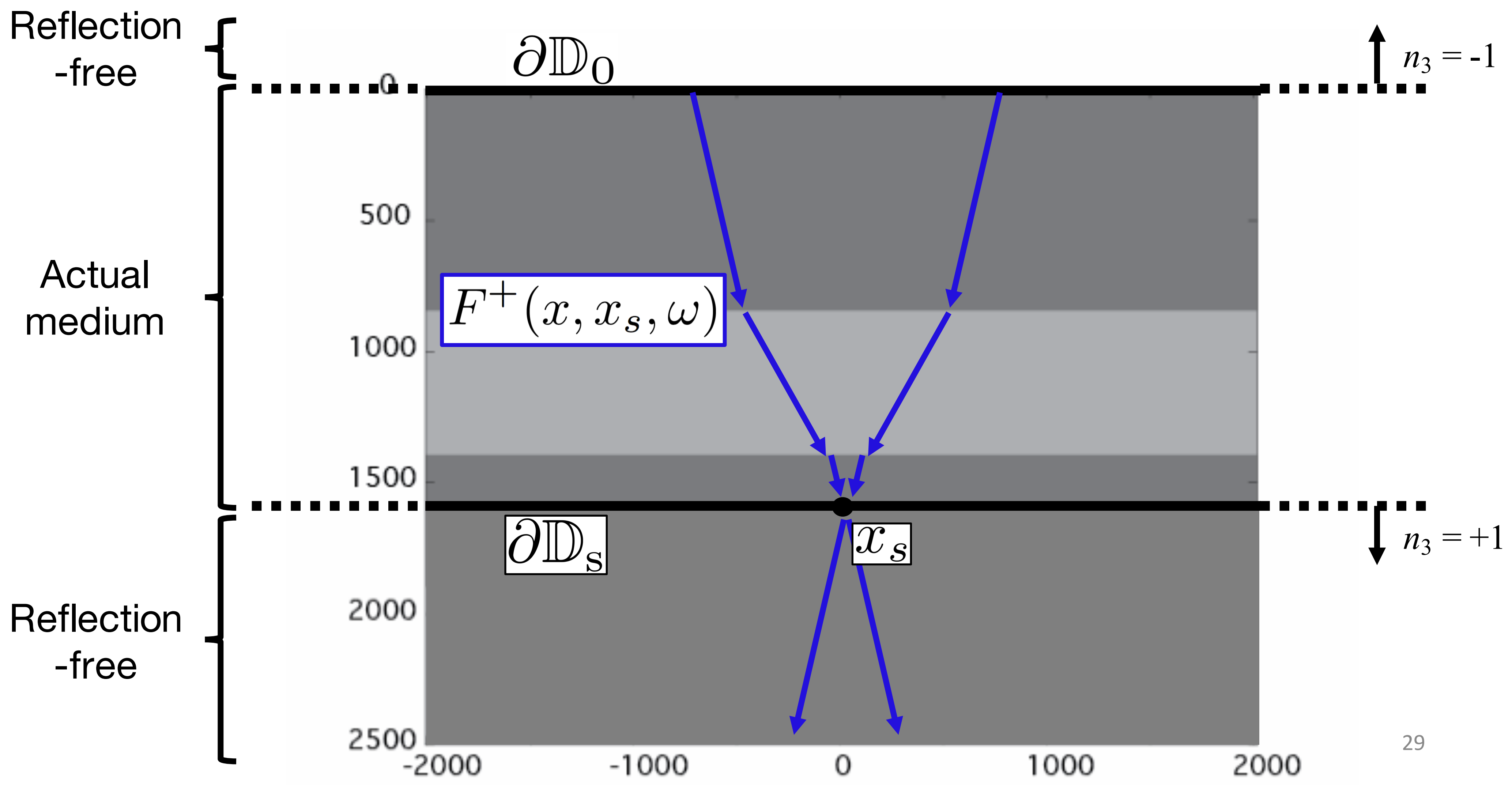
WAVES

Thank you!

