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Marchenko Redatuming of the Hdvs Signal

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Summary

Heterodyne distributed vibration sensing (hDVS) is one of the techniques for distributed acoustic sensing (DAS) to record seismic data using an optical fibre. The conventional hDVS data processing method suffers from nonlinear response of the hDVS system to strain measurement. To improve the hDVS data processing, we investigate the numerical application of Marchenko redatuming, which retrieves the Green's response to a virtual source inside the unknown medium from single-sided reflection measurement. We build a forward model for the recorded hDVS signal. It is heterodyned down to the intermediate frequency from the optical frequency, where the laser pulse propagates physically inside the fibre and multiple scattering has strong impacts. We derive the Marchenko equations for the hDVS signal and find that the same causality argument holds. Without knowledge of the fibre medium, we iteratively retrieve the Green's response to a desired virtual source inside the fibre from the hDVS signal recorded at one end of the fibre. The redatumed Green's functions have the potential to clean up hDVS raw data, handle multiple scattering, and improve strain estimation.

Introduction

Heterodyne distributed vibration sensing (hDVS) is a technique for distributed acoustic sensing (DAS) to record seismic data using an optical fibre (Hartog et al., 2013). The conventional hDVS processing method suffers from nonlinear response of the hDVS system to strain measurement, which may result, in part, from the complex waveform interference and multiple-scattering process inside the fibre. Recent research on Marchenko redatuming has shown that the Green's response to a virtual source inside the medium can be retrieved from single-sided reflection measurement. Becker et al. (2016) first perform Marchenko focusing physically in a simple 1D sound wave tube. In this paper, we investigate Marchenko redatuming numerically on a 1D fibre model with complex inhomogeneity distributions. The redatumed Green's response can potentially improve hDVS data processing.

Modelling the hDVS system

We derive a forward model of the hDVS signal recorded from a static single-mode fibre. For simplicity, all the phase drift and noise associated with the real hDVS experiment are neglected. In a typical hDVS acquisition system (Figure 1), a laser source emits monochromatic light at the optical frequency ω_0 (e.g., 193 THz). The Fourier spectrum of this carrier signal is

$$C(\omega) = \frac{1}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \tag{1}$$

where δ is the Dirac delta function. An acousto-optic modulator then shapes the carrier signal to a short pulse. This process is modelled as windowing the carrier signal with a Gaussian-shaped envelope, whose Fourier spectrum is $E(\omega)$ and its maximum frequency $\omega_E \ll \omega_0$:

$$C_E(\omega) = C(\omega) * E(\omega) = \frac{1}{2}[E(\omega - \omega_0) + E(\omega + \omega_0)] \tag{2}$$

where $*$ denotes convolution. The acousto-optic modulator also increases the frequency of the modulated pulse C_E by the intermediate frequency $\Delta\omega$ (e.g., 110 MHz), so the output is

$$X(\omega) = C_E(\omega - sgn(\omega)\Delta\omega) = \frac{1}{2}[E(\omega - \omega_0 - \Delta\omega) + E(\omega + \omega_0 + \Delta\omega)]. \tag{3}$$

The interrogation pulse X is then sent into the optical fibre where it is partially reflected by natural impurities. Taking the fibre as a 1D acoustic medium, we model the impurities as a normal distribution of small-scale scatters with random density contrasts relative to the fibre core material (Figure 2a). With a source (solid star) and a receiver (solid triangle) at one end of the fibre, its impulse reflection response R is modelled by the propagator matrix method (Margrave, 2015), in which single-scattering (primaries only) and multiple-scattering (primaries and internal multiples) effects can be modelled separately. From Figure 2b, we can see that the single-scattering model breaks down at about 10^9 Hz and higher, where the amplitude of the reflection coefficients becomes larger than 1. The optical frequency regime has strong multiple-scattering effects compared to the intermediate frequency regime. We may set the density contrasts too large, making the scattering effects much stronger than the real case, but the same physical principle holds. The backscattered signal is

$$Y(\omega) = X(\omega)R(\omega) = \frac{1}{2}[E(\omega - \omega_0 - \Delta\omega) + E(\omega + \omega_0 + \Delta\omega)]R(\omega). \tag{4}$$

