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Using a Marchenko-redatumed Reflection Response as an Exact Boundary Condition

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SUMMARY

Marchenko Redatuming enables the creation of both virtual sources and receivers at an arbitrary depth level in the subsurface using only reflection data recorded at the surface and an estimate of the background velocity model. In this work, we perform full-waveform, target oriented modeling for a sub-domain of a 1D synthetic model using a combination of Marchenko-redatumed reflection responses and exact boundary conditions. We apply Marchenko redatuming to the two ends of the target region yielding Green's functions for virtual sources and receivers at these positions, which illuminate the unknown embedding medium from below and above, respectively. Applying some simple processing steps to these Green's functions enables us to use them as so-called exact boundary conditions for the target region. This procedure is aimed at establishing a link between the numerically modelled target region and the embedding background medium in order to include all interactions with the background medium. It appears that we can correctly account for the interactions without knowing the background medium in detail.

Introduction

Marchenko redatuming is a recently developed method to image a target zone below a scattering overburden. It allows one to create virtual sources and receivers in the subsurface using only reflection data recorded at the surface and an estimate of the background velocity model. In contrast to seismic interferometry, Marchenko redatuming does not require any actual sources or receivers in the subsurface, yet it successfully accounts for all internal multiples. In this work, we use Marchenko redatuming to perform a target-oriented modelling below an arbitrary overburden. We model the target area (i.e. a truncated sub-region) which is linked with the full (background) model via the results of Marchenko redatuming. The method that enables us to establish this link is the so-called exact boundary condition (EBC) method, which was introduced by van Manen et al. (2007). It is based on the Kirchhoff integral and a combination of a non-reflecting boundary and a re-injection of interactions with the background medium on that boundary. On the boundary, outgoing waves are cancelled to avoid reflections from that (non-physical) boundary. The interactions of these outgoing waves with the background medium, i.e. outgoing waves that are reflected back, are re-injected at the boundary using pre-computed Green's functions. In the EBC method described by van Manen et al. (2007) the Green's functions that are needed for the boundary condition are obtained by performing forward finite-difference modelling of the full medium (including the background medium). In this work, however, we use the Green's functions $R^{\cap}$ and $R^{\cup}$ obtained by Marchenko redatuming as EBCs on a truncated part of a 1D model. For this example, synthetic reflection data are produced for a source and a receiver at the top of the model. Marchenko redatuming yields the two Green's functions $R^{\cap}$ at $z_{f,1}$ (for imaging from below) and $R^{\cup}$ at $z_{f,2}$ (for imaging from above), with $z_{f,1} < z_{f,2}$. With a few simple processing steps, these Green's functions can be modified to be used as EBCs for finite difference modelling of the region between $z_{f,1}$ and $z_{f,2}$. Without knowing the exact background medium above $z_{f,1}$ and below $z_{f,2}$ all interactions with this background medium are captured correctly.

Method and Theory

The 1D Marchenko equation and its solution allow one to use the reflection response $R_{surface}$ measured at z_0 on one side of a lossless 1D medium to derive the Green's function between a virtual source at arbitrary depth z_f and z_0 (Wapenaar et al. 2014). Moreover, a downgoing focusing function f_1^+ (and its upgoing reflection response f_1^-) at the surface can be derived in such a way that, when emitted into the medium at the surface, it focuses at depth z_f and continues as a purely downgoing wave. These focusing functions f_1^+ and f_1^- are related via the reflection response $R^{\cup}$, which is the Green's function for a source and a receiver located at z_f in a reference medium, which is equal to the actual medium below z_f and homogenous above (Figure 1b). Using Marchenko redatuming to derive $R^{\cup}$, which includes a deconvolution step, is referred to as *imaging from above* and it redatums the source and the receiver from the surface to z_f in a medium which is reflection-free above z_f. In a similar way, the focusing function f_2^- propagates upwards through the medium, focuses at the surface and continues as purely upgoing wave. f_2^+ and f_2^- are directly related via the reflection response $R^{\cap}$ which is the Green's function for a source and receiver located at z_f in a reference medium that is reflection-free below z_f (Figure 1c). This alternative geometry is denoted hereinafter as *imaging from below*. Note that both $R^{\cup}$ and $R^{\cap}$ are derived from the same reflection data that is recorded with a source and a receiver at the medium surface (Figure 1a).