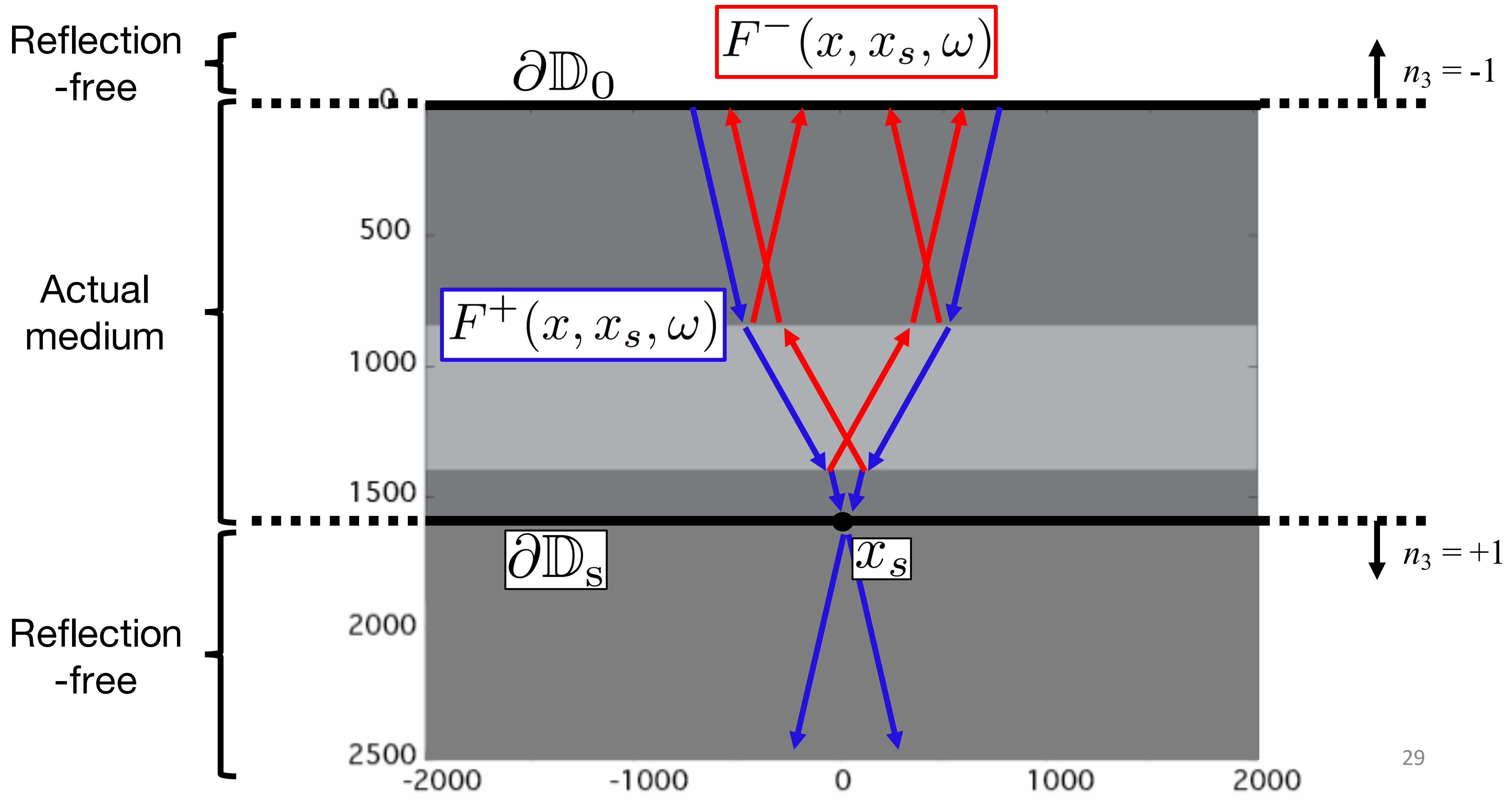
Focusing function: $\mathbf{F}(x, x_s, \omega) = \begin{pmatrix} F^+ \\ F^- \end{pmatrix}(x, x_s, \omega)$