It is then mixed with the carrier signal and a balanced receiver measures its intensity

$$I(\omega) = [Y(\omega) + C(\omega)] * [Y(\omega) + C(\omega)]. \tag{5}$$

The balanced receiver strips out the DC component and a bandpass filter (B) removes all the components at the optical frequencies for sampling

$$H(\omega) = B[I(\omega)] = \frac{1}{2}[E(\omega - \Delta\omega)R(\omega + \omega_0) + E(\omega + \Delta\omega)R(\omega - \omega_0)]. \tag{6}$$

Since the recorded signals are real in the time domain, we only need to specify their Fourier coefficients for positive frequencies. And by defining $\frac{1}{2}E(\omega - \Delta\omega)$ as an equivalent wavelet $W(\omega)$ at the frequency $\Delta\omega$, we model the recorded hDVS signal heterodyned from the optical frequency as

$$\hat{R}(\omega) = W(\omega)R(\omega + \omega_0),\ \omega > 0. \tag{7}$$

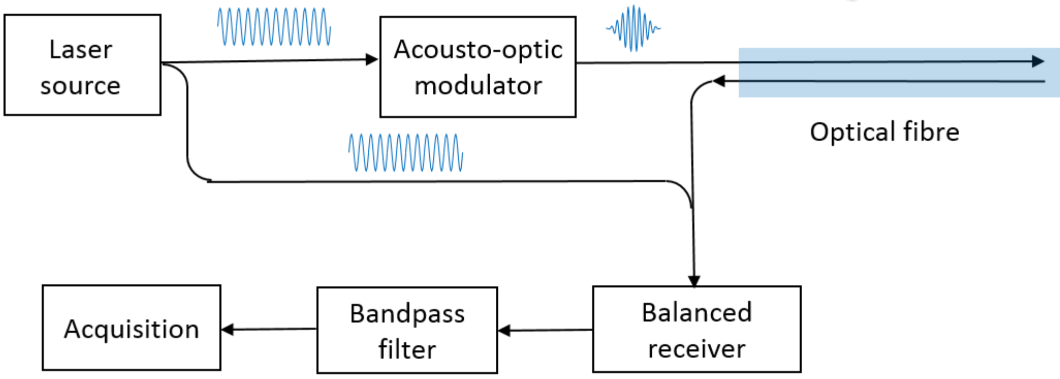

Figure 1 *Block diagram of the hDVS acquisition system.*

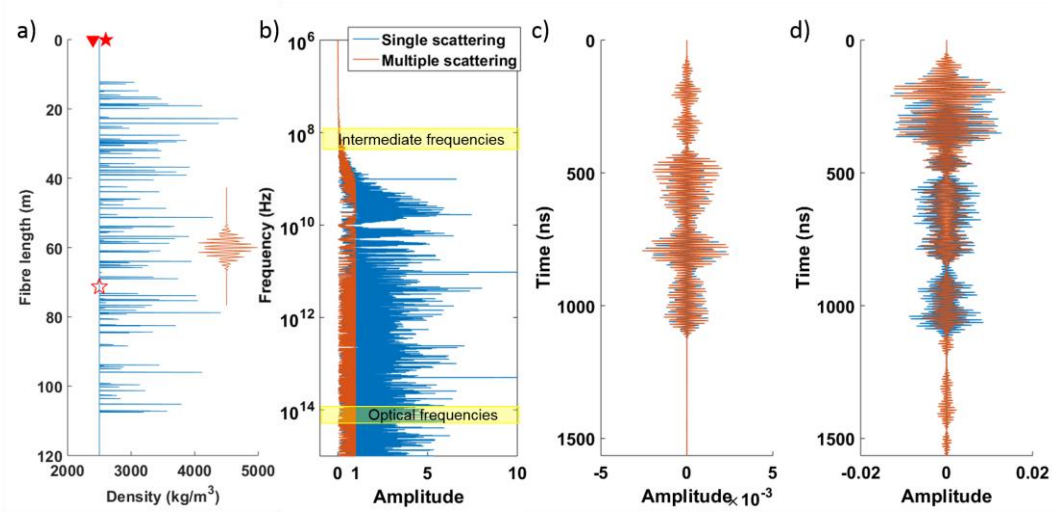

Figure 2 *(a) Fibre model with random scatter distribution. (b) Fibre impulse reflection response in the frequency domain. (c) Signals at Δω of single and multiple scattering without heterodyning. (d) hDVS signals of single and multiple scattering heterodyned from the optical frequency.*

For comparison, assuming that we send the equivalent wavelet into the fibre and record the backscattered signal directly, the signal at $\Delta\omega$ without heterodyning is modelled as

$$\tilde{R}(\omega) = W(\omega)R(\omega). \tag{8}$$

Figure 2a plots the red equivalent wavelet in depth. Figure 2c plots the time-domain signals at $\Delta\omega$ showing weak multiple-scattering effects. Figure 2d plots the time-domain hDVS signals heterodyned from the optical frequency showing much stronger multiple-scattering effects.