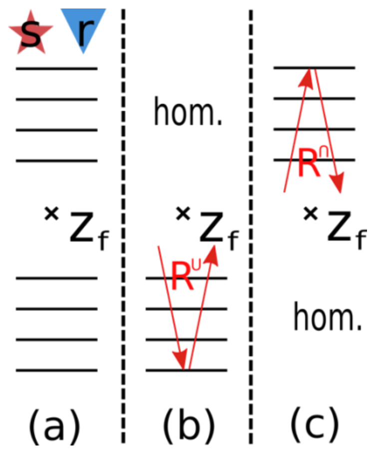

Figure 1 *(a) Geometry to record the reflection data $R_{surface}$ on one side of the medium, (b) imaging from above, (c) imaging from below. z_f indicates the focusing position, s and r denote the source and receiver, respectively.*

Wapenaar et al. (2014) show the relations between the reflection response recorded at the surface $R_{surface}$, the focusing functions f_1 and f_2 and the reflection responses $R^{\cup}$ and $R^{\cap}$ used for imaging from above and below, respectively. The first input required for this method is $R_{surface}$, which we take here free of surface multiples. The second input required by the Marchenko method is the direct arrival of the inverse of the transmission response from the focusing position z_f to the surface, T_d^{inv}. The latter can be approximated by the time reversal of the direct arrival of the Green's function, for the computation of which a smooth velocity model suffices.

The iterative Marchenko scheme represents a data-driven approach to derive the focusing function f_1^+. To verify this focusing function, additionally a model-driven approach is performed. It is based on Wapenaar (1993) and consists of a recursive scheme that back-propagates a single (downgoing) pulse at the focusing position to the surface. The initial pulse at z_f is backward propagated in time and upward propagated in space just below the lowermost interface between the surface and z_f. Subsequently, boundary conditions for the pressure and the particle velocity are applied. Above the interface, this yields a downgoing (incident) wave and an upgoing (reflected) wave. The incident wave is backward propagated in time and the reflected wave is forward propagated in time through the current layer. This propagates both waves upward in space just below the next interface, where the same scheme is applied. In this way the number of events doubles at each interface. The focusing function f_1^+ consists of all downgoing events at the source position.

The EBC requires two surfaces for each side of the truncated domain. One is the emitting surface which coincides with the domain boundary and is chosen to be at z_f. The second one is the receiving surface (s_{rec}) from where the wavefield is extrapolated to the emitting surface using Green's functions. Following van Manen et al. (2007), this requires Green's functions for both monopole and dipole sources. We choose the emitting boundary to be a rigid boundary, so the Green's functions need to predict the particle velocity. The reflection response obtained by the iterative Marchenko scheme predicts the pressure at z_f corresponding to a monopole source. To use it as exact boundary condition the following four steps are required:

- redatum the source from the emitting surface to the recording surface in 1D (this equals a time shift governed by the velocity between the two surfaces);
- derive the particle velocity v_z from the pressure p using the relation $v_z = p/(\rho c)$, where ρ is the density and c the seismic velocity;
- use reciprocity: the particle velocity obtained from a monopole source equals the pressure obtained from a dipole source;
- add the direct arrival to the Green's function, since it is not included in the Marchenko reflection response.

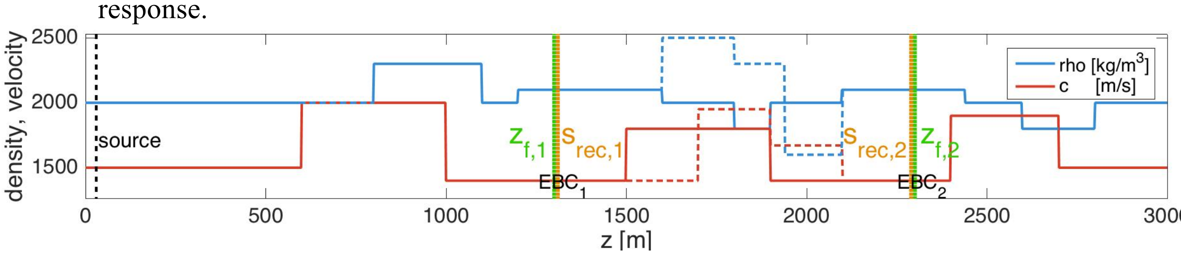

Figure 2 *Velocity and density model of the one-dimensional example. The vertical dashed lines indicate the location of the source for the synthetic reflection data, the focusing positions z_f (= emitting surfaces) and the recording surfaces for the EBCs s_{rec}. The perturbed model, indicated by the dashed velocity and density profiles, is used to obtain the data shown in Figure 6.*

Examples

The method described above is tested on a simple one-dimensional finite-difference model of 3000 m length. The velocity and density profiles are displayed in Figure 2. The source and the receiver to acquire the reflection data $R_{surface}$ on the 'surface' are placed at z_0 = 30 m depth. The focusing position $z_{f,1}$ for imaging from below is chosen to be at 1300 m and the focusing position $z_{f,2}$ for imaging from above is located at 2300 m. These depths coincide with the location of the emitting surfaces of the EBCs when modelling the truncated domain between these depths. The recording surfaces $s_{rec,1}$ and

$s_{rec,2}$ are located 10 m below and above the corresponding emitting surfaces, respectively. In all of the following examples, very efficient absorbing boundaries are used on both ends of the 1D model, i.e. at 0 m and at 3000 m, to achieve reflection-free boundaries (these boundary conditions are also based on EBCs, but this shall not be discussed further in this work).

