Elastodynamic Marchenko Imaging

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Introduction

- The Marchenko method can retrieve virtual recordings inside a medium from reflection data $\mathbf{R}$ recorded at the surface (see *Figure 1*).

Earth is elastic ! → Elastic extension of Marchenko is needed !

- da Costa (2014) and Wapenaar (2014) have started to extend the theory of the Marchenko method to the elastic case.
- The numerical example in *Figure 2* demonstrates the potential of elastic Marchenko redatuming:
  - Using reflection data $\mathbf{R}$ recorded at the surface of the medium in *Figure 1* the Green's function of a virtual source inside the medium was retrieved (see *Figure 2a*).
  - *Figure 2b* shows the difference between the retrieved and a mod elled Green's function.
  - The Marchenko method has correctly retrieved all events including converted waves and internal multiples.

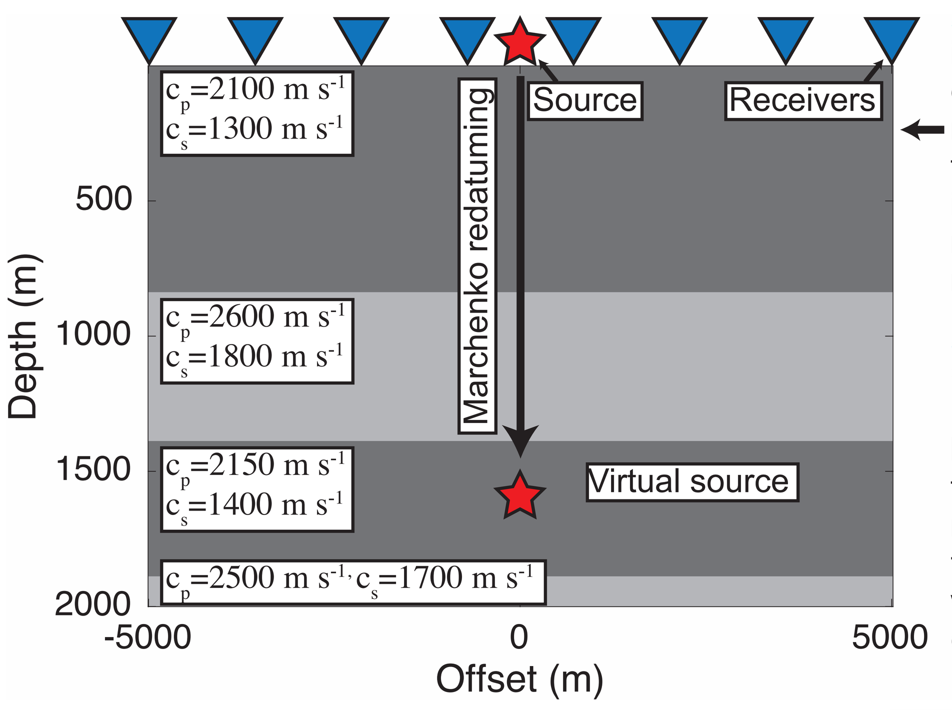
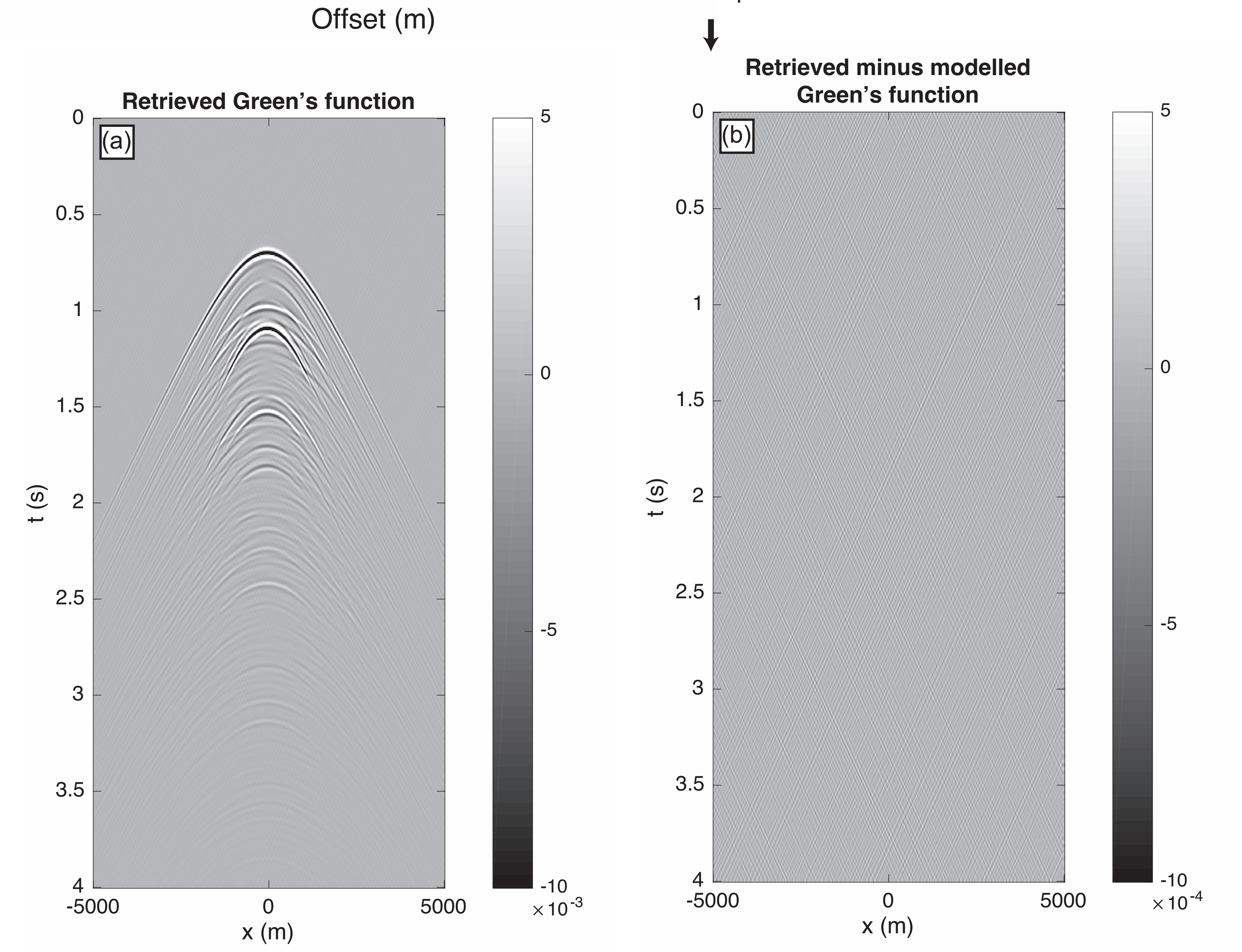