Marchenko redatuming of the hDVS signal

Wapenaar et al. (2013) introduce the so-called focusing functions $f_1^{\pm}$ in Marchenko redatuming based on a truncated medium, which is the same as the actual medium above a desired virtual source (hollow star in Figure 2a) and is reflection-free below it. The focusing functions are defined as

$$T(\omega)f_1^{+}(\omega) = 1, \tag{9}$$
$$R_A(\omega)f_1^{+}(\omega) = f_1^{-}(\omega) \tag{10}$$

where T and R_A are the broadband transmission and reflection response of the truncated medium. The focusing functions relate the impulse reflection response R measured at one end of the fibre to the broadband upgoing and downgoing Green's response $G^{\pm}$ to a desired virtual source inside the fibre:

$$G^{-}(\omega) = R(\omega)f_1^{+}(\omega) - f_1^{-}(\omega), \tag{11}$$
$$G^{+}(\omega) = -R(\omega)f_1^{-*}(\omega) + f_1^{+*}(\omega). \tag{12}$$

Here, R is the fibre response with the physical multiple-scattering effects. We can write the first Marchenko equation for the signal at the intermediate frequency as

$$\tilde{G}^{-}(\omega) = R(\omega)\tilde{f}_1^{+}(\omega) - \tilde{f}_1^{-}(\omega) \tag{13}$$

with $\tilde{G}^{-}(\omega) = G^{-}(\omega)W(\omega)$ and $\tilde{f}_1^{\pm}(\omega) = f_1^{\pm}(\omega)W(\omega)$ being the Green's and focusing functions at the intermediate frequency without heterodyning. This is the conventional Marchenko equation for bandlimited data. To redatum the hDVS signal, we rewrite the first Marchenko equation as

$$\hat{G}^{-}(\omega) = R(\omega + \omega_0)\hat{f}_1^{+}(\omega) - \hat{f}_1^{-}(\omega) \tag{14}$$