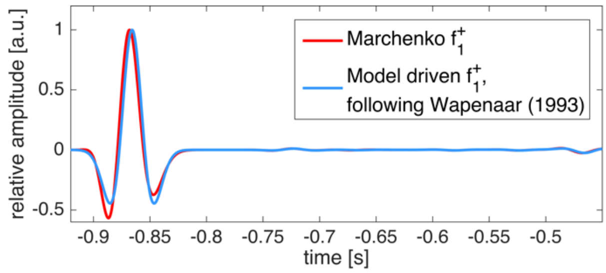

Figure 3 *Comparison between the focusing function f_1^+ obtained by using the Marchenko scheme and the model-driven function following Wapenaar (1993).*

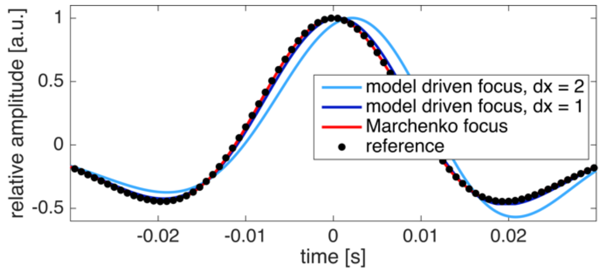

Figure 4 *Re-emitting the focusing function produces a single pulse at the focusing depth at t = 0 s. Model-driven focus for different grid sizes compared with the Marchenko focus and a reference Ricker wavelet.*

In Figure 3, the data-driven focusing function f_1^+ obtained with the iterative Marchenko scheme is compared against a reference solution computed using the recursive method of Wapenaar (1993). Since there are significant differences, we test their performance by re-emitting them into the medium. As can be seen in Figure 4, the waveform obtained at the focusing position using the Marchenko derived function perfectly resembles a single Ricker pulse at t = 0 s, as expected. Interestingly, the reference focusing function yields a pulse with a small positive time shift and an asymmetric energy distribution. When the cell size for the forward calculation is halved the result is much closer to the reference Ricker pulse and the energy following the pulse reduced, indicating that these effects are due to numerical dispersion. The Marchenko focusing function also suffers from numerical dispersion, but since the time-reversed direct arrivals used to start the scheme were also computed using finite-differences, they act as a matched filter and improve the focusing significantly.

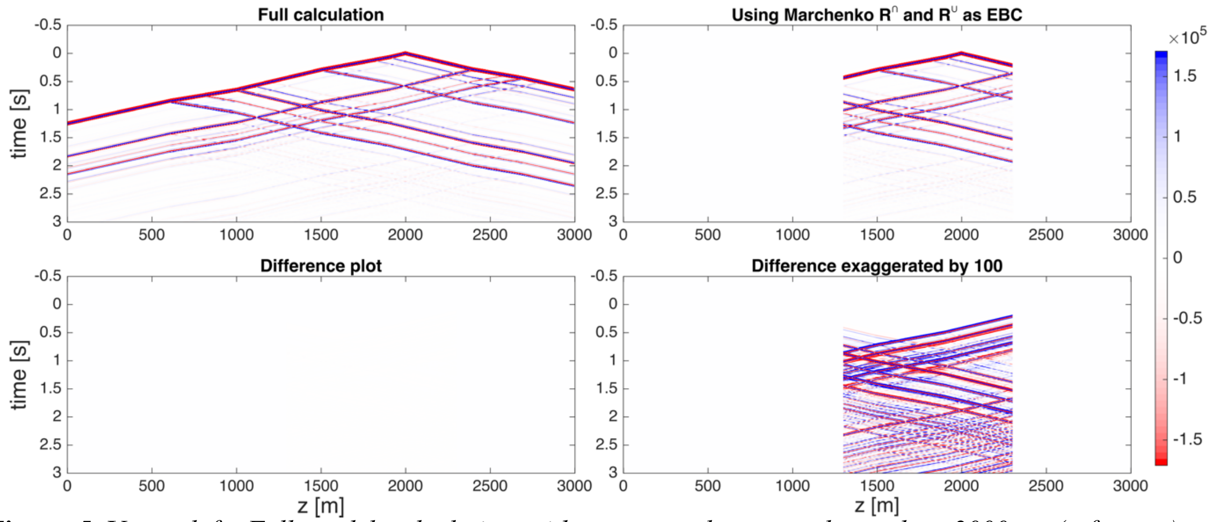

Figure 5 *Upper left: Full model calculation with a monopole source located at 2000 m (reference). Upper right: Finite difference modelling of the truncated model (between $z_{f,1}$ and $z_{f,2}$) using $R^\cap$ and $R^\cup$ at the EBCs. Lower left: Difference between the two plots in the top row in the truncated domain (from 1300 m to 2300 m). Lower right: difference exaggerated by 100.*