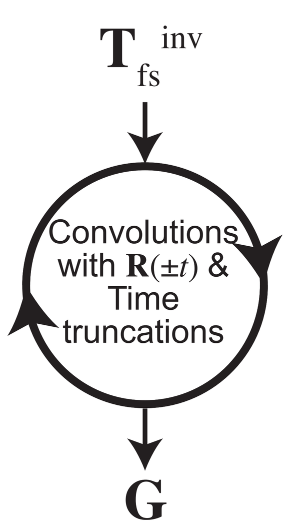
Figure 1: A layered elastic medium is used to illustrate the Marchenko method. Marchenko redatuming creates virtual recordings inside the medium using reflection data recorded at the surface. The P- and S-wave velocities c_p and c_s are indicated in the model. Dark and light grey colors represent densities of $1000\ \mathrm{kg\ m^{-3}}$ and $2000\ \mathrm{kg\ m^{-3}}$ respectively.

Figure 2: (a) Via Marchenko redatuming the Green's function of a virtual source located at $(x,z)=(0\ \mathrm{m}, 1600\ \mathrm{m})$ is retrieved. (b) The difference between the retrieved and a modelled Green's function demonstrates that the Marchenko method takes into account primary, multiply scattered and converted waves. The shown wavefields are a superposition of flux-normalised compressional and shear wavefields.



What does the elastic Marchenko scheme require?

- The elastic Marchenko method requires as an input:
  (1) Reflection data $\mathbf{R}$ ✓
  (2) An estimate of the so-called inverse forward-scattered transmission $\mathbf{T}_{\mathrm{fs}}^{\mathrm{inv}}$?



The initial wavefield $\mathbf{T}_{\mathrm{fs}}$

- The forward-scattered transmission $\mathbf{T}_{\mathrm{fs}}$ includes direct and forward-scattered waves from the recording surface to the virtual source location (see *Figure 3*).
  - The direct waves can be estimated using a smooth velocity model.
  - Currently we investigate how to estimate the forward-scattered events from the reflection data or a smooth velocity model.