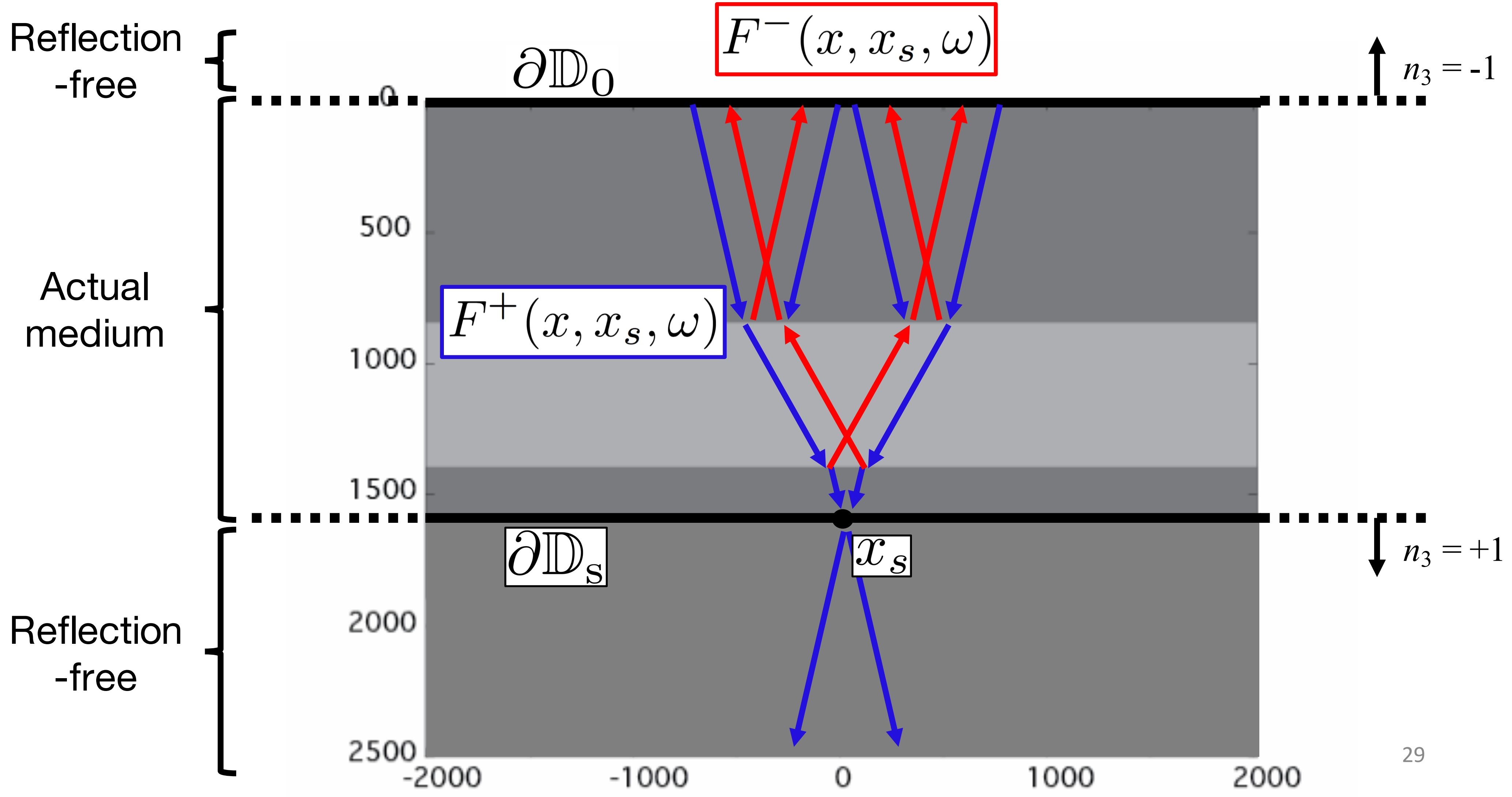
Focusing function $\mathbf{F}(x, x_s, \omega) = \begin{pmatrix} F^+ \\ F^- \end{pmatrix} (x, x_s, \omega)$