with $\hat{G}^{-}(\omega) = G^{-}(\omega + \omega_0)W(\omega)$ and $\hat{f}_1^{\pm}(\omega) = f_1^{\pm}(\omega + \omega_0)W(\omega)$ being the Green's and focusing functions heterodyned from the optical frequency. Similar equations can be derived for the second Marchenko equation. To clearly show the Marchenko focusing wavefield, we simplify the fibre model to contain two scatters only and the equivalent wavelet to have shorter wavelength (Figure 3a). Assuming the truncated fibre model was known, we compute the focusing functions and reconstruct the Green's wavefields for the signal at the intermediate frequency without heterodyning (Figure 3b) and for the hDVS signal with heterodyning (Figure 3c). In either case, the wavefield at negative time is the acausal Green's function retrieved by Marchenko redatuming whereas the wavefield at positive time is the causal Green's function directly modelled by igniting a real source. Their antisymmetry about time zero verifies that Marchenko redatuming retrieves Green's functions correctly for the signal at the intermediate frequency as well as the hDVS signal. Note that the wavepaths are the same kinematically in both cases but the latter seems to have rapidly varying phase.

Iterative Marchenko scheme of the hDVS signal

In real hDVS acquisition, we have no access to the truncated fibre information, but it is possible to solve the underdetermined Marchenko equations using the causality properties. We retrieve the Green's response to the virtual source inside the more realistic fibre model (Figure 2a) by iterative substitution (van der Neut et al., 2015) assuming the fibre model is unknown. As a benchmark, we directly calculate the heterodyned focusing functions $\hat{f}_1^{\pm}$ (Figures 4a and 4b, blue) and Green's function $\hat{G}$ (Figure 4c, blue) assuming the truncated fibre model is known. The traveltime of the transmitted direct arrival from the virtual source to the real source/receiver is denoted as t_d. As we can see, $\hat{f}_1^{+}$ is composed by a direct arrival $\hat{f}_{1d}^{+}$ centred at $-t_d$ and a following coda $\hat{f}_{1m}^{+}$ between $-t_d$ and t_d; $\hat{f}_1^{-}$ arrives roughly between $-t_d$ and t_d (a few events after t_d due to the relatively long wavelength in this example); all the events of $\hat{G}$ appear at and after t_d. Thus, the causality also holds for the heterodyned signals. A windowing operator $\theta\{\cdot\}$ applied to the coupled Marchenko equations for the heterodyned signals removes all the events arriving at and after t_d and all the acausal events.

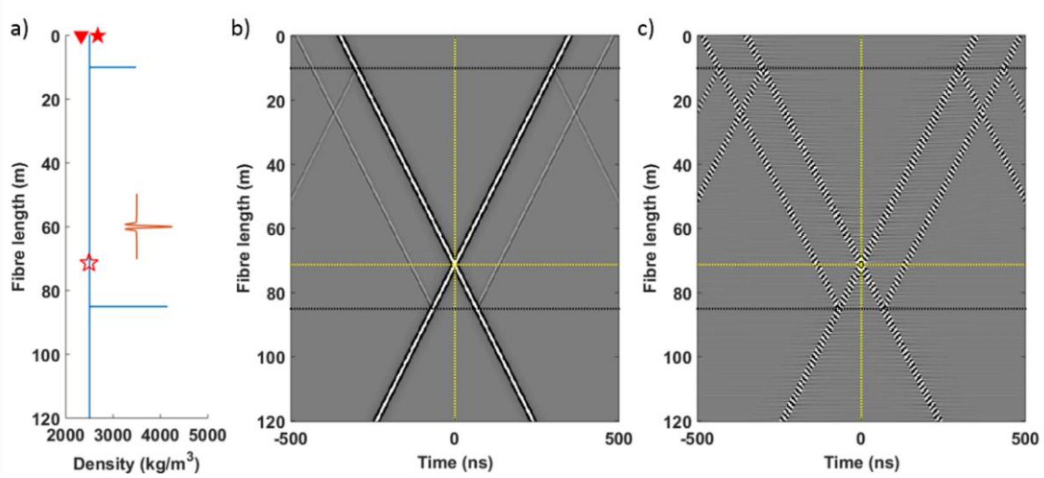

***Figure 3** (a) Simplified fibre model. (b) Green's wavefield to a source inside the fibre for the signal at $\Delta\omega$ by Marchenko redatuming (acausal) and by direct modelling (causal). (c) Green's wavefield to a source inside the fibre for hDVS signal by Marchenko redatuming (acausal) and by direct modelling (causal).*