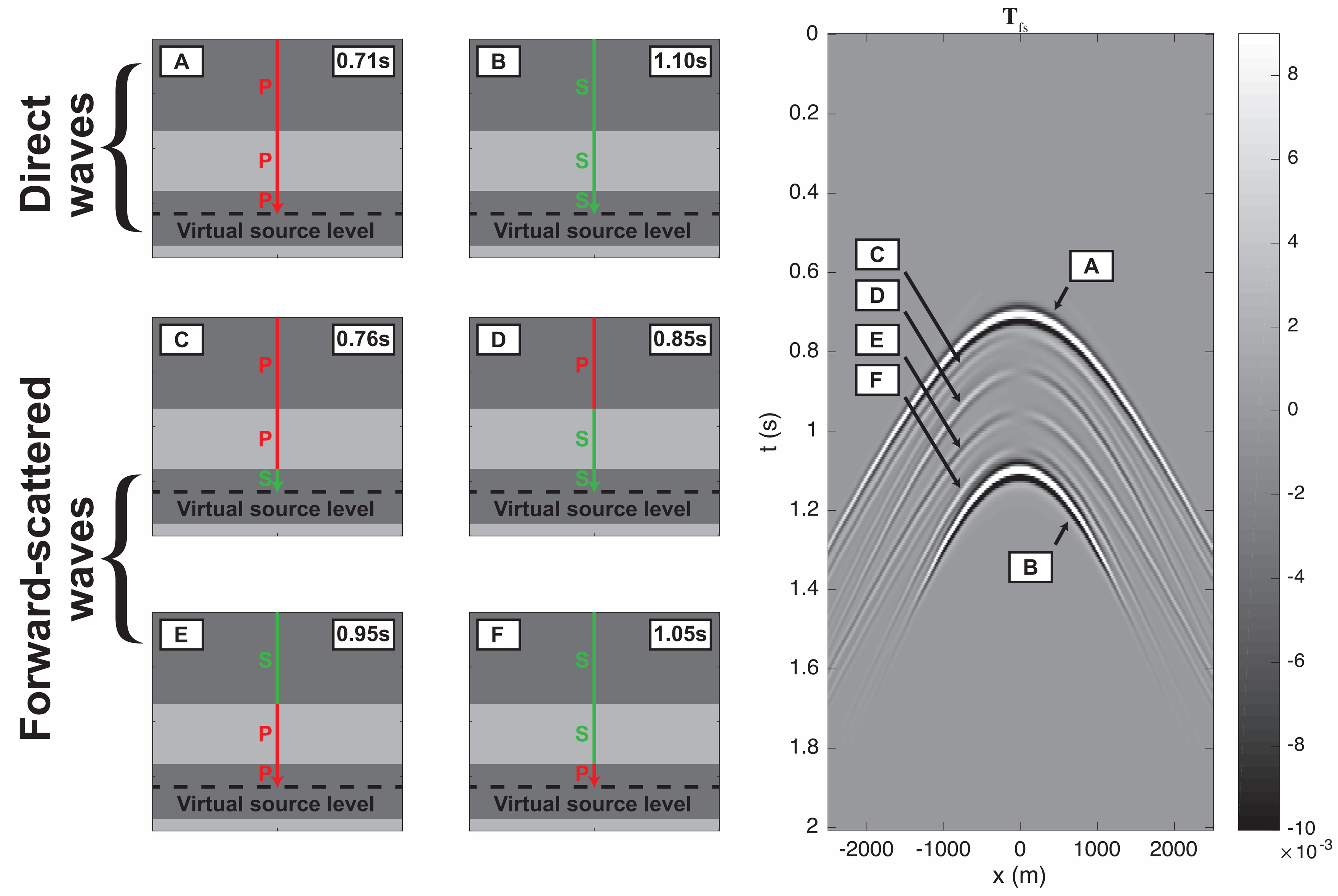


Figure 3: The two columns on the left side illustrate that the forward-scattered transmission $\mathbf{T}_{\mathrm{fs}}$ includes direct P- and S-waves (A and B) as well as forward-scattered waves (C-F). The zero-offset travel time from the surface to the virtual source location is indicated for each event. The right column shows the forward-scattered tranmission $\mathbf{T}_{\mathrm{fs}}$ of the numerical example in *Figures 1-2*. The shown wavefields are a superposition of flux-normalised compressional and shear wavefields.

How can we get $\mathbf{T}_{\mathrm{fs}}$?

- The correct inverse forward-scattered transmission $\mathbf{T}_{\mathrm{fs}}^{\mathrm{inv}}$ and the correct retrieved forward-scattered Green's function $\mathbf{G}_{\mathrm{fs}}^{++}$ are mutually related;

$$\mathbf{G}_{\mathrm{fs}}^{++}(k_H,\omega)\mathbf{T}_{\mathrm{fs}}^{\mathrm{inv}}(k_H,\omega)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \qquad (1)$$

- An incorrect estimate of the inverse forward-scattered transmission $\overline{\mathbf{T}}_{\mathrm{fs}}^{\mathrm{inv}}$ can be corrected by multiplication with a matrix $\mathbf{A}$;

$$\mathbf{T}_{\mathrm{fs}}^{\mathrm{inv}}(k_H,\omega)=\overline{\mathbf{T}}_{\mathrm{fs}}^{\mathrm{inv}}(k_H,\omega)\mathbf{A}(k_H,\omega) \qquad (2)$$