From the focusing functions obtained by the Marchenko scheme, the reflection responses $R^\cup$ and $R^\cap$ are derived. The processing steps described in the previous section are performed to obtain the Green's functions for EBCs at $z_{f,1}$ and $z_{f,2}$. Subsequently, the propagation in the medium between 1300 m and 2300 m is modelled with EBCs on either side. Through $R^\cap$ and $R^\cup$ obtained with Marchenko redatuming, the EBCs provide the link between the numerically simulated truncated model and the full model. The result for a monopole source at 2000 m is shown in the upper right corner of Figure 5. A reference solution computed in the full domain is shown in the upper left corner. In the simulation of the truncated model, the waves emitted by the monopole source do not exhibit any immediate reflections at $z_{f,1}$ and $z_{f,2}$. This indicates that reflections from these non-physical boundaries are suppressed successfully. Similarly, reflections from interactions with the background medium are re-emitted correctly into the truncated model. To confirm the correct timing of these events, the difference between both simulations in the region between the two focusing positions (from 1300 m to 2300 m) is shown in the lower left corner of Figure 5. In the domain of interest, the difference between both simulations is very close to zero. An exaggerated difference plot is shown in the lower right corner of Figure 5, which reveals that the calculations are not exact. We can further implement model alterations in the truncated domain and still use the same boundary conditions, as long as we make sure that the estimated travel time from the surface to the focusing position $z_{f,2}$ does not change (i.e., to ensure consistency). The altered model indicated by the dashed lines in Figure 2 fulfils this requirement and is used to compute the trace shown in Figure 6. The trace is identical to a reference trace calculated in the full (altered) model until approximately 0.6 s. At this time, which is when reflections from the background medium appear, differences start to show up. However, these differences are very small.

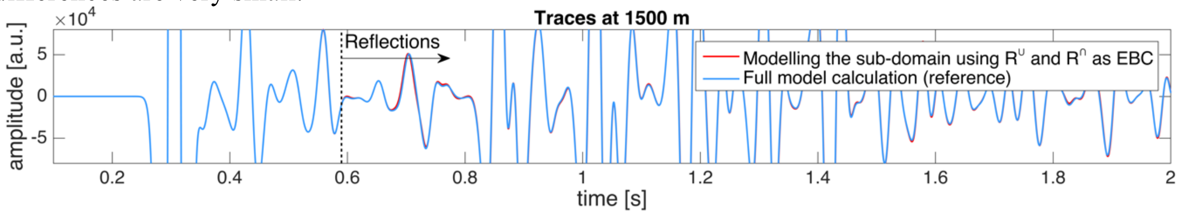

Figure 6 *Traces at 1500 m taken from the calculation that uses the Marchenko-derived reflection responses as EBCs, and a reference calculation in the full domain. Both calculations are performed in a medium which is perturbed in the region between* $z_{f,1}$ *and* $z_{f,2}$ *(dashed profiles in Figure 2).*

Discussion & Conclusions

In a 1D lossless medium which is only accessible from the surface, the iterative Marchenko scheme successfully provides a focusing function that focuses at an arbitrary depth z_f. This function appears little affected by the discretization. Further, Marchenko redatuming yields reflection responses that illuminate the medium from an arbitrary focusing position z_f up- or downwards. We can use these reflection responses to link a finite difference calculation of a truncated model with the surrounding full model using EBCs. The combination of these two methodologies allows us to perform target-oriented modelling with only an approximate knowledge of the model outside the target area. Moreover, arbitrary model alterations within the truncated domain are possible, as long as this does not affect the travel time from the surface to the lower focusing position. Because the simulations demonstrated in this work are performed in 1D, the synthetic reflection data is recorded with full aperture. This will be different for our future work, when we will implement this scheme in 2D. A major challenge will be the finite aperture which is inherent to 2D data and that will affect the Green's functions on the EBCs. Further, these Green's functions will be sensitive to errors in the smoothed velocity model used for the computation of the first arrival and this will be investigated.

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