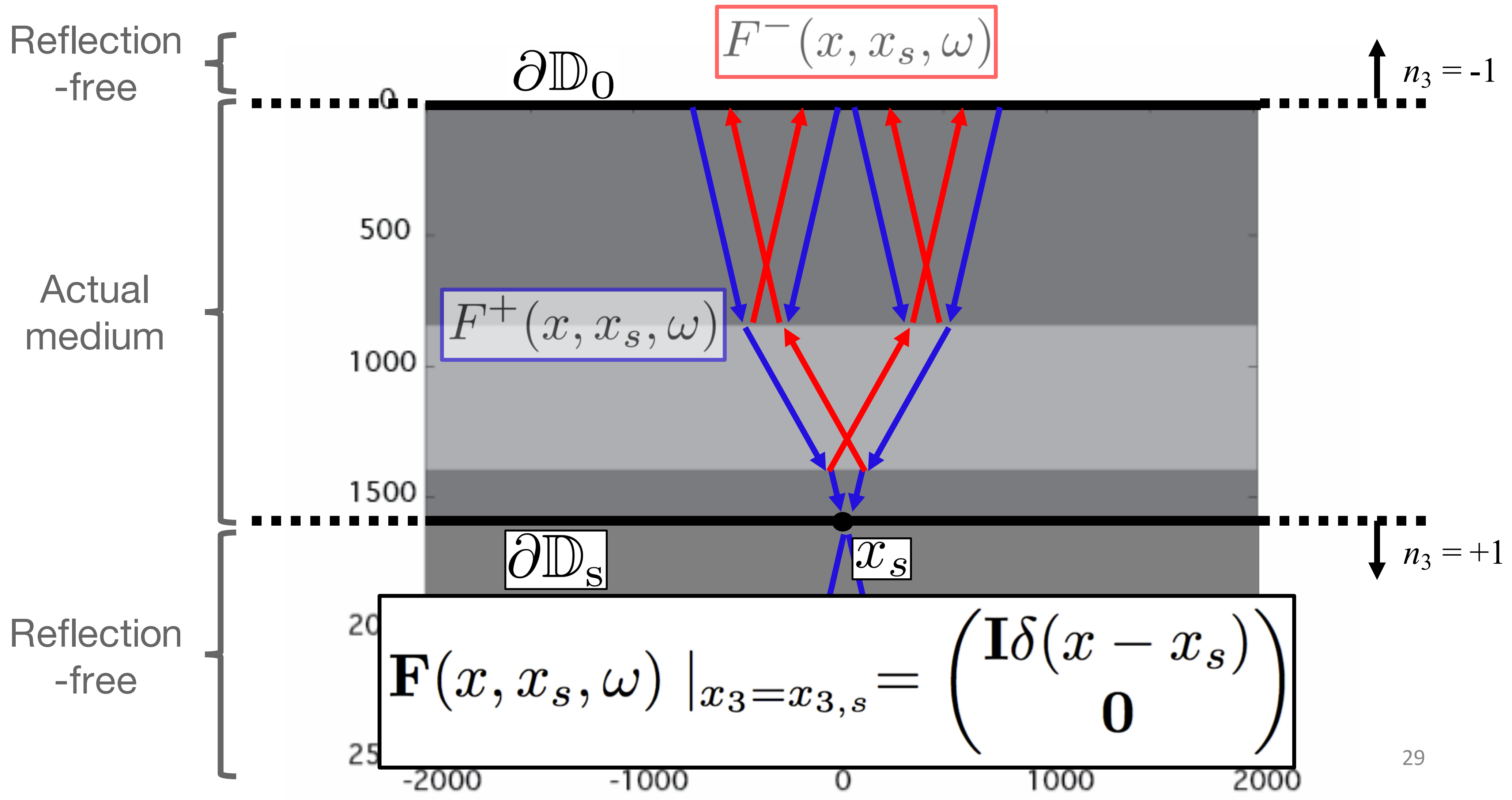
Focusing function $\mathbf{F}(x, x_s, \omega) = \begin{pmatrix} F^+ \\ F^- \end{pmatrix} (x, x_s, \omega)$



Focusing function $\mathbf{F}(x, x_s, \omega) = \begin{pmatrix} F^+ \\ F^- \end{pmatrix}(x, x_s, \omega)$