- If the incorrect forward-scattered transmission $\overline{\mathbf{T}}_{\mathrm{fs}}$ is inserted into the Marchenko scheme the identity of equation (1) transforms to;

$$\overline{\mathbf{G}}_{\mathrm{fs}}^{++}(k_H,\omega)\overline{\mathbf{T}}_{\mathrm{fs}}^{\mathrm{inv}}(k_H,\omega)=\left(\mathbf{A}(k_H,\omega)\mathbf{A}^{\dagger}(k_H,\omega)\right)^{-1} \qquad (3)$$

where $\overline{\mathbf{G}}_{\mathrm{fs}}^{++}$ is the retrieved but incorrect forward-scattered Green's function.

→ We aim to retrieve $\mathbf{A}$ from equation (3) to update the estimated inverse forward-scattered transmission $\overline{\mathbf{T}}_{\mathrm{fs}}^{\mathrm{inv}}$.

Conclusions

- The theory of the elastic Marchenko method was validated in a numerical example.
- For practical applications we need to find an estimate of the required input data, in particular $\mathbf{T}_{\mathrm{fs}}$, with limited knowledge of the medium.

References

- da Costa Filho, Carlos Alberto, et al. "Elastodynamic Green's function retrieval through single-sided Marchenko inverse scattering." Physical Review E 90.6 (2014): 063201.
- Wapenaar, Kees. "Single-sided Marchenko focusing of compressional and shear waves." Physical Review E 90.6 (2014): 063202.