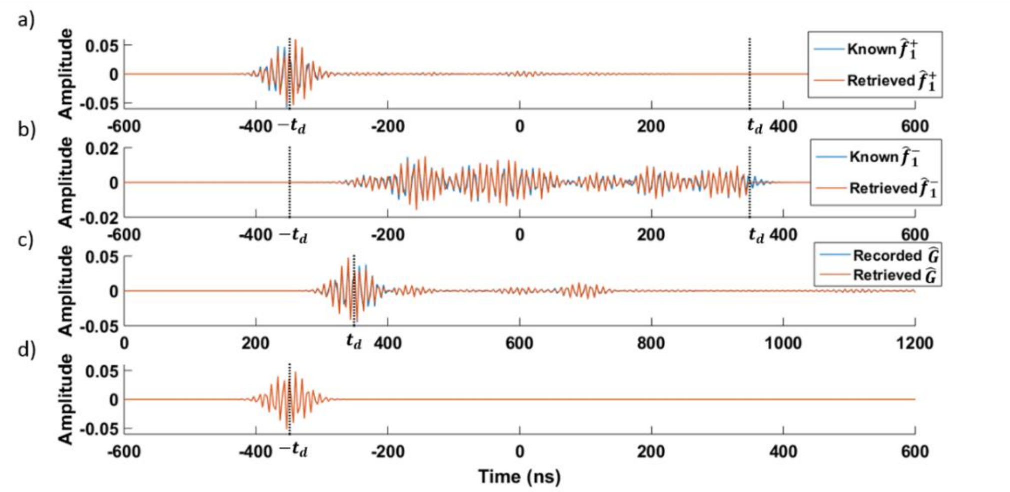

Figure 4 *(a) Retrieved and known $\hat{f}_1^+$. (b) Retrieved and known $\hat{f}_1^-$. (c) Retrieved and directly modelled $\hat{G}$. (d) Initial focusing function $\hat{f}_{1d}^+$. The vertical dotted lines denote the time t_d and $-t_d$.*

$$\theta\{R(\omega+\omega_0)[\hat{f}_{1d}^+(\omega)+\hat{f}_{1m}^+(\omega)]\} = \hat{f}_1^-(\omega), \tag{15}$$
$$\theta\{R(\omega+\omega_0)\hat{f}_1^{-*}(\omega)\} = \hat{f}_{1m}^{+*}(\omega). \tag{16}$$

Taking the equivalent wavelet being shifted to $-t_d$ as a crude estimate of $\hat{f}_{1d}^+$ (Figure 4d), we solve (15) and (16) iteratively. During each iteration, we propagate an updated focusing function into the fibre at the optical frequency, heterodyne the output signal down to the intermediate frequency, and apply the windowing operator. This numerical example converges after eight iterations. The retrieved $\hat{f}_1^{\pm}$ (Figures 4a and 4b, red) and $\hat{G}$ (Figure 4c, red) are compared to the known ones. Their difference may result from the scaling error of the initial focusing function estimation. Also, the windowing operator cannot precisely separate the focusing functions and Green's functions at $-t_d$ or t_d during each iteration, due to the embedded complex waveform.

Conclusions

We model the hDVS signal as a time-domain convolution of an equivalent wavelet at the intermediate frequency and the fibre impulse response being shifted by the optical frequency, where waves propagate physically inside the fibre and multiple scattering has strong effects. We derive the Marchenko equations for the hDVS signal and find that the same causality argument holds. By iterative substitution, we retrieve the Green's response to a desired virtual source inside the fibre from the hDVS signal recorded at one end of the fibre. The redatumed Green's functions have the potential to clean up hDVS raw data, handle multiple scattering and improve strain estimation.

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