Focusing function $\mathbf{F}(x, x_s, \omega) = \begin{pmatrix} F^+ \\ F^- \end{pmatrix}(x, x_s, \omega)$



$$\mathbf{F}(x, x_s, \omega)\,|_{x_3 = x_{3,s}} = \begin{pmatrix} \mathbf{I}\delta(x - x_s) \\ \mathbf{0} \end{pmatrix}$$

Elastodynamic single-sided homogeneous Green's function representation

- Step 1: Virtual source creation

$$\mathbf{G}_1(x, x_s, \omega) = \int_{\partial\mathbb{D}_0'} \mathbf{R}_h(x, x', \omega)\mathbf{F}(x', x_s, \omega)\mathbf{I}_1^t \mathrm{d}^2 x'$$

$$\mathbf{G}_h(x, x_s, \omega) = \mathbf{G}_1(x, x_s, \omega) - \mathbf{K}\mathbf{G}^*{}_1(x, x_s, \omega)\mathbf{K}$$

(Wapenaar, Kees, Joost van der Neut, and Evert Slob. "Unified double- and single-sided homogeneous Green's function representations." *Proc. R. Soc. A*. Vol. 472. No. 2190. The Royal Society, 2016.)

TU Delft

WAVES

Elastodynamic single-sided homogeneous Green's function representation

- Step 1: Virtual source creation

$$\mathbf{G}_1(x, x_s, \omega) = \int_{\partial\mathbb{D}_0'} \mathbf{R}_h(x, x', \omega)\mathbf{F}(x', x_s, \omega)\mathbf{I}_1^t \mathrm{d}^2 x'$$

$$\mathbf{G}_h(x, x_s, \omega) = \mathbf{G}_1(x, x_s, \omega) - \mathbf{K}\mathbf{G}^*{}_1(x, x_s, \omega)\mathbf{K}$$

- Step 2: Virtual receiver creation

$$\mathbf{G}_2(x_r, x_s, \omega) = \int_{\partial\mathbb{D}_0} \mathbf{I}_2\mathbf{F}^t(x, x_r, \omega)\mathbf{N}\mathbf{G}_h(x, x_s, \omega)\mathrm{d}^2 x$$

$$\mathbf{G}_h(x_r, x_s, \omega) = \mathbf{G}_2(x_r, x_s, \omega) - \mathbf{K}\mathbf{G}^*{}_2(x_r, x_s, \omega)\mathbf{K}$$

(Wapenaar, Kees, Joost van der Neut, and Evert Slob. "Unified double- and single-sided homogeneous Green's function representations." *Proc. R. Soc. A*. Vol. 472. No. 2190. The Royal Society, 2016.)

Wavefield Composition

- The $\mathcal{L}$-operators are integral operators in the space-frequency domain

$$\begin{pmatrix} G_{v_1,f_1} & G_{v_1,f_3} & G_{v_1,h_1} & G_{v_1,h_3} \\ G_{v_3,f_1} & G_{v_3,f_3} & G_{v_3,h_1} & G_{v_3,h_3} \\ G_{\tau_{31},f_1} & G_{\tau_{31},f_3} & G_{\tau_{31},h_1} & G_{\tau_{31},h_3} \\ G_{\tau_{33},f_1} & G_{\tau_{33},f_3} & G_{\tau_{33},h_1} & G_{\tau 33,h_3} \end{pmatrix}(x, x_s, \omega) = \mathcal{L} \begin{pmatrix} G^{+,+}_{p,p} & G^{+,+}_{p,s} & G^{+,-}_{p,p} & G^{+,-}_{p,s} \\ G^{+,+}_{s,p} & G^{+,+}_{s,s} & G^{+,-}_{s,p} & G^{+,-}_{s,s} \\ G^{-,+}_{p,p} & G^{-,+}_{p,s} & G^{-,-}_{p,p} & G^{-,-}_{p,s} \\ G^{-,+}_{s,p} & G^{-,+}_{s,s} & G^{-,-}_{s,p} & G^{-,-}_{s,s} \end{pmatrix}(x, x_s, \omega)\mathcal{L}^{-1